Pricing Adverse Development Cover with option pricing methods

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IAA AAI ASTIN Non-Life Insurance

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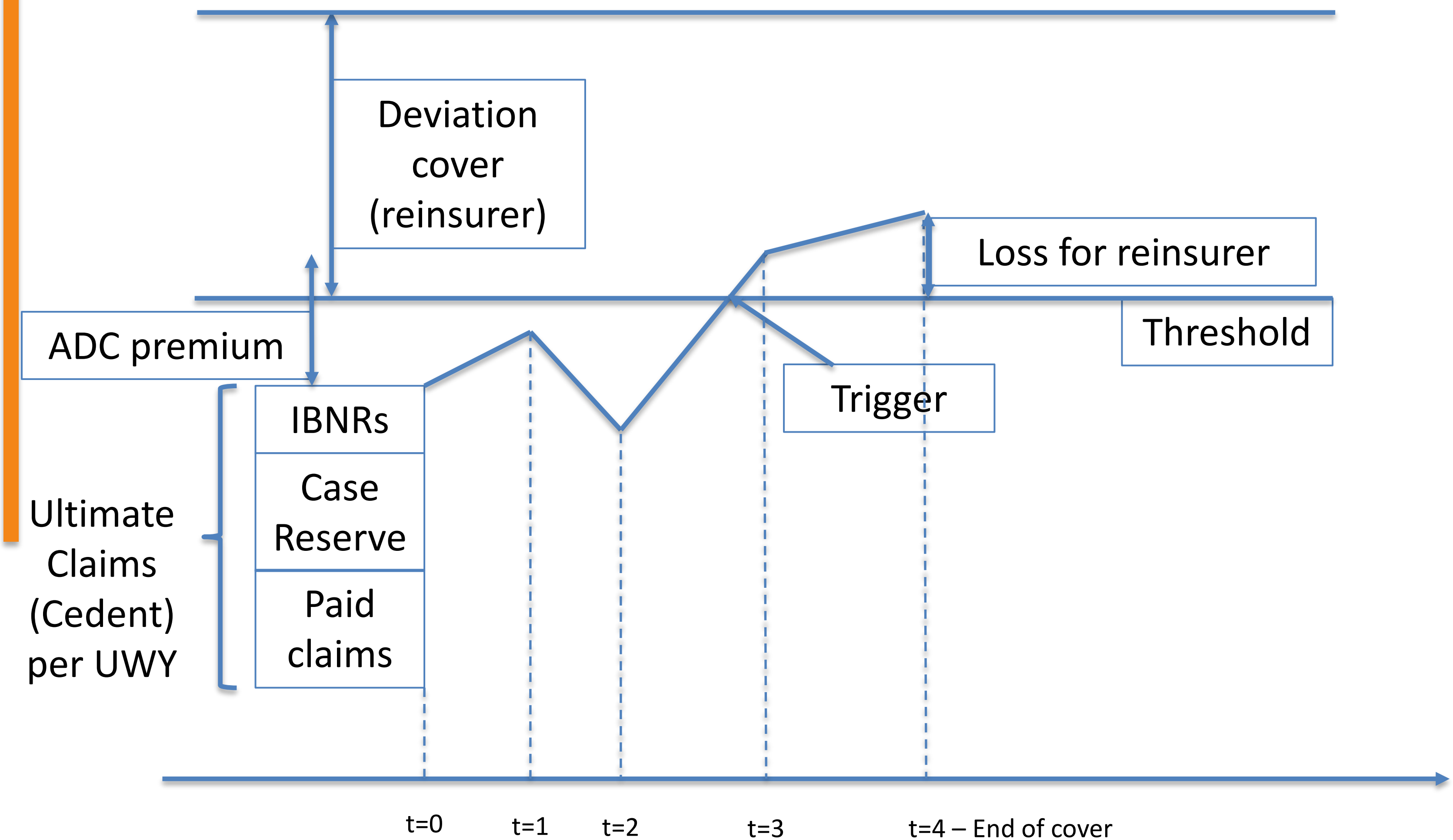
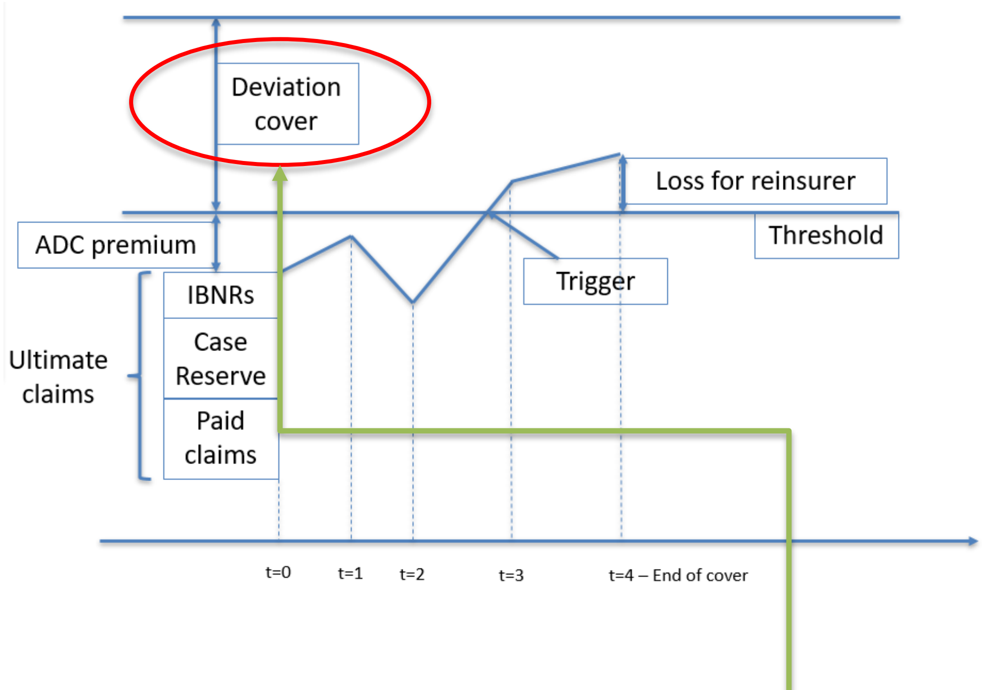
Agenda

- Introduction to Adverse Development Covers (“ADC”)
- P&C triangles and the Mack model
- Mack model vs Constant Elasticity of Variance model
- ADC as a call option
- ADC valuation in a P&C framework
- Numerical examples
- Conclusion

Introduction to Adverse Development Cover

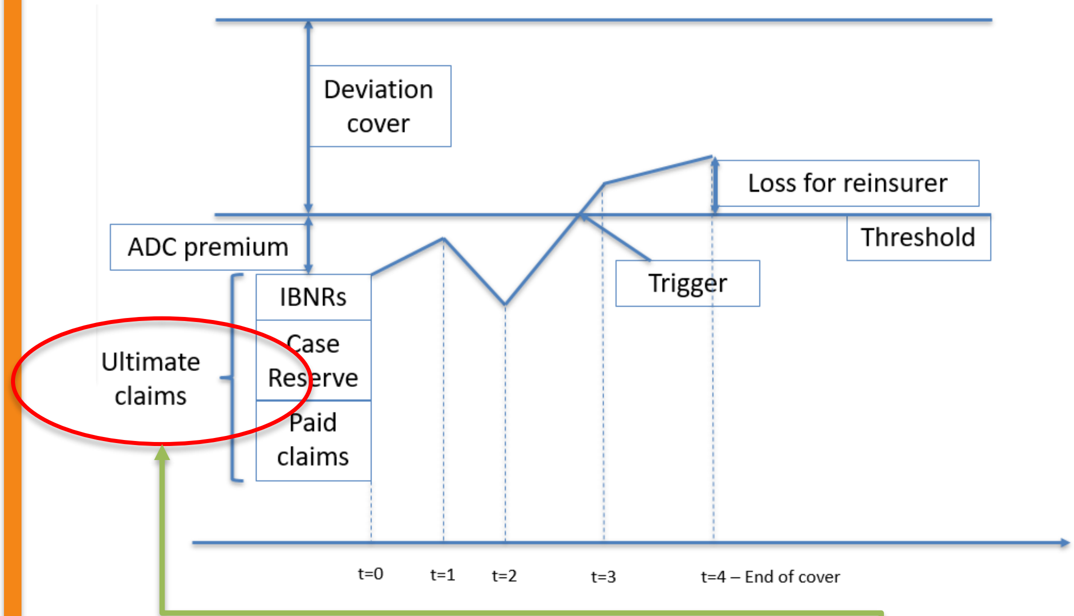


Introduction to Adverse Development Cover



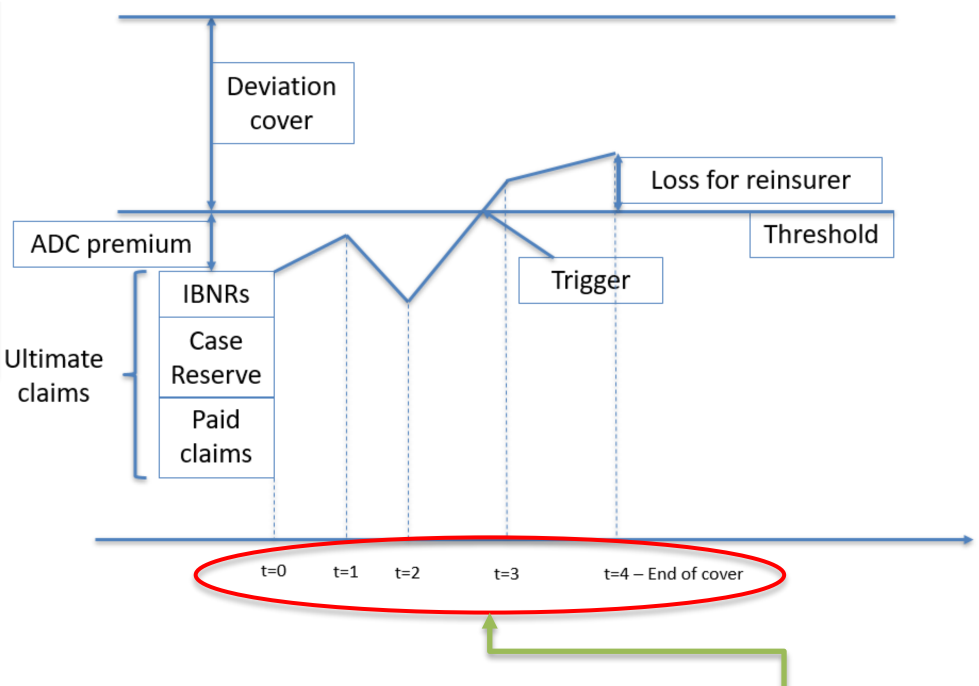
- Deviation cover usually limited
- For this presentation, to ease calculations, it will be considered unlimited

Introduction to Adverse Development Cover



- Ultimate claims refer to a few Underwriting Years
- Consider independence between the covered Underwriting Years
- Not all the reserve portfolio is covered by the ADC
- Reserves = Case reserve + IBNR

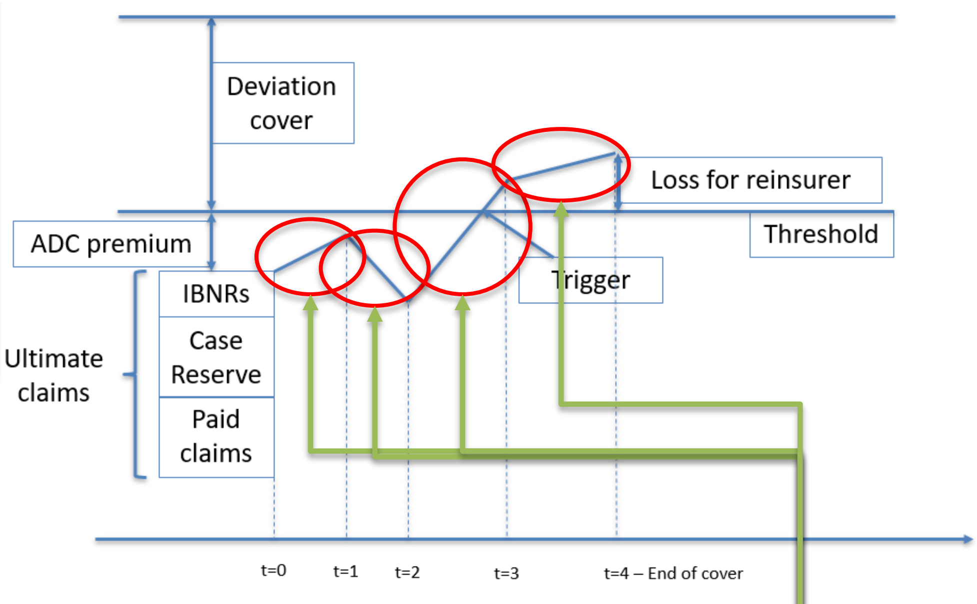
Introduction to Adverse Development Cover



- Covered deviation of ultimate claims limited in time
- Usually:
  - First 2 or 3 years, no commutation possible by the cedent
  - After 2/3 years, each year, commutation possible
  - Cover ends after a few years

To ease calculation, we consider a possible commutation each year.

Introduction
to
Adverse Development Cover



- European option
  - Term = 1 year (commutation each year)
  - Trigger = Threshold
  - Underlying asset = Ultimate claims
- Questions
  - Which option model to use ?
  - What calibration ?

P&C triangles
and
the Mack model

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UWY	Development year **k**									
i	1	2	3	4	5	6	7	8	9	10
1	21'003	21'021	20'503	20'909	22'767	18'989	18'960	25'485	25'743	25'721
2	26'469	28'808	40'931	40'396	47'347	46'895	45'554	54'546	54'495	
3	27'317	26'467	26'020	25'655	22'188	22'872	26'820	26'795		
4	32'835	30'980	30'596	36'693	36'035	46'326	46'272			
5	28'941	29'113	25'227	25'828	31'059	31'046				
6	32'413	34'340	33'363	37'887	37'885					
7	33'081	32'202	34'068	34'069						
8	29'976	31'444	31'715				$C_{i,k}$			
9	31'718	31'452								
10	35'939									

$$\hat{f}_k = \frac{\sum_{j=1}^{I-k} C_{j,k+1}}{\sum_{j=1}^{I-k} C_{j,k}}, 1 \leq k \leq I\text{-}1$$

Chain-Ladder coefficient

TRIANGLE of ULTIMATES !

P&C triangles
and
the Mack model

UWY	Development year **k**									
i	1	2	3	4	5	6	7	8	9	10
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Chain-Ladder coefficient

$$\hat{\sigma}_k^2 = \frac{1}{I-k-1} \sum_{i=1}^{I-k} C_{i,k} \left(\frac{C_{i,k+1}}{C_{i,k}} - \hat{f}_k \right)^2 \quad for\ 1 \le k \le I\text{-}2$$

Mack Volatility

P&C triangles and the Mack model

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10	35'939									

$$\hat{f}_k = \frac{\sum_{j=1}^{I-k} C_{j,k+1}}{\sum_{j=1}^{I-k} C_{j,k}}, 1 \le k \le I\text{-}1$$

Chain-Ladder coefficient

$$C_{i,k+1} = f_k C_{i,k}$$

$$Var(C_{i,k+1}) = C_{i,k+1} \frac{\sigma_k^2}{f_k}\left(1 + \frac{C_{i,k}}{\sum_{j=1}^{n-k} C_{j,k}}\right)$$

$$\hat{\sigma}_k^2 = \frac{1}{I-k-1} \sum_{i=1}^{I-k} C_{i,k} \left(\frac{C_{i,k+1}}{C_{i,k}} - \hat{f}_k\right)^2 \quad for\ 1 \le k \le I\text{-}2$$

Mack Volatility

Mack model
vs
Constant Elasticity of Variance model

$C_{i,k+1} = f_k C_{i,k}$ ← Translation in continuous time

$$\Delta C_{i,k} = C_{i,k+1} - C_{i,k} = f_k C_{i,k} - C_{i,k} = (f_k - 1)\, C_{i,k}$$

$$dC_{i,k} = log(f_k)\, C_{i,k}$$

Mack model
vs
Constant Elasticity of Variance model

$$Var(C_{i,k+1}) = C_{i,k+1}\frac{\sigma_k^2}{f_k}\left(1+\frac{C_{i,k}}{\sum_{j=1}^{n-k} C_{j,k}}\right)$$

$$\sigma(C_{i,k+1}) = \gamma_k\sqrt{C_{i,k+1}}$$

Where

$$\gamma_k = \sigma_k\sqrt{\frac{1}{f_k}\left(1+\frac{C_{i,k}}{\sum_{j=1}^{n-k} C_{j,k}}\right)}$$

Mack model
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$$dC_{i,k} = log(f_k)\, C_{i,k}$$

$$dC_{i,k} = log(f_k)\, C_{i,k} dk + \gamma_{k-1}\sqrt{C_{i,k}}\, dW_k$$

CEV model !

$$Var(C_{i,k+1}) = C_{i,k+1}\frac{\sigma_k^2}{f_k}\left(1 + \frac{C_{i,k}}{\sum_{j=1}^{n-k} C_{j,k}}\right)$$

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Mack model
vs
Constant Elasticity of Variance model

UWY i	Development year **k** 1	2	3	4	5	6	7	8	9	10
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$$\hat{f}_k = \frac{\sum_{j=1}^{I-k} C_{j,k+1}}{\sum_{j=1}^{I-k} C_{j,k}}, 1 \le k \le I\text{-}1$$

Chain-Ladder coefficient

$$\hat{\sigma}_k^2 = \frac{1}{I-k-1} \sum_{i=1}^{I-k} C_{i,k} \left(\frac{C_{i,k+1}}{C_{i,k}} - \hat{f}_k \right)^2 \quad for\ 1 \le k \le I\text{-}2$$

Mack Volatility

$$C_{i,k+1} = f_k C_{i,k}$$

$$Var(C_{i,k+1}) = C_{i,k+1} \frac{\sigma_k^2}{f_k} \left(1 + \frac{C_{i,k}}{\sum_{j=1}^{n-k} C_{j,k}} \right)$$

$$dC_{i,k} = log(f_k) C_{i,k}\, dk \; + \; \gamma_{k-1} \sqrt{C_{i,k}} dW_k$$

Mack model
vs
Constant Elasticity of Variance model

- Implementation in R - Packages “Sim.Diffproc” and “ChainLadder”

```
mack<-MackChainLadder(tri, est.sigma="Mack")
MackSigma<-mack$sigma^2                          Mack Sigma from ChainLadder package

for(k in 1:m1) {
 S<-S0[k]                                        Latest diagonal of the triangle
 for(i in k:m1){
fi<-r[i]                                         Chain-Ladder coefficient from ChainLadder package
f <- expression( log(fi)*x )
s <- MackSigma[i] *(1+S/SumCol[i])
g <- expression( sqrt(s*x/fi) )                  Expression for γk

mod <- snssde1d(N=simul,drift=f,diffusion=g,x0=S,M=simul,T=1)     Stochastic Differential Equation
```

Mack model
vs
Constant Elasticity of Variance model

- Tests

Triangle 1 - Expected Value (000's)

UWY	Mack projection – Development year 1	2	3	4	5	6	7	8	9	Mack vs CEV – Development year 1	2	3	4	5	6	7	8	9
1	-	-	-	-	-	-	-	-	54'448									0.0%
2	-	-	-	-	-	-	-	26'864	26'841								-0.1%	-0.1%
3	-	-	-	-	-	-	54'120	54'259	54'213							-0.5%	-0.5%	-0.5%
4	-	-	-	-	-	31'627	36'991	37'086	37'054						0.4%	0.5%	-0.3%	-0.4%
5	-	-	-	-	39'486	40'224	47'046	47'167	47'127					0.0%	0.0%	0.0%	0.4%	0.4%
6	-	-	-	35'872	37'387	38'086	44'546	44'660	44'622				1.0%	0.9%	0.6%	-0.2%	0.4%	0.4%
7	-	-	33'330	35'093	36'575	37'259	43'579	43'691	43'653			0.1%	0.1%	0.3%	0.5%	0.1%	1.0%	1.0%
8	-	32'532	34'189	35'998	37'518	38'220	44'702	44'817	44'779		0.0%	0.0%	-0.8%	-1.1%	-1.4%	-0.8%	-1.8%	-1.8%
9	36'222	37'466	39'374	41'456	43'208	44'015	51'481	51'613	51'569	0.3%	0.3%	0.3%	-0.3%	0.2%	0.4%	0.0%	-1.7%	-1.7%

Triangle 1 - Std Deviation (000's)

UWY	Mack projection – Development year 1	2	3	4	5	6	7	8	9	Mack vs CEV – Development year 1	2	3	4	5	6	7	8	9
1	-	-	-	-	-	-	-	-	33									-1.1%
2	-	-	-	-	-	-	-	276	276								-0.8%	-0.9%
3	-	-	-	-	-	-	6'968	6'999	6'994							1.2%	1.1%	1.7%
4	-	-	-	-	-	2'926	6'432	6'457	6'452						-1.9%	2.8%	2.4%	1.8%
5	-	-	-	-	6'226	7'189	10'538	10'572	10'563					3.2%	2.4%	3.9%	0.6%	0.8%
6	-	-	-	4'374	7'558	8'365	11'546	11'582	11'572				3.8%	2.5%	-2.3%	-1.5%	-2.3%	-2.3%
7	-	-	2'829	5'247	8'081	8'843	11'979	12'016	12'006			-3.6%	-0.1%	1.4%	3.1%	3.2%	6.6%	6.6%
8	-	5'298	6'264	7'919	10'227	10'923	14'176	14'218	14'205		-0.8%	-0.6%	3.2%	4.3%	1.9%	3.3%	3.7%	3.4%
9	1'652	5'985	7'017	8'788	11'273	12'028	15'599	15'645	15'632	-1.1%	0.8%	2.4%	5.1%	4.8%	3.7%	2.9%	2.6%	2.7%

ADC as a call option

For one Underwriting Year:

$$dC_{i,k} = log(f_k)C_{i,k}\, dk \; + \; \gamma_k C_{i,k}^{\beta} dW_k \quad \text{where } \beta\text{=0.5}$$

Closed-form formula for a call option based on the following parameters:

- $C_{i,k}$ is the price of the asset at time k,
- K is the strike,
- g_k is defined as: $g_k = \frac{1}{2}\left(C_{i,k} + K\right)$
- a_k is defined as:

$$a_k = \gamma_k \sqrt{\frac{exp\left(2\,(1-\beta)\,\sum_{i=1}^{k} log(f_i)\right)\; - 1}{2\,(1-\beta)\,\sum_{i=1}^{k} log(f_i)}}$$

The Black and Scholes model can be used on replacing an implied volatility defined as:

$$\sigma_{B\,k} = \frac{\sum_{i=1}^{k} a_i}{g_k^{1-\beta}}\left(1 + \frac{1}{24}(1-\beta)(2+\beta)\left(\frac{C_{i,k} \; - \; K}{g_k}\right)^2 + \frac{1}{24}\frac{(1-\beta)^2\left(\sum_{i=1}^{k} a_i\right)^2}{g_k^{2(1-\beta)}}\right)$$

For the period k – k+1 (i.e. one year), the value VC of the Call option is then calculated using the standard Black and Scholes formula:

$$VC_k = C_{i,k}\, \mathrm{N}(d_1) \; - \; \mathrm{K}\, e^{-log(f_k)}\, \mathrm{N}(d_2)$$

- where:

$$d_{1/2} = \frac{log\left(\frac{C_{i,k}}{K}\right) \; +/- \; \left(log(f_k) + \frac{\sigma_{B\,k}^2}{2}\right)}{\sigma_{B\,k}}$$

ADC as a call option

For one development period i, over the period of the Adverse Development Cover (e.g. n years), the value of the contract will be estimated as:

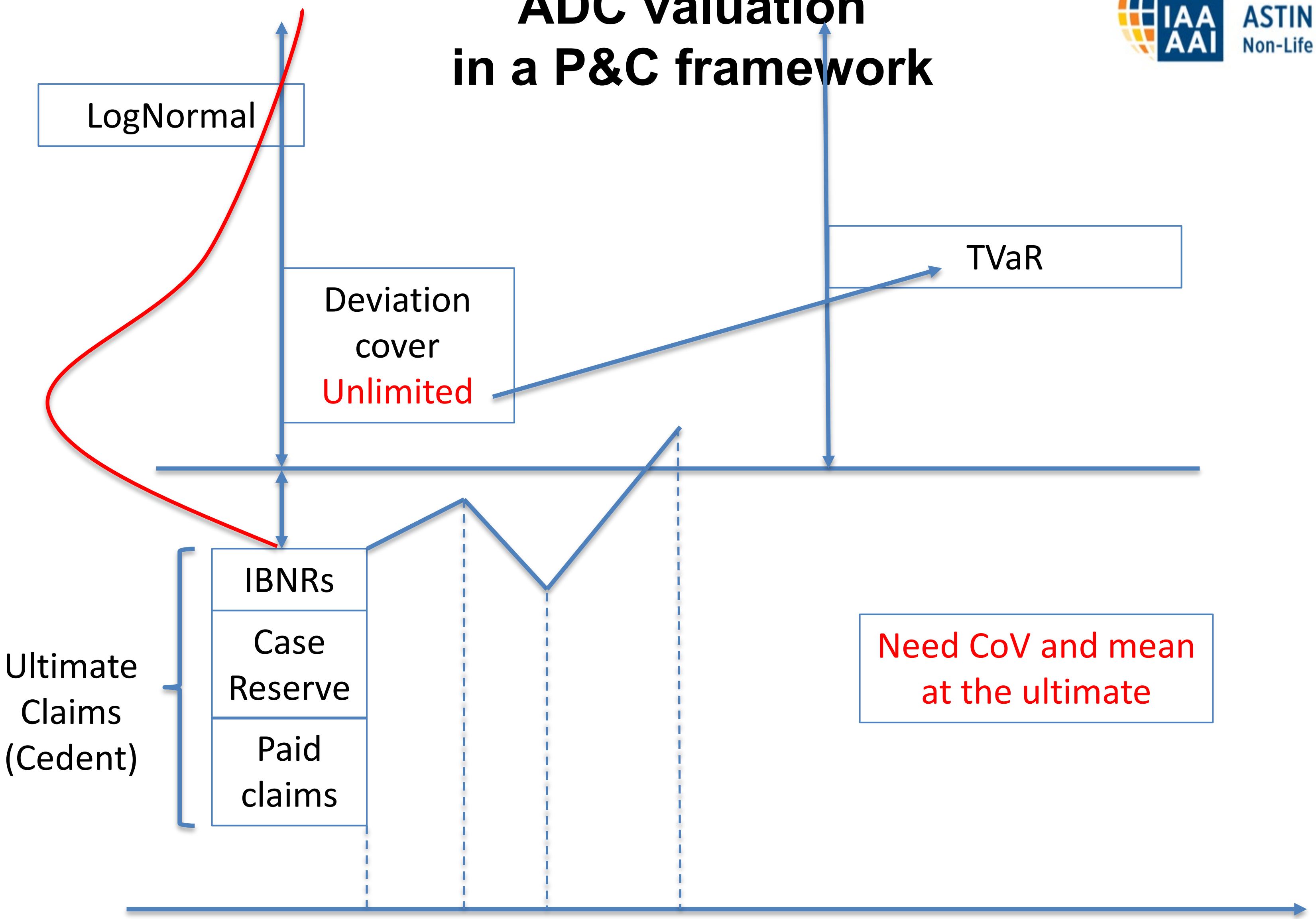
- $Contract\ value = \sum_{j=1}^{n} \frac{VC_j}{(1+ZC_j)^j}$

where ZC_j corresponds to the Zero-Coupon interest rate at time j.

Adverse Development Cover – Simplifications:

- Cedent can commute the contract at the end of each development period.
- Usually, an Adverse Development Cover will first have
  - a period of 2 to 3 years where the cedent can not commute
  - followed by a period of a few years where the cedent can commute at each anniversary date or whenever it wants.
- Zero-Coupon interest rate is nil at any time.

ADC valuation in a P&C framework



ADC valuation
in a P&C framework

INCURRED TRIANGLE (in 000's)

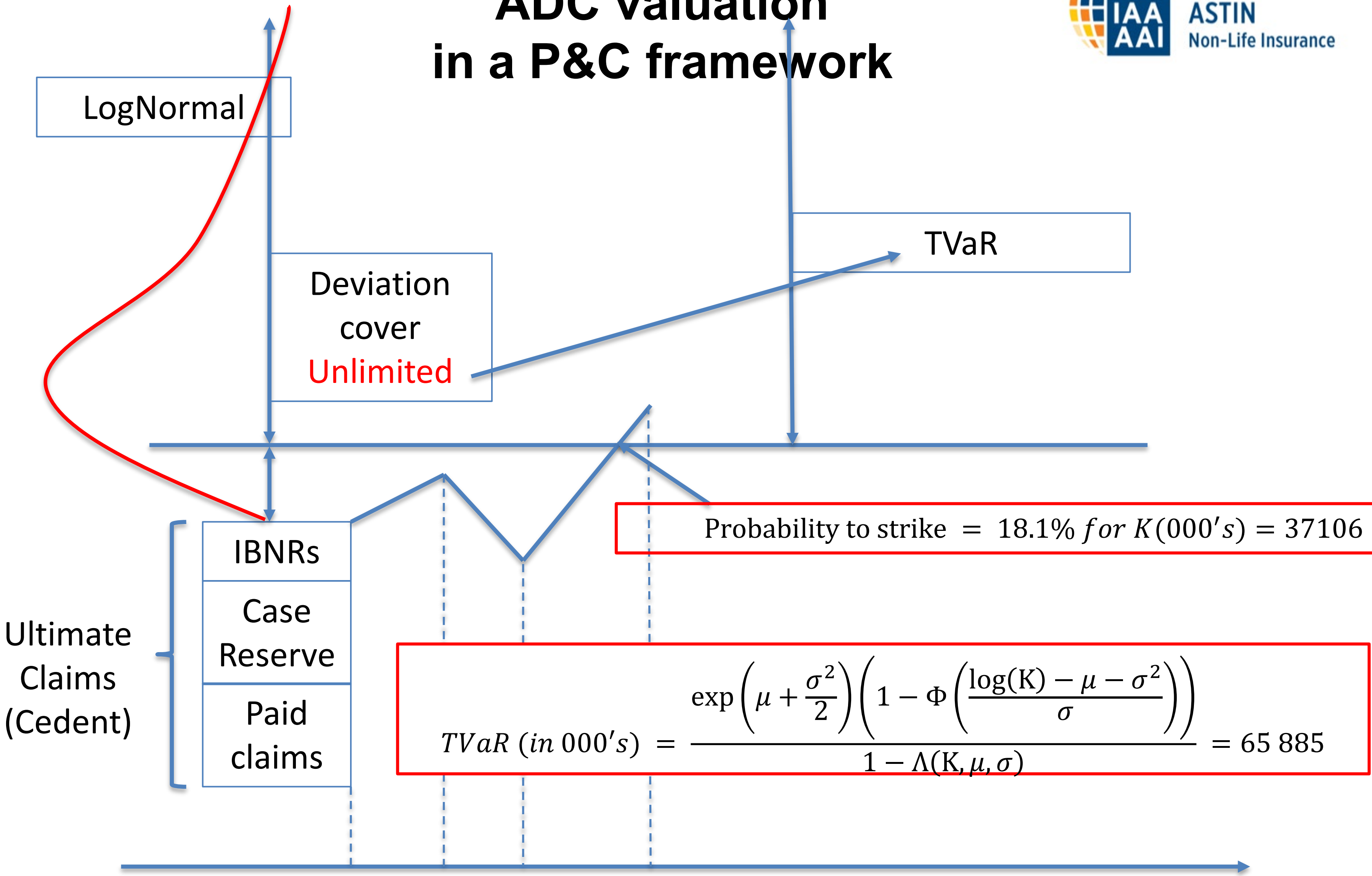
UWY	Development year **k**									
i	1	2	3	4	5	6	7	8	9	10
1	352	1 119	2 122	2 561	3 578	5 199	5 503	15 484	16 272	16 417
2	305	7 185	23 764	23 731	27 389	27 694	27 629	33 920	37 876	
3	1 464	2 761	4 011	3 451	4 282	5 344	5 446	6 905		
4	3 833	5 147	5 880	21 458	21 771	30 403	29 849			
5	1 603	3 345	4 605	5 970	7 278	8 356				
6	312	3 682	4 219	5 934	7 565					
7	1 619	4 480	3 865	5 714						
8	2 949	4 233	5 485							
9	10 292	6 928								
10	183									

- Mack model for UWY 9
  - Coefficient of variation on IBNR (ultimate) = 108%
  - Ultimate = 31 452
  - IBNR = 31 452 – 6928=24 525

- Strike = 37 106

- Lognormal distribution
  - $\sigma = \sqrt{log(1 + CoV^2)} = 87.78\%$
  - $\mu = log(24525) - \frac{\sigma^2}{2}$ = 16.63
  - $\text{Probability to strike} \;=\; 18.1\%$

ADC valuation in a P&C framework



Probability to strike $= 18.1\%\ for\ K(000's) = 37106$

$$TVaR\ (in\ 000's) = \frac{\exp\left(\mu + \frac{\sigma^2}{2}\right)\left(1 - \Phi\left(\frac{\log(\mathrm{K}) - \mu - \sigma^2}{\sigma}\right)\right)}{1 - \Lambda(\mathrm{K}, \mu, \sigma)} = 65\ 885$$

ADC value covering ultimate UWY 9 =18.1% x 65 885 = 11 941

Numerical examples

ASTIN Non-Life Insurance

	UWY	Ultimate	Strike Ultimate	IBNR	Strike IBNR	CoV IBNR	Proba. Strike	TVaR	Price GI	Price SDE
Triangle 1	6	37'885'292	53'039'409	30'320'074	45'474'191	137%	18.2%	91'403'012	16'618'857	5'802'242
	7	34'069'353	47'697'094	28'355'471	41'983'212	132%	18.6%	82'955'332	15'427'436	10'252'025
	8	31'714'670	44'400'538	26'230'117	38'915'985	146%	18.3%	81'021'253	14'837'265	14'605'261
	9	31'452'444	44'033'422	24'524'838	37'105'816	108%	18.1%	65'884'921	11'941'273	21'342'187
Tri. 2	10	35'291'175	42'349'410	19'382'543	26'440'778	78%	21.3%	41'593'210	8'869'135	39'575
	11	38'571'224	46'285'469	14'229'060	21'943'305	59%	14.4%	30'039'732	4'321'144	55'298
	12	36'800'886	44'161'063	31'106'955	38'467'132	114%	24.5%	74'726'799	18'325'642	230'282
Tri. 3	9	28'166'178	45'065'885	5'221'849	22'121'556	134%	2.7%	35'032'246	935'208	26'487
	10	22'481'268	35'970'029	5'761'423	19'250'184	90%	2.5%	26'795'430	679'105	589'042
	11	19'206'010	30'729'616	9'465'070	20'988'676	78%	6.7%	29'540'552	1'965'631	800'161

- For the triangles 1 and 3, both methods seem to provide prices which are in the same order of magnitude.
- However, for triangle 2, the differences are significant.

Numerical examples

	UWY	CoV development as %ultimate											
Triangle 1	6				12%	20%	22%	26%	26%	26%			
	7			8%	15%	22%	24%	27%	28%	28%			
	8		16%	18%	22%	27%	29%	32%	32%	32%			
	9	5%	16%	18%	21%	26%	27%	30%	30%	30%			
Tri. 2	10			5%	9%	11%	14%	16%	18%	19%	21%	23%	26%
	11		5%	7%	11%	13%	15%	17%	19%	19%	22%	24%	27%
	12	7%	9%	10%	13%	14%	16%	18%	20%	20%	22%	24%	27%
Tri. 3	9			18%	19%	24%	33%	47%	48%	49%	49%	49%	
	10		6%	20%	21%	26%	36%	51%	52%	53%	53%	53%	
	11	18%	19%	25%	26%	29%	36%	49%	49%	50%	51%	51%	

- Triangle 2: Such differences are explained by the relatively slow increase of the volatility along the development years

⇒ The price of the cover should be smaller in the first years of the ADC.

Conclusions

- The market of ADC/Loss Portfolio Transfer is increasing.
- Application of option pricing for ADC requires Triangles of ultimates
  - NEW REQUIREMENT !
- The volatility pattern affects the value of the ADC
  - Like for option pricing !

Model and data

- **Model and data available at:**

https://drive.google.com/open?id=16wPMzdZQtpccDi2bFhTYE-uYpMBI0UOo

- **Article available in www.ssrn.com**

- **The article is currently under peer-review for possible publication in the Journal of Derivatives.**

Questions ?

Thank you

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