

# Pay-as-you-go financing: the volatility of the financial equilibrium of the Pension Funds

by Cinzia FERRARA

## 1. Introduction<sup>1</sup>

In the field of compulsory social insurance, financed by the pay-as-you-go system, the economic assumptions adopted to establish the relevant equilibrium have a particular dominance.

In fact, as contributions and benefits are calculated as percentages of wages, any variation from the economic assumptions may cause a financial unbalance. In this paper, by means of a simplified mathematical model, some indexes are proposed in order to measure the long-term effects on the annual financial results of the following variables:

- inflation;
- employment;
- real rates of increase in the wage.

Moreover, the variability of the equilibrium conditions is analysed by taking into consideration the following two cases: 1) a pension fund where the benefits are related to the final remuneration; 2) the “contributory system” as introduced in Italy in 1995.

It must be said that the 1995 Italian reform provided that pensions will be calculated on the basis of the sum of contributions paid (re-valued according to the GDP variations) instead of taking into account the final salary, while, the financial system adopted is still the pay-as-you-go.

## 2. Mathematical model for Pension funds

In order to simplify let us consider a compulsory pension fund, which provides only old-age pensions at the attainment of the retirement age. It is assumed that in any year  $t-h$  all members enter the scheme at age  $x$  and the number of the new entrants is equal to  $N_{t-h}$ . Under these conditions, at the time  $t > \bar{x}$  (stationary state), the amount of contributions and benefits will be expressed as follows:

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$$C(t) = \alpha \cdot \sum_{h=0}^{n-1} N_{t-h} \cdot X_t(h) \cdot {}_h p_x \quad (1)$$

$$O(t) = \sum_{h=n}^{\omega-x} N_{t-h} \cdot Y_t(h) \cdot {}_h p_x \quad (2)$$

where:

- = retirement age;
- $n$  =  $\xi - x$ ;
- = maximum age;
- $h$  = seniority in respect to the registration into the fund;
- = contribution rate;
- ${}_h p_x$  = probability that a member of age  $x$  is still alive at age  $x+h$ ;
- $C(t)$  = random variable representing contribution amounts in year  $t$ ;
- $O(t)$  = random variable representing benefit amounts in year  $t$ ;
- $X_t(h)$  = random variable representing the salary in year  $t$  of a member registered into the fund in year  $t-h$ ;
- $Y_t(h)$  = random variable representing the benefits in year  $t$  of a member registered into the fund in year  $t-h$ ;
- $N_{t-h}$  = random variable representing the number of members registered into the fund in year  $t-h$ .

Obviously, each r.v. depends on the economic assumptions, as employment rates, remuneration increases and inflation. In fact the number of new entries into the fund will depend on the employment condition, as shown in the following formula:

$$N_{t-h} = N_o \cdot \prod_{i=1}^{t-h} e^{\rho_i} \quad (3)$$

where the r.v.  $e^{\rho_i}$  represents the employment rate at time  $i$  and  $N_o$  the number of new entrants in year  $t=0$ .

By assuming that the r.v.  $e^{\rho_i}$  be mutually independent and with the same expected value  $e^{\bar{\rho}}$ , the expected number of the entries in year  $t-h$  will be:

$$\overline{N}_{t-h} = N_o \cdot e^{\bar{\rho}(t-h)} \quad (4)$$

As for the salary, let us assume that it will increase in every year  $i$  according to the rates:

- real general increase  $e^r$
- inflation  $e^d$
- career salary scale  $e^c$

As far as the pensions are concerned, their amount will be calculated as a percentage  $\gamma$  of the final remuneration and re-valued thereafter according to the inflation index.

Hence, by denoting by  $s_o$  the initial salary in the year 0, we will obtain the following quantities:

$$X_t(h) = s_o \cdot \prod_{i=1}^t e^{r_i} \prod_{i=1}^t e^{d_i} \prod_{k=1}^h e^{c_k} \quad (5)$$

$$Y_t(h) = \gamma \cdot X_{t-h+n}(n) \cdot \prod_{i=t-h+n+1}^t e^{d_i} = s_o \cdot \gamma \cdot \prod_{i=1}^{t-h+n} e^{r_i} \prod_{i=1}^t e^{d_i} \prod_{k=1}^n e^{c_k} \quad (6)$$

By assuming that the r.v.  $e^r$ ,  $e^d$  and  $e^c$  are mutually independent and have the same expected value  $\bar{e}^r$ ,  $\bar{e}^d$  and  $\bar{e}^c$ , the expected values relevant to salaries and benefits are:

$$\bar{X}_t(h) = s_o \cdot \bar{e}^{r \cdot t} \cdot \bar{e}^{\bar{d} \cdot t} \cdot \bar{e}^{\bar{c} \cdot h} \quad (7)$$

$$\bar{Y}_t(h) = s_o \cdot \gamma \cdot \bar{e}^{\bar{r} \cdot (t-h+n)} \cdot \bar{e}^{\bar{d} \cdot t} \cdot \bar{e}^{\bar{c} \cdot n} \quad (8)$$

Then the expectations of the quantities (1) and (2), relevant to contribution amount and benefit payment in the year  $t$ , may be expressed as:

$$\bar{C}(t) = \alpha \cdot \sum_{h=0}^{n-1} \bar{N}_t(h) \cdot \bar{X}_t(h) = \alpha \cdot N_o \cdot s_o \cdot \sum_{h=0}^{n-1} \bar{e}^{\rho(t-h)} \bar{e}^{r \cdot t} \cdot \bar{e}^{\bar{d} \cdot t} \cdot \bar{e}^{\bar{c} \cdot h} \cdot {}_h p_x \quad (9)$$

$$\bar{O}(t) = \alpha \cdot \sum_{h=n}^{\omega-x} \bar{N}_t(h) \cdot \bar{Y}_t(h) = \gamma \cdot N_o \cdot s_o \cdot \sum_{h=n}^{\omega-x} \bar{e}^{\rho(t-h)} \bar{e}^{\bar{r} \cdot (t-h+n)} \cdot \bar{e}^{\bar{d} \cdot t} \cdot \bar{e}^{\bar{c} \cdot n} \cdot {}_h p_x \quad (10)$$

where  $\bar{N}_t(h)$  represents the expected value in year  $t$  of a cohort which entered in  $t-h$ .

### 3. The volatility of the financial equilibrium

Let us suppose that the fund is financed according to the pay-as-you-go system. Moreover we will assume that at the valuation time  $T=0$ , the fund is expected to be in financial equilibrium when the stationary status is attained. We will consider the situation, in each year  $t > T + \Delta x$ , where the following condition holds:

$$W(t) = \frac{\bar{C}(t) - \bar{O}(t)}{\bar{C}(t)} = 1 - \frac{\bar{O}(t)}{\bar{C}(t)} = 0 \quad (11)$$

In other words, it is supposed that, under the assumed economic conditions, contribution amount is equal to benefit out-flow in each of the years following the attainment of the stationary status. Our purpose is to analyse the effect on the variable  $W(t)$ , representing the operating surplus as a percentage of the contribution amount paid-in, caused by additive shifts in economic assumptions. It is assumed that those variations occur after the point  $T=0$ .

In this respect, let us denote with  $\Delta \rho$  the variation in the employment force  $\rho_i$  and assume that this occurrence does not change the distributions of  $\rho_i$ . Because of the assumed independence of the r.v.  $\rho_i$ , the expected values of contribution and benefit amounts may be expressed as follows:

$$\bar{C}(t; \Delta \rho) = \alpha \cdot \sum_{h=0}^{n-1} \bar{N}_t(h) \cdot \bar{X}_t(h) \cdot e^{\Delta \rho(t-h)} \quad (12)$$

$$\bar{O}(t; \Delta \rho) = \sum_{h=n}^{\omega-x} \bar{N}_t(h) \cdot \bar{Y}_t(h) \cdot e^{\Delta \rho(t-h)} \quad (13)$$

If  $\Delta \rho = 0$  then  $\bar{C}(t; \Delta \rho) = \bar{C}(t)$  and  $\bar{O}(t; \Delta \rho) = \bar{O}(t)$ .

As a first approximation, the ratio  $W(t)$  (considering the employment rate variations) will vary according to:

$$\Delta_{\rho} W(t) = \left\{ \frac{\bar{C}'(t)}{\bar{C}(t)} - \frac{\bar{O}'(t)}{\bar{O}(t)} \right\} \Delta \rho \quad (14)$$

which represents the Taylor series expansion up to the second term, while  $\bar{C}'(t)$  and  $\bar{O}'(t)$  are the partial derivatives of the (12) and (13) with respect to  $\Delta \rho$  calculated at the point  $\Delta \rho = 0$ . Since:

$$\frac{\bar{C}'(t)}{\bar{C}(t)} = t - \frac{1}{\bar{C}(t)} \sum_{h=0}^{n-1} h \cdot \bar{N}_t(h) \cdot \bar{X}_t(h) \cdot \alpha = t - Mc \quad (15)$$

$$\frac{\bar{O}'(t)}{\bar{O}(t)} = t - \frac{1}{\bar{O}(t)} \sum_{h=n}^{\omega-x} h \cdot \bar{N}_t(h) \cdot \bar{X}_t(h) = t - Mo \quad (16)$$

Then the formula (14) will be:

$$\Delta_\rho W(t) = (Mo - Mc)\Delta\rho = I(\rho)\Delta\rho \quad (16 \text{ bis})$$

where the indexes  $Mo$  and  $Mc$  represent the weighted average of the membership seniorities into the fund. The weights are given by the benefit amounts and the contribution amounts in the year  $t$  respectively.

As  $Mo > Mc$ , the index  $I(\rho)$  will be positive, the function  $W(t)$  will increase according to the employment force  $\rho$  and the fund will present a financial surplus.

On the other hand, the index  $I(\rho)$  measures the elasticity of the variable  $W(t)$  with respect to the employment condition. In fact, the higher the value of the index, the larger the surplus (or deficit) of the fund will be.

Moreover, the values  $Mo$  and  $Mc$  may be expressed as:

$$Mo = \frac{\sum_{h=n}^{\omega-x} h \cdot e^{-h(\bar{r} + \bar{\rho})} {}_h P_x}{\sum_{h=n}^{\omega-x} e^{-h(\bar{r} + \bar{\rho})} {}_h P_x} \quad (18)$$

$$Mc = \frac{\sum_{h=0}^{n-1} h \cdot e^{-h(\bar{\rho} - \bar{c})} {}_h P_x}{\sum_{h=0}^{n-1} e^{-h(\bar{\rho} - \bar{c})} {}_h P_x} \quad (19)$$

In this form the indexes may be considered as the average durations respectively of an  $n$ -year deferred life annuity at the constant force of interest equal to  $i = \bar{r} + \bar{\rho}$  and an  $n$ -year temporary life annuity at the constant force of interest equal to  $i = \bar{\rho} - \bar{c}$ .

A similar approach may be used to analyse the variations relevant to the inflation force ( $d$ ), to the real salary increase ( $r$ ) and to the career salary scale ( $c$ ). The following indexes are then obtained:

$$\Delta_d W(t) = 0 \quad (20)$$

$$\Delta_r W(t) = (Mo - n)\Delta r = I(r)\Delta r \quad (21)$$

$$\Delta_c W(t) = (Mc - n)\Delta c = I(c)\Delta c \quad (22)$$

From the previous formulae and by taking into account that  $Mo > Mc$ ;  $Mo > n$  and  $Mc < n$ , we note that the annual financial result expressed as a percentage of the contribution amount will be:

- independent from inflation;
- increasing according to the employment rate, the relevant elasticity being equal to the difference between the weighted averages of membership seniorities, where the weights are given by benefit and contribution amounts respectively;
- increasing according to the real remuneration rate, the relevant elasticity being equal to the difference between the weighted average of membership seniorities where the weights are given by benefit amounts and the number of years  $n = \xi - x$ ;
- decreasing according to the career salary scale rate, the relevant elasticity being equal to the difference between the number of years  $n = \eta - x$  and the weighted averages of membership seniorities, where the weights are given by contribution amounts.

Eventually, we can state that the risk involved in financing pension funds by pay-as-you-go system and relevant to the volatility of economic assumptions may be measured, as a first approximation, by the variables  $Mo$ ,  $Mc$  and  $n$ .

In particular, if the equilibrium conditions have been determined on the basis of static assumptions, that is:

$$\bar{r} = \bar{\rho} = \bar{c} = 0$$

the indexes expressed by (18) and (19) will be equal to:

$$Mo = \frac{\sum_{h=n}^{\omega-x} h \cdot h P_x}{\sum_{h=n}^{\omega-x} h P_x} \quad (23)$$

$$Mc = \frac{\sum_{h=0}^{n-1} h \cdot {}_h P_x}{\sum_{h=0}^{n-1} {}_h P_x} \quad (24)$$

Then the indexes will depend only on the surviving probabilities and represent the average seniorities of the active members and of the retirees existing in year  $t > \bar{x}$ .

The formulae (16 bis), (21) and (22) represent a first approximation of the  $W(t)$  variation. By considering an approximation of the Taylor series expansion up to the third term, we obtain:

$$\Delta_\rho W(t) = I(\rho)\Delta\rho - [Mo_{,2} - Mc_{,2} + 2(Mc)^2 - 2McMo] \frac{(\Delta\rho)^2}{2} = I(\rho)\Delta\rho + I_2(\rho)(\Delta\rho)^2$$

$$\Delta_r W(t) = I(r)\Delta r - [Mo_{,2} + n^2 - 2nMo] \frac{(\Delta r)^2}{2} = I(r)\Delta r + I_2(r)(\Delta r)^2 \quad (25)$$

$$\Delta_c W(t) = I(c)\Delta c - [Mc_{,2} - n^2 - 2(Mc)^2 + 2nMc] \frac{(\Delta c)^2}{2} = I(c)\Delta c + I_2(c)(\Delta c)^2$$

where  $Mo_{,2}$  and  $Mc_{,2}$  are defined by the quantities:

$$Mo_{,2} = \frac{\sum_{h=n}^{\omega-x} h^2 \cdot e^{-h(\bar{r}+\bar{\rho})} {}_h P_x}{\sum_{h=n}^{\omega-x} e^{-h(\bar{r}+\bar{\rho})} {}_h P_x} \quad (26)$$

$$Mc_{,2} = \frac{\sum_{h=n}^{\omega-x} h^2 \cdot e^{-h(\bar{\rho}-\bar{c})} {}_h P_x}{\sum_{h=n}^{\omega-x} e^{-h(\bar{\rho}-\bar{c})} {}_h P_x} \quad (27)$$

Of course, if the equilibrium conditions have been determined on the basis of static assumptions, the (26) and (27) can be transformed into:

$$Mo_{,2} = \frac{\sum_{h=n}^{\omega-x} h^2 \cdot {}_h P_x}{\sum_{h=n}^{\omega-x} {}_h P_x}$$

$$Mc_{,2} = \frac{\sum_{h=0}^{n-1} h^2 \cdot {}_h P_x}{\sum_{h=0}^{n-1} h P_x}$$

In this case the indexes represent the second moment of the seniority distribution of the members existing in year  $t$  ( $t > \bar{x}$ ).

#### 4. Some considerations on the Italian contribution system.

Unlike the majority of national insurance systems in Europe in which the pension is linked to the final salaries, the reform law n.335/1995 provides that for newly employed persons the amount is a function of the accumulated paid-in contributions re-valued according to the growth rate of the gross national product.

Moreover, the reform, while introducing into pension calculation the typical principles of the individual funding, has left in force the pay-as-you-go system in which the financial equilibrium, being guaranteed in each year by the balance between income from contributions and outflow in pension payments, does not involve the accumulation of invested funds.

The new, so-called called “contributory system”, can be analysed from a strictly theoretical point of view by using the model and the simplified hypotheses laid out in the second paragraph.

For this purpose, it should be considered that the pension is no longer equal to a proportion of the final salary, as expressed in formula (8), but to a quota  $\bar{\gamma}^*$  of the accumulated sum of paid-in contributions revaluated at a rate equivalent to the sum of the rates of increment of the real gross national product ( $e^{p_i}$ ) and inflation ( $e^{d_i}$ ). In particular, considering that for each year  $i$  the rate  $e^{p_i}$  can be expressed as the sum of growth in employment, real general wage increase and a residual rate  $e^{\lambda_i}$ , i.e. assuming that:

$$e^{p_i} = e^{\rho_i} + e^{r_i} + e^{\lambda_i}$$

and assuming that the r.v.  $e^{\rho_i}$ ,  $e^{r_i}$  and  $e^{\lambda_i}$  are independent, formula (8) will become:

$$\bar{Y}_i^*(h) = \bar{\gamma}^* \cdot e^{\bar{d} \cdot t} \sum_{u=0}^{n-1} \alpha \cdot s_o \cdot e^{\bar{r}(t-h+u)} \cdot e^{\bar{c}u} \cdot e^{\bar{p}(n-u)} = \bar{\gamma}^* \cdot \alpha \cdot s_o \cdot e^{\bar{d} \cdot t} \cdot e^{\bar{r}(t-h+n)} \cdot e^{n(\bar{\rho}+\bar{\lambda})} \sum_{u=0}^{n-1} e^{u(\bar{c}-\bar{\rho}-\bar{\lambda})}$$

where  $e^{\bar{\lambda}}$  represents the average value of  $e^{\lambda_i}$ .



The above formula substituted in formula (10) gives the outflow of the “contributory system”, that is:

$$\begin{aligned}\bar{O}^*(t) &= \sum_{h=n}^{\omega-x} \bar{N}_t(h) \bar{Y}_t^*(h) = \\ &= N_o \gamma^* \alpha_s \alpha_o e^{\bar{d} \cdot t} e^{n \cdot (\bar{\rho} + \bar{\lambda} + \bar{r})} \sum_{h=n}^{\omega-x} e^{(\bar{\rho} + \bar{r})(t-h)} {}_h p_x \sum_{u=0}^{n-1} e^{u(\bar{c} - \bar{\rho} - \bar{\lambda})}\end{aligned}\quad (28)$$

Therefore, by serial expansion to the second term of the new function  $W^*(t)$ , the following indicators of the variability of the “contributory system” can be obtained:

$$\begin{aligned}\Delta_\rho W^*(t) &= [Mo - Mc + Ds - n] \Delta\rho = [I(\rho) - (n - Ds)] \Delta\rho = I^*(\rho) \cdot \Delta\rho \\ \Delta_r W^*(t) &= [Mo - n] \Delta r = I^*(r) \cdot \Delta r \\ \Delta_c W^*(t) &= [Mc - Ds] \Delta c = [I(c) + n - Ds] \Delta c = I^*(c) \cdot \Delta c \\ \Delta_\lambda W^*(t) &= [Ds - n] \Delta \lambda = I^*(\lambda) \cdot \Delta \lambda\end{aligned}\quad (29)$$

where  $Ds$ , representing the average financial duration calculated at the rate  $\bar{\rho} + \bar{\lambda} - \bar{c}$ , is given by:

$$Ds = \frac{\sum_{u=0}^{n-1} u \cdot e^{u(\bar{c} - \bar{\rho} - \bar{\lambda})}}{\sum_{u=0}^{n-1} e^{u(\bar{c} - \bar{\rho} - \bar{\lambda})}}\quad (30)$$

Given  $Ds < n$  and bearing in mind that  $I(c) < 0$ , in comparison with “traditional systems” the new system presents the following features:

- a lower level of risk in relation to reductions in the employment rate equal to the difference between  $n$  and  $Ds$ ;
- the same level of risk in the case of changes in real salary increments due to renewals of salary agreements;
- a lower level of risk in relation to unbalances due to career salary scale increase;
- a dependence on the rate  $\lambda$  (equal to the difference between the GNP growth rate and the earnings volume subject to contribution payments) that may cause financial deficit when the rate is higher than that estimated at the time of evaluation.

In particular, if the initial equilibrium was obtained taking into consideration static hypotheses, that is assuming  $\bar{\rho} = \bar{\lambda} = \bar{c} = \bar{r} = 0$ , then:

$$Ds = \frac{\sum_{u=0}^{n-1} u}{\sum_{u=0}^{n-1} 1} = \frac{n-1}{2}$$

and the previous indicators of the variations in the annual financial results become:

$$\begin{aligned} I^*(\rho) &= \left[ I(\rho) - \frac{n+1}{2} \right] \\ I^*(r) &= I(r) \\ I^*(c) &= \left[ I(c) + \frac{n+1}{2} \right] \\ I^*(\lambda) &= -\frac{n+1}{2} \end{aligned} \tag{31}$$

## 5. Numerical illustrations

In order to evaluate the goodness of fit of the indicators relating to the volatility of the equilibrium conditions in a pay-as-you-go system, we take into consideration a pension fund that provides pension payments calculated on the basis of the final salary. The following features are considered:

$$x = 25;$$

$$\square = 65;$$

$$\gamma = 0.8;$$

$$\alpha = 26.62\%$$

${}_h p_x$  = life expectancy of males – Italian census, 1991 (ISTAT).

Under these hypotheses, and in a static economic condition, the fund in year  $t \geq \square$  (stationary state) will be in a condition of equilibrium with annual financial results equal to 0 and will present the following values:

$$Mc = 18.98$$

$$M_0 = 49.36$$

$$n = 40$$

$$M_{c,2} = 491.24$$

$$M_{o,2} = 2,485.67$$

In table 1 the values of the annual financial results in the stationary status are presented by assuming given variations in the economic conditions. The results are presented as percentages of the contribution amounts and a comparison is made between the actual results and those estimated by using the previously described indicators.

In particular, the average index indicates the variation obtained on the basis of formulas (16 bis), (21) and (22), while the quadratic indices refer to formula (25).

The results illustrated in the table show how the average indicators provide a good approximation as regards variations in instantaneous rates relevant to general wage and to career salary scale increases. As for the instantaneous employment rates, only when small variations are involved, i.e. when the new assumptions are similar to the initial hypotheses ( $\Delta = \pm 0.005$ ), the average indicators give sound results. On the other hand, the quadratic indexes prove in all cases to be excellent indicators in evaluating the financial equilibrium of pay-as-you-go system.

*Table 1: Annual financial results expressed as percentages of the contribution amount (% values)*

Economic variables	Exact Value %	Approximated value %	
		Average Index	Quadratic Index
Employment	-35.0	-30.4	-34.6
	-16.3	-15.2	-16.2
	+14.2	+15.2	+14.1
	+26.5	+30.4	+26.2
General wage increase	-10.1	-9.4	-10.1
	-4.9	-4.7	-4.9
	+4.5	+4.7	+4.5
	+8.7	+9.4	+8.7
Career and salary scale	+19.5	+21.0	+19.5
	+10.1	+10.5	+10.1
	-10.9	-10.5	-10.9
	-22.6	-21.0	-22.6

## 6. Conclusions

On the basis of the obtained results, it can be concluded that in a pension fund financed by pay-as-you-go system and in which the pension is equal to a percentage of the final salary, the variations in the economic hypotheses compared with those formulated in order to achieve equilibrium, lead, in the stationary status, to:

- a deficit in relation to employment rate decreases with an elasticity approximately equal to the difference between “the average seniorities” in terms of pension payments and income from contributions;
- the maintenance of equilibrium in the case of variations in the inflation rate;
- an increasing deficit in the case of reductions in the rate of general real wage increase, the relevant percentage variation being equal to the difference between the “average seniorities” in terms of pension benefits paid out and the number of years spent in activity;
- an unbalance between pension payments and contributions with respect to increases in career advancement rates. The relevant elasticity is equal to the difference between the maximum length of activity and the average length of fund membership in terms of paid-in contributions.

The study also illustrates how the volatility of equilibrium in pay-as-you-go system basically depends on the following three indicators calculated on the basis of the initial economic hypotheses that assured the financial equilibrium of the fund:

- $M_o$  equal to the average length of membership to the fund weighted with the value of pension payments;
- $M_c$  equal to the average length of membership to the fund weighted with the value of contributions;
- $n$  equal to the difference between the retirement age and the age on entry.

In particular, when the initial conditions of equilibrium were determined on the basis of static economic hypotheses,  $M_o$  and  $M_c$  will represent the average membership age of pensioners and contributors, hence the elasticity will depend essentially on the distribution according to the term of membership in the year  $t$  of the stationary status.

The study also made use of corrective factors that take into account the convexity of the pension payment-contribution curve and that improve the degree of approximation of the previous results.

As far as the new Italian system is concerned, it can be observed that in comparison with the so-called “income-based systems” the “contributory system” presents the following aspects:

- the same elasticity in relation to variations in the inflation rate and salary rises due to contract renewals;
- a lower level of risk in the case of reductions in the employment rate;
- reduced sensitivity to variations in career advancement;
- a dependence on the rate  $\lambda$  (equal to the difference between the real growth of the GNP and the earnings income) measured by the difference between the number of years of activity ( $n$ ) and the average financial duration  $D_s$  of paid-in contributions prior to retirement evaluated at the rate  $e^{\bar{\rho} + \bar{\lambda} - c}$ .

The results of the study were obtained, for simplicity of exposition, taking into consideration only one entry age and benefits concerning only old age pension.

In addition, it can be demonstrated that the results remain substantially valid even if the hypotheses underlying the simplified model are generalised. In such cases the elasticity of the financial result will depend on the average length of membership in terms of pension payments and contributions calculated taking into account invalidity and survivor benefits, while the indicator  $n$  will represent the average length of membership in terms of new pensions payments in the year  $t$  of the stationary state.

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