JAB Chain

Long-tail claims development

ASTIN - September 2005

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Outline

Chain Ladder : comments

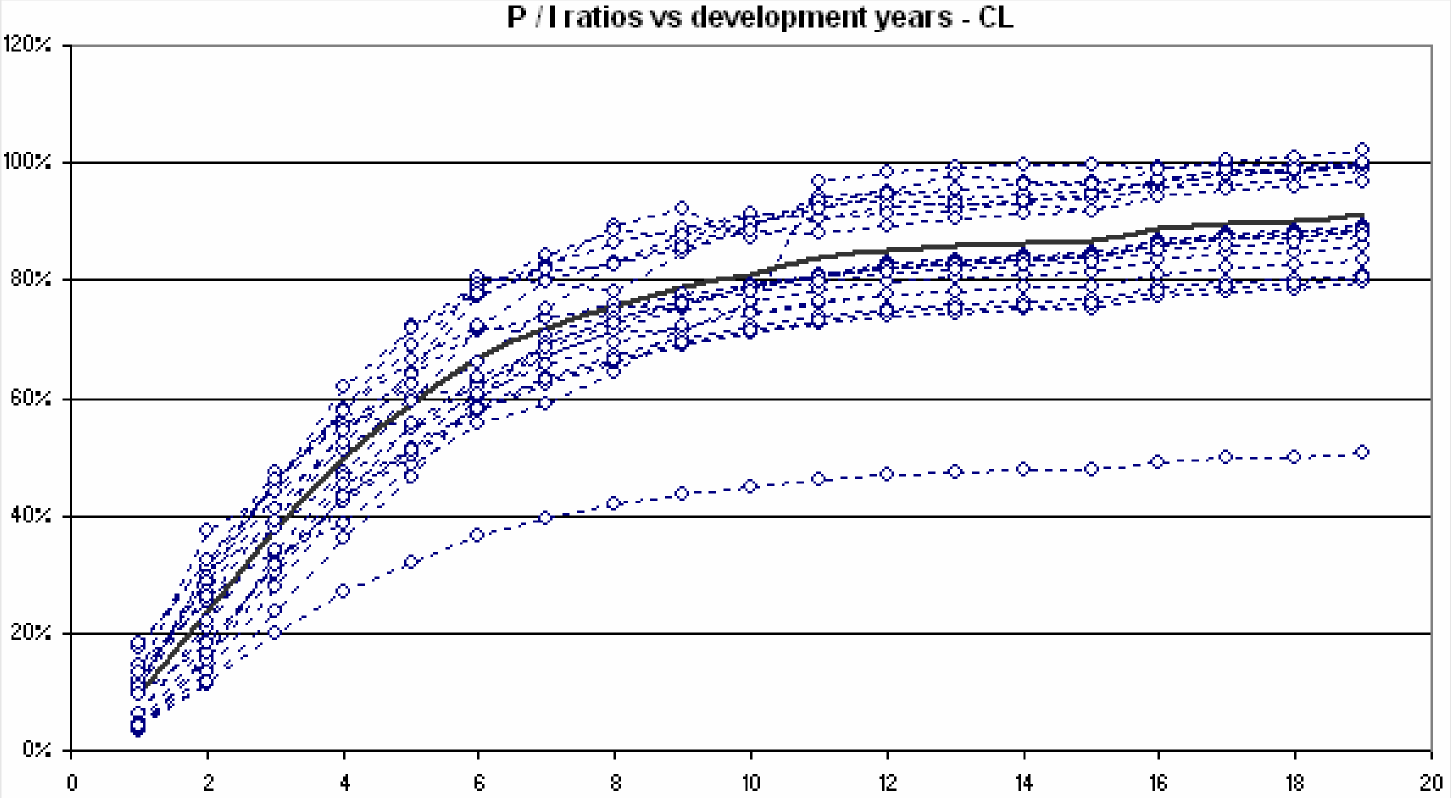
A first solution: Munich Chain Ladder

JAB Chain

Chain Ladder: Comments

Black line: average paid to incurred (P/I) ratio

CL keeps a below or above-average P/I ratio



Outline

Chain Ladder: comments

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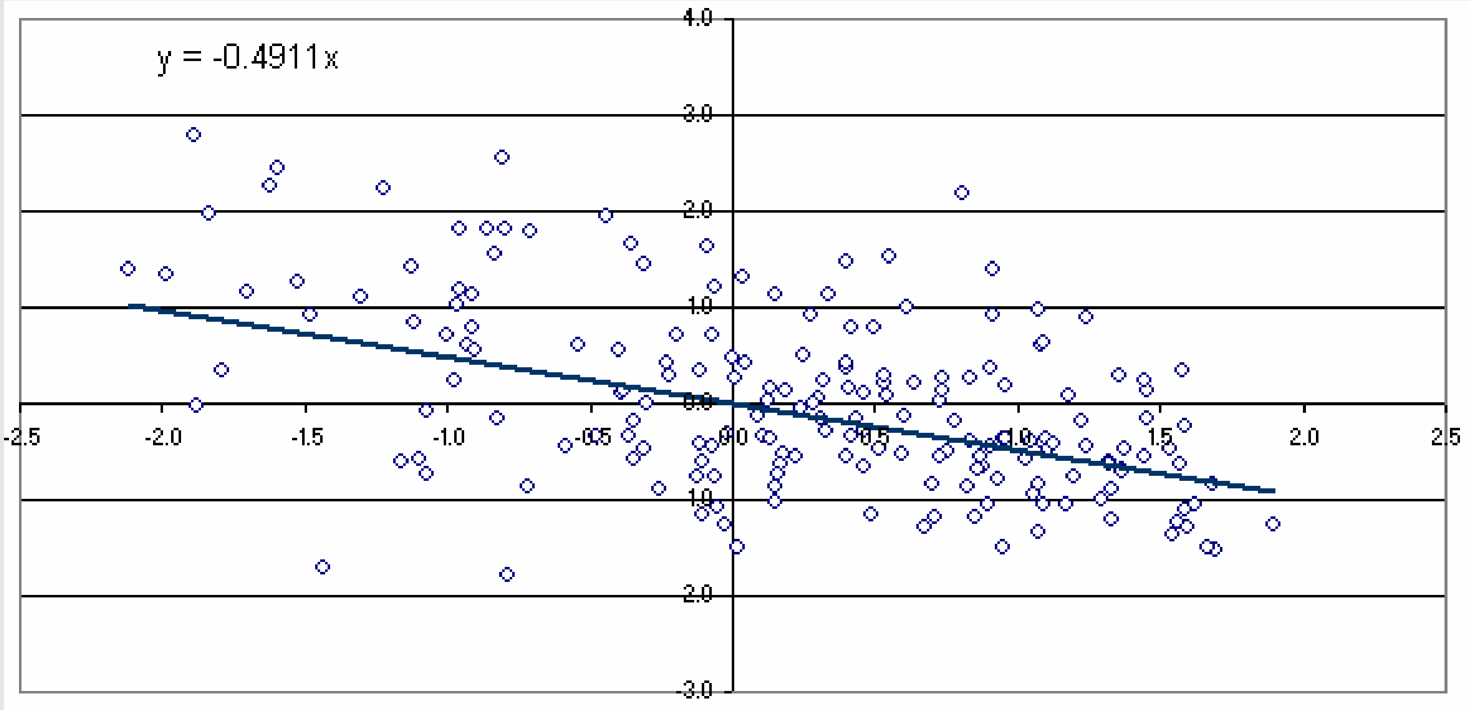
A first solution : Munich Chain Ladder

Idea: apply a correction to the Chain Ladder development factors (CL DF), depending on the P/I ratio.

This intends to increase the DF (for UW year i and DY j) if the previous P/I (for UW year i and DY j-1) is lower than the average and vice-versa:

Low P/I ratio → Relat. high incurred → Higher future payments → High DF

A first solution : Munich Chain Ladder



Y-axis: $f_{i,j} - \bar{f}_j$ Where $\bar{f}_j$ is the DF average, ie CL.

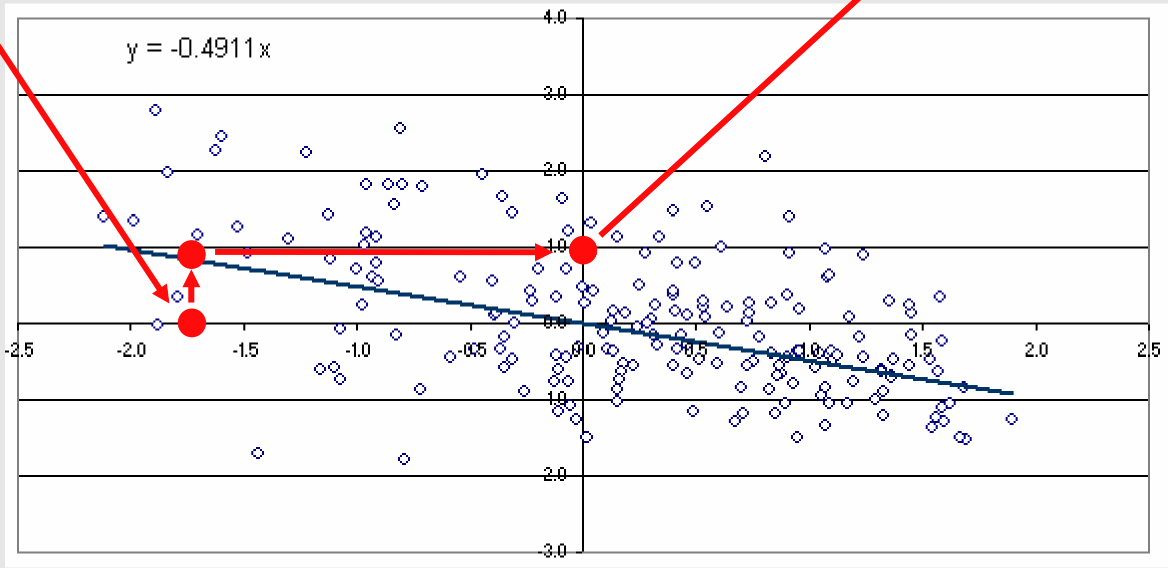
X-axis: $P/I_{i,j} - \overline{P/I}_j$ Where $\overline{P/I}_j$ is the P/I average ratio.

A first solution : Munich Chain Ladder

DF residual = DF – Average yields DF = Average + DF Residual

Corrected DF = CL + Correction

and, given the P/I ratio residual…



A first solution : Munich Chain Ladder

To be accurate…

- MCL does the same for incurred and paid but uses I/P ratios for the paid process and the P/I ratio for the incurred process.
- The formula is:

$$\mathbf{E}\left(\frac{P_{i,s+1}}{P_{i,s}} \mid \mathbf{B}_i(s)\right) = f_s^P + \lambda^P \cdot \frac{\sigma\left(\frac{P_{i,s+1}}{P_{i,s}} \mid \mathbf{P}_i(s)\right)}{\sigma\left(Q_{i,s}^{-1} \mid \mathbf{P}_i(s)\right)} \cdot \left(Q_{i,s}^{-1} - \mathbf{E}\left(Q_{i,s}^{-1} \mid \mathbf{P}_i(s)\right)\right)$$

Expected paid development factors, assuming that both paid and incurred processes (**B**$_i$(s)) are known up to DY s

CL DF

Regression line slope

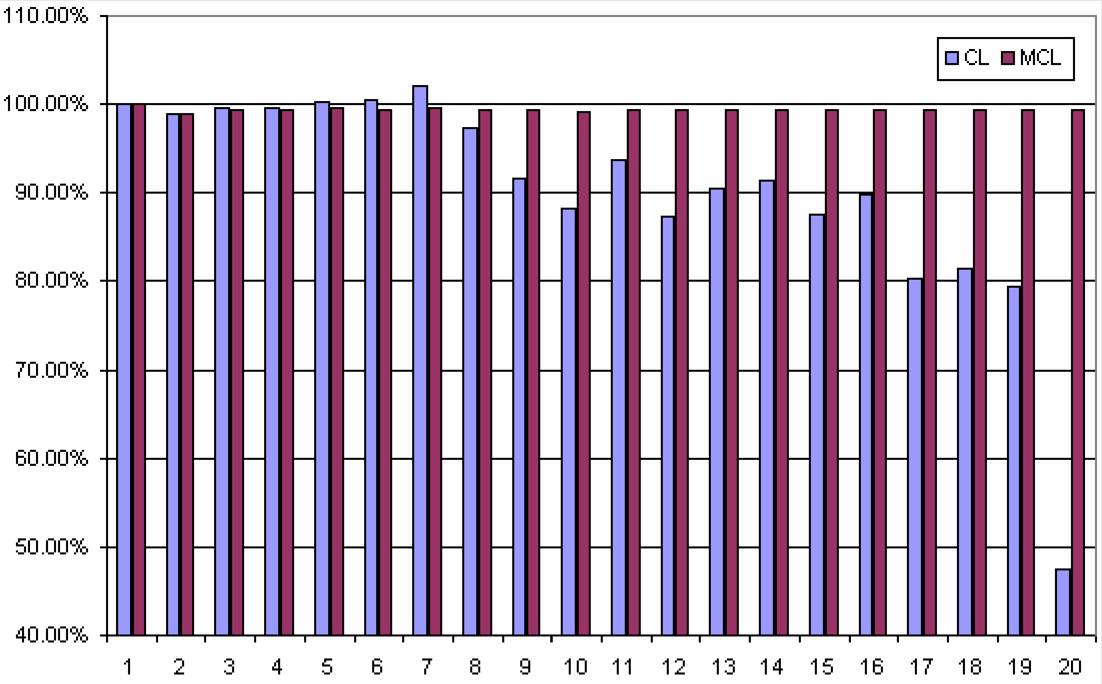
Variance factor (Scale parameter)

I/P residual

Munich Chain Ladder: Results

Ultimate P/I ratios converging towards 100%

→ consistent development of paid and incurred.



Outline

Chain Ladder: comments

A first solution: Munich Chain Ladder

JAB Chain

JAB Chain

We further developped the MCL method to the JAB method by taking into account:

- time-varying slopes
- integration into one model
- joint estimation of all factors

Time-varying slopes

– MCL uses only one slope in the correction term: this might be a little bit too rough. We expect the incurred process more informative during the first DYs, when the paid are still low. Thus, having a single coefficient is “averaging” the correction over the whole development period.

– On the other hand, estimating a different coefficient for each DY may be difficult for the last DYs (few data) → smoothing.

Integration into one model

– the incurred process is thought to be informative to the paid process, but the latter does not necessary add some relevant information to the incurred process.

– the incurred process is modeled separately and differently, e.g. by incorporating also calendar year effects.

Joint estimation of all factors

- MCL lies on regression residual analysis, and thus implies estimating some parameters, calculating the residuals, then deducing the slopes to finally predict the future paid and incurred amounts.
- We adopted a single model:
  - simultaneous estimation of all parameter, taking into account possible dependencies between them.
  - direct calculations for model validation and standard error

JAB Chain

We propose a model in accordance with the three previous remarks:

- the correction applied to the DF depends on the DY
- the incurred process is modeled separately
- we use a "one-step model"

Chain ladder

Chain ladder
JAB

$$\begin{cases} I_{i,j+1} = I_{i,j} \cdot f_j^I + \varepsilon_j^I \\ P_{i,j+1} = P_{i,j} \cdot \left(\alpha_j^P + \beta_j \left(Q_{i,j} - q_j\right)\right) + \varepsilon_{i,j}^P \end{cases}$$

This last equation can be rewritten to:

$$\frac{P_{i,j+1}}{P_{i,j}} = \alpha_j + \beta_j \left(Q_{i,j} - q_j\right) + \frac{\varepsilon_{i,j}^P}{P_{i,j}} \quad \text{where} \quad \begin{cases} i, j = 1 \ldots n-1 \\ \left\{\varepsilon_{i,j}\right\}_{j=1\ldots n-1} \sim \left[0, \sigma_j^{P\,2} \cdot P_{i,j}\right] \end{cases}$$

heteroscedasticity: the variance is proportional to the payments $P_{i,j}$

JAB Chain

The model allows us to assume smoothingly varying α_j and β_j coefficients. The sequence of states is defined by two linear transition equations:

$$\alpha_{j+1} = \alpha_j + \varepsilon_{\alpha_j} \quad \text{where} \quad \varepsilon_{\alpha_j} \sim \left[o, \sigma_{\alpha}^2\right]$$

$$\beta_{j+1} = \beta_j + \varepsilon_{\beta_j} \quad \text{where} \quad \varepsilon_{\beta_j} \sim \left[0, \sigma_{\beta}^2\right]$$

α_j and β_j are defined as random walks, while the noises are assumed to be white noise processes, such that their expected value is zero and their variance is constant.

Parameters estimation

We try to minimize the following quantity:

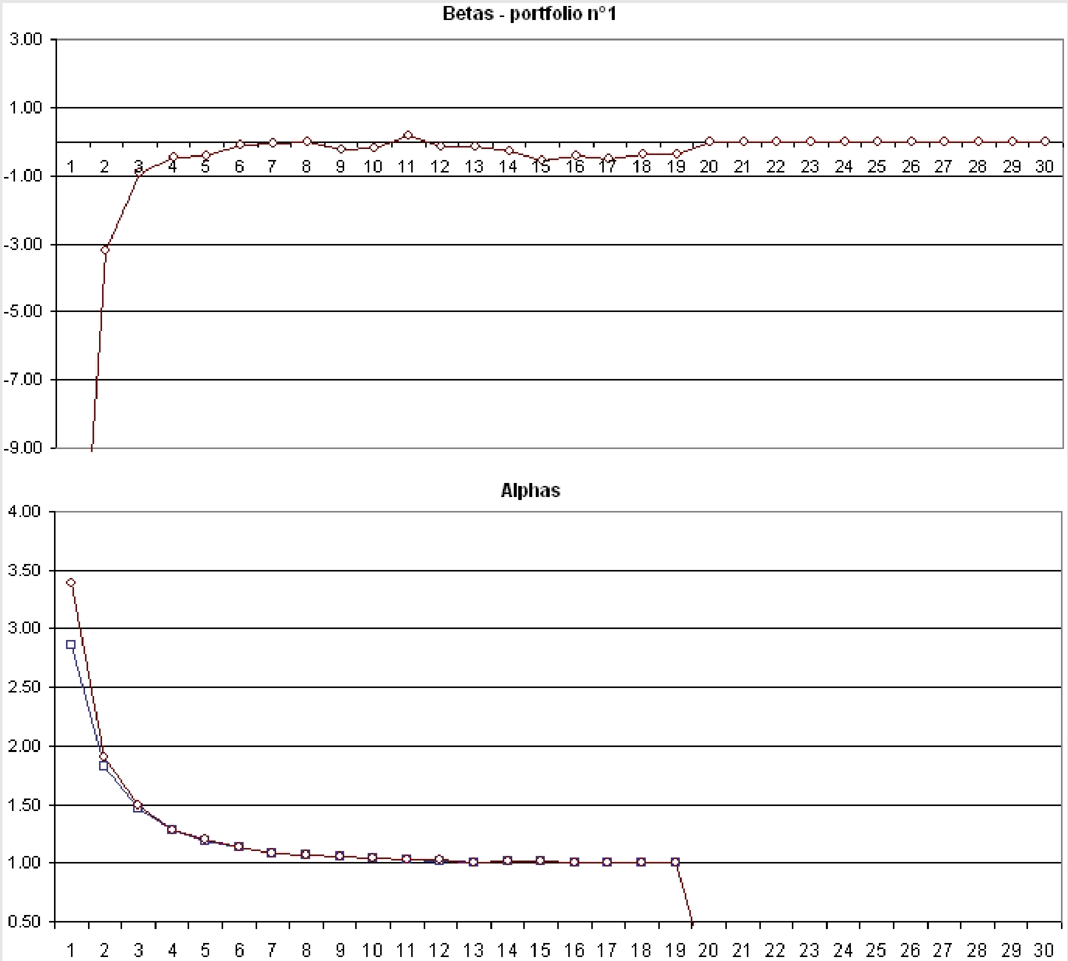
$$M = \sum_{i,j} \omega_{i,j} \Big(P_{i,j+1} - \underbrace{\beta_j \cdot (Q_{i,j} - q_j) \cdot P_{i,j} - \alpha_j \cdot P_{i,j}}_{\hat{P}_{i,j+1}} \Big)^2 + \frac{1}{\sigma_\alpha^2} \sum_{i,j} (\alpha_{j+1} - \alpha_j)^2 + \frac{1}{\sigma_\beta^2} \sum_{i,j} (\beta_{j+1} - \beta_j)^2$$

- First term: measures the fit between the model predictions and reality
- Last two terms: penalizes volatile estimations of β_j and α_j

This can be achieved in a “simple” way by rewriting this into a matrix form.

JAB Chain: results

Parameters estimation: α_j β_j

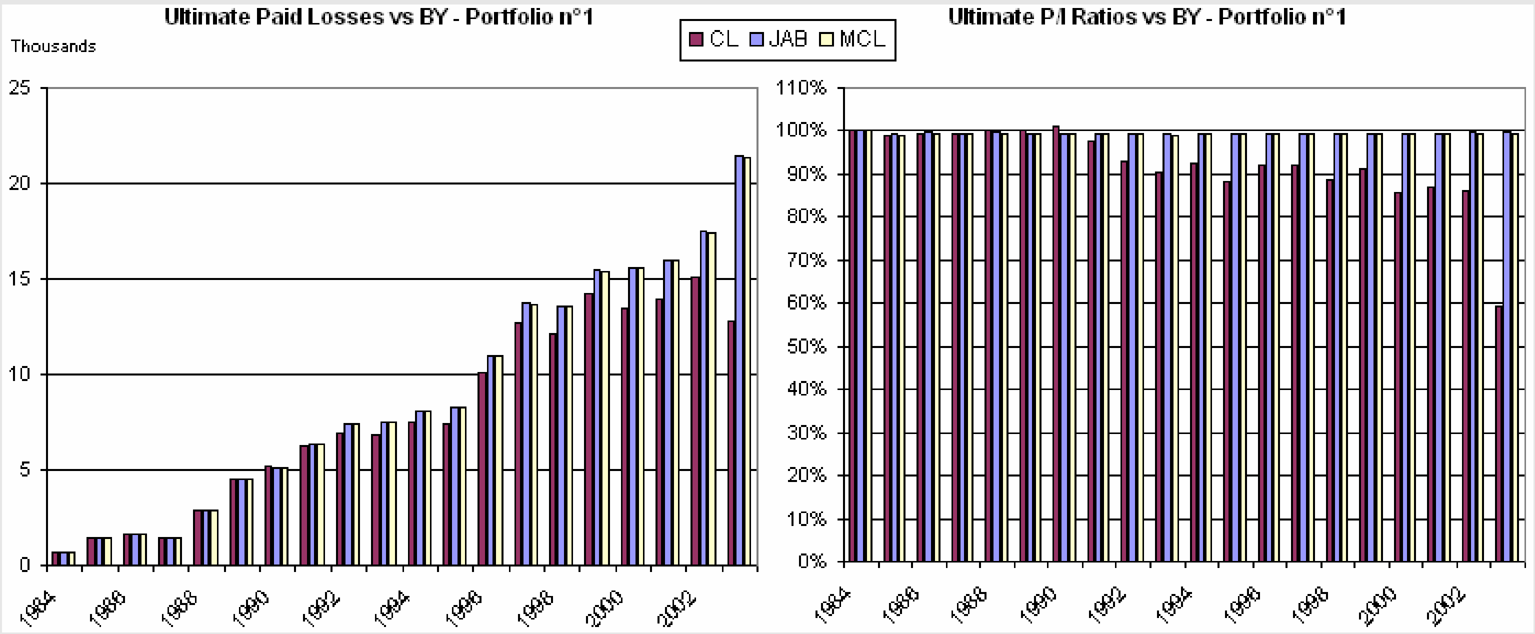


- β_j not constant
- growing β_j (in absolute value)
- β_j converging towards 0

JAB Chain: results

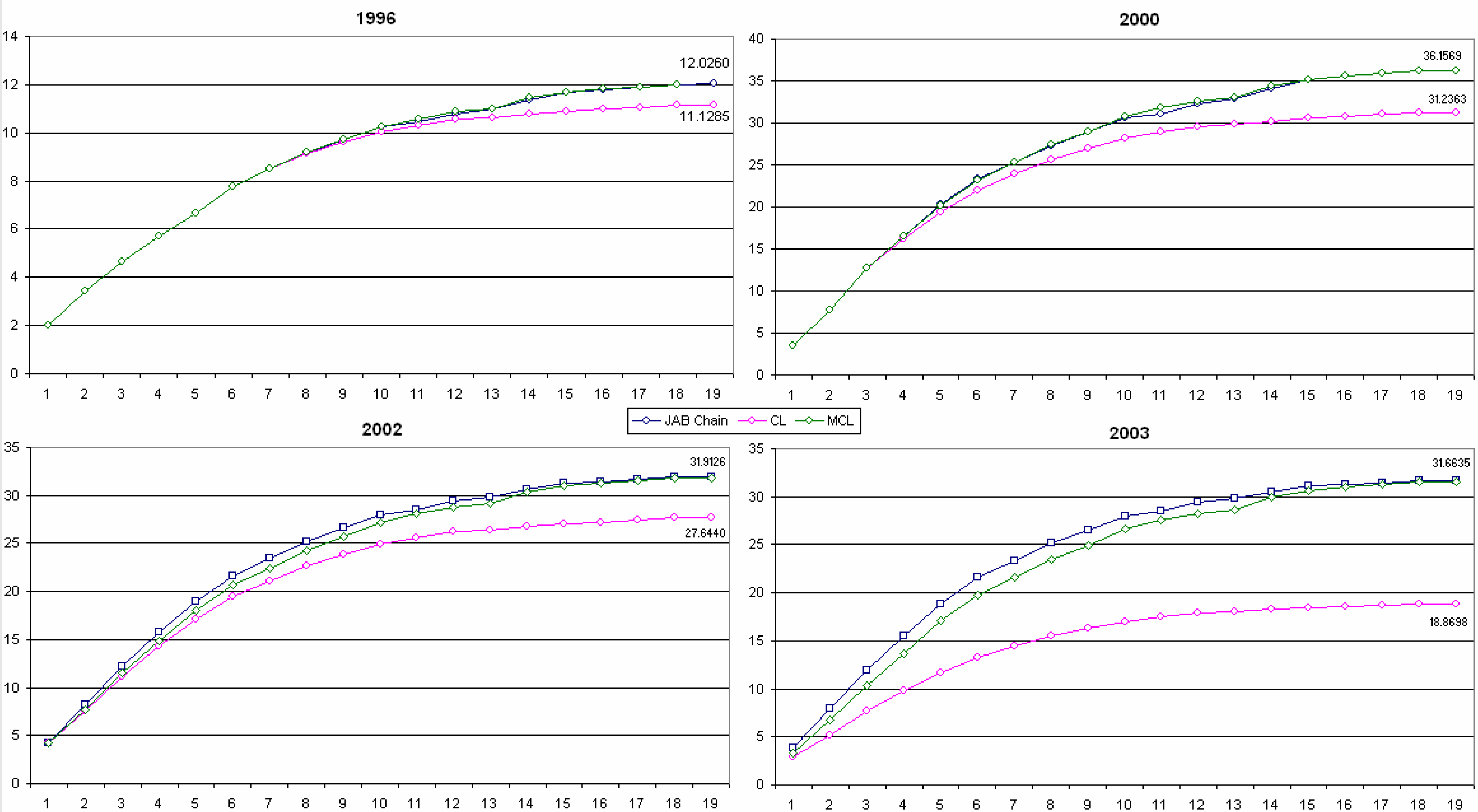
Ultimate paid and ultimate P/I ratios

- JAB predictions close to the MCL one, greater than CL
- ultimate P/I ratios around 100% → consistent development of paid and incurred processes



JAB Chain: results

Patterns



Outlook

This idea, first with Munich Chain Ladder and now with our method, is relatively recent and the field is still to be explored. Here are some potential developments, among others:

– incurred DF could be incorporated and estimated simultaneously in another model.

– extension to non-gaussian paid and incurred processes

– instead of assuming the coefficients to vary over the DY, the coefficient β_j may also be assumed to vary a different time scales or even other metric variables.