Covid-19
Actuarial Modelling

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Multiple State Models

Living

L(t)

Dead

D(t)

t is either time or age
Continuous, differential equations
\[ \frac{dL(t)}{dt} = -\mu(t).L(t) \]
\[ \frac{dD(t)}{dt} = +\mu(t).L(t) \]

Discrete, difference equations
\[ L(t+1) = L(t) - q(t).L(t) \]
\[ D(t+1) = D(t) + q(t).L(t) \]
Other examples:

Marriage: Single, Married, Widowed, Divorced, Dead

Pensions:
Active, Early leaver, Ill health, Retired, Dead
Income Protection (Sickness) Insurance

Healthy age x
  ↓ μ(x)
  Dead

Sick age x, duration z
  ↓ v(x,z)
  Dead

σ(x) →

ρ(x,z) ←
Infections

Epidemiologists: S-I-R (no dead)
  in simple model no duration

Actuarial: H-I-D
  Healthy-Infected-Dead
  Time t and duration infected, d
  \( H(t) \quad I(t, d) \quad D(t) \)
Simple model:
One population
  Ignores sex, age, occupation
  Ignores other deaths
All onto one Excel spreadsheet
Time \( t \) and duration of infection \( d \) measured in days
One line per day of time \( t \)
Basic parameters:

- $r(t)$: basic rate of infection time $t$ per day
- $j(d)$: relative infectiousness of infected at duration $d$
- $m(d)$: mortality rate for infected at duration $d$ per day
Additional:

\[ f(t) \text{ fraction of infection who are recorded as new cases at duration day } K \text{ (e.g. 6)} \]

New cases recorded day \( t \):

\[ NCR(t) = f(t).I(t, K) \]
Initial conditions:

\[ H(0) \quad \text{Healthy population at time 0} \]

\[ I(t_1) \quad \text{Number (usually 1) of new infected on time day } t_1 \]
Transitions from box to box all straightforward except infections:
Healthy to Infected on day $t$ from those infected for $d$ days:

$$HI(t,d) = r(t).j(d).g(t).I(t,d)$$

where $g(t) = H(t)/(H(t)+I(t))$

$$HI(t) = \sum_d HI(t,d)$$

$I(t+1,1) = HI(t)$
System works for “what if” projections

But we would like to compare with actual cases and deaths with those expected by model.
Various ways of fitting

Use 14 (21, 7) days overlapping periods.
Choose fixed \( r(t) \) for 14 days, say 0 to 14.
Also set \( m(d) = m \) for days \( L_1 \) to \( L_2 \)
say 6 to 15; fixed for all \( t \)

Compare Actual (A) with Expected (E) Cases and Deaths, days \( t = K \) to \( K+20 \).
Set \( r \) and \( m \) so that \( \sum_c A = \sum_c E \) & \( \sum_d A = \sum_d E \)
Set \( r(7) = r \)
Note value of \( m \) as \( m(7) \)
Move up one day
At start set r for all days before first period equal to first r calculated.

At end carry on last r calculated till end of data.
Various trials of lesser parameters to get best structure.

e.g. \( j(d) = 1 \) for \( d = 1 \) to \( 5 \)

\[ = 0.5 \] for \( d = 6 \) to \( 15 \)

or \[ = 0.25 \] for \( d = 6 \) to \( 25 \)

\( m(d) \neq 0 \) for \( d = 6 \) to \( 15 \), or \( 11 \) to \( 20 \)

\( f(t) = 10\% \) for UK and Italy

or \( = 20\% \) for Switzerland

Very variable results
ECDC (European Centre for Disease Control) data, minus 2 days

Very variable numbers from day to day
Probably reporting very unevenly
Seven day sum of new CASES as percentage of maximum

- Italy
- Switzerland
- UK
- Austria
Seven day sum of new DEATHS as percentage of maximum

- **Italy**
- **Switzerland**
- **UK**
- **Austria**
Switzerland: New cases reported

- **Expected by Model**
- **Actual**
Switzerland: New deaths reported

Expected by model

Actual
UK: New deaths reported

- Expected by model
- Actual
Italy: New cases reported

- **Expected by Model**
- **Actual**
Italy: New deaths reported

- Expected by model
- Actual
Austria: New cases reported

- Expected by Model
- Actual
Austria: New deaths reported

- Expected by model
- Actual
Further calculations

\[ p(d) = 1 \quad p(d+1) = p(d).(1 - m(d)) \]

Total mortality rate:

\[ M(t) = 1 - p(N) \]

Epidemiologist’s \( R_0 \)

\[ Q = \sum_d p(d).j(d) \]

\[ R_0(t) = r(t).Q \]

Mean time:

\[ T(t) = \frac{\sum_d d.p(d).j(d)}{Q} \]
Switzerland, UK, Italy, Austria: Fitted $R_0(t)$

- **Switzerland**
- **UK**
- **Italy**
- **Austria**
If NO changes:

Day when Expected Reported Cases < 1

<table>
<thead>
<tr>
<th></th>
<th>CH</th>
<th>Italy</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date</td>
<td>26 Jul</td>
<td>9 Oct</td>
<td>28 Dec</td>
</tr>
<tr>
<td>Add Deaths</td>
<td>36</td>
<td>1,667</td>
<td>9,849</td>
</tr>
<tr>
<td>Deaths now</td>
<td>1,656</td>
<td>33,340</td>
<td>38,376</td>
</tr>
</tbody>
</table>
If NO changes: Austria

Maximum expected future:
- Cases 238.7 on 22 Jan 2022
- Deaths 9.7 on 28 Jan 2022

Expected Reported Cases < 1
- Date 23 Mar 2026
- Add Deaths 8,100
- Deaths now 668
Conclusions:
Model fits past data moderately well with adjustments
Can project future IF $r(t)$ unchanged

Cannot predict:
Governments
Public responses
Effects on $r(t)$
END

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