Two binomial methods for evaluating the aggregate claims distribution in De Pril's individual risk model

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De Pril's (AB 1989) individual risk model

Independent policies

Each policy can have at most one claim during the period

Two-way model

Cell (i, j) $(i = 1, 2, \ldots, I;\ j = 1, 2, \ldots, J)$ has

- n_{ij} policies
- claim probability π_j
- severity distribution (probability function) h_i on $\{1, 2, \ldots, m_i\}$

Problem: Find the aggregate claims distribution f of the portfolio

Some notation

Compound distribution with counting distribution p and severity distribution h:

$$p \vee h = \sum_{n=0}^{\infty} p(n)\, h^{n*}$$

Cell (i, j):

- Claim number distribution: $p_j(1) = \pi_j = 1 - p_j(0)$
- Aggregate claims distribution for policy: $g_{ij} = p_j \vee h_i$
- Aggregate claims distribution for cell: $f_{ij} = g_{ij}^{n_{ij}*}$

Aggregate claims distribution for all policies with fixed i: $f_i = *_{j=1}^{J} f_{ij}$

J_i = number of non-empty cells (i, j) with fixed i

Evaluate $f = *_{i=1}^{I} f_i$

Known methods

- Brute force convolution

- De Pril's (AB 1989) two exact methods based on De Pril transforms:

  1. Recursion for the De Pril transform of each g_{ij}

  2. Closed-form expression for the De Pril transform of each g_{ij}

- Dhaene-Vandebroek's (IME 1995) method

Comparison

How do we compare the methods?

Although not perfect, by counting dot operations

Brute force convolution and De Pril's second method usually inefficient

Intuitively, an optimal method should utilise the information in the two-way structure

Not the case with De Pril's first method and Dhaene-Vandebroek's method

Could we do better?

Two new binomial methods where the information is used

Binomial methods

f_i compound distribution with severity distribution h_i and counting distribution convolution of J_i binomial distributions

$$f_i(0) = \prod_{j=1}^{J} \left(1 - \pi_j\right)^{n_{ij}}$$

Let

$$c_i(y) = \sum_{u=\{y/m_i\}}^{\min(J_i, y)} a_i(u)\, h_i^{u*}(y)\,;\; d_i(y) = y \sum_{u=\{y/m_i\}}^{\min(J_i, y)} \frac{b_i(u)}{u} h_i^{u*}(y)\,.\;\; (y = 1, 2, \ldots, J_i m_i)$$

For evaluation of a_i and b_i, see paper.

First binomial method

Evaluate f_i by

$$f_i(x) = \sum_{y=1}^{\min(J_i m_i, x)} \left(c_i(y) + \frac{d_i(y)}{x} \right) f_i(x-y). \qquad (x = 1, 2, \dots)$$

Evaluate $f = *_{i=1}^{I} f_i$ by brute force convolution.

Second binomial method

Evaluate f by

$$f(x) = \frac{1}{x} \sum_{i=1}^{I} \psi_i(x) \qquad (x = 1, 2, \dots)$$

with

$$\psi_i(x) = \sum_{y=1}^{\min(J_i m_i, x)} \left((y c_i(y) + d_i(y)) f(x-y) + c_i(y) \psi_i(x-y) \right).$$

$$(x = 1, 2, \dots; \; i = 1, 2, \dots, I)$$

Assumptions for comparison of methods

$m_i = \infty$ for all i

Number of dot operations for evaluating $f(x)$ for $x = 0, 1, 2, \ldots., s$

Let s go to infinity

Results

1. De Pril's exact methods and second binomial method discarded.

2. Brute force convolution discarded unless only one or two cells with more than one policy.

3. First binomial method better than Dhaene-Vandebroek's method when $J_\bullet > 5I - 4$.

Very close race between binomial methods, De Pril's first method, and Dhaene-Vandebroek's method

Combined methods: Dhaene-Vandebroek when $J_i < 5$; binomial when $J_i \geq 5$.

Combined methods

Combination always at least as good as Dhaene-Vandebroek
Better than first binomial when

$$\sum_{\{i:J_i<5\}} (5 - J_i) > 4$$

Equally good with each of the binomial methods, but simpler programing with second