Modeling the Underwriting Cycle

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Underwriting Cycles Are a Reality

- US liability crisis mid 1980s.
- Property catastrophe reinsurance crisis early 1990s.
- Significant hardening of the reinsurance market early 2000s.
- ...

Causes of Underwriting Cycles

- Irrational expectations
- Institutional interventions
- Feedback of profit and losses into surplus
- External factors
- Underwriting profits follow an AR(2) process

Model Assumptions

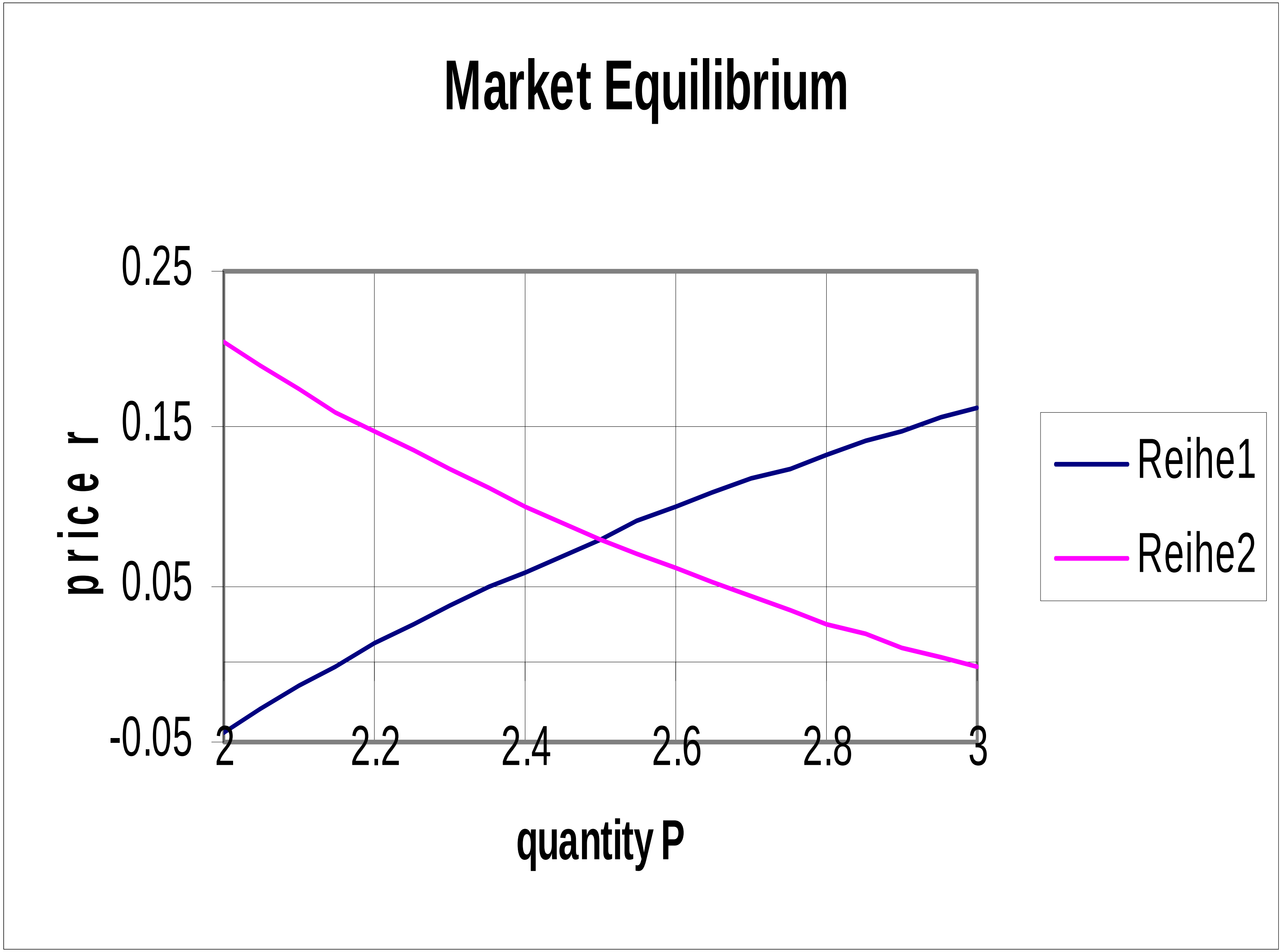
- Insurance markets are driven by supply and demand
- Supply is driven by the amount of capital
- Demand is driven by the size of the economy
- Profits are defined by the equilibrium of supply and demand
- Profits feedback into capital

Model

- Insurance quantity = pure risk premium (P)
- RBC = P/leverage
- Insurance price = rate of return on RBC (r)
- Supply function
- Demand function

$$P - \kappa^s C = \varepsilon^s \left(r \frac{P}{\kappa^s} - r^s \frac{P}{\kappa^s}\right)$$

$$P - P^0 = -\varepsilon^d \left(r \frac{P}{\kappa^s} - r^d \frac{P}{\kappa^d}\right)$$


Market Equilibrium
0.25
0.15
0.05
-0.05
price r
2
2.2
2.4
2.6
2.8
3
quantity P
Reihe1
Reihe2

Market Equilibrium

$$r^m = \frac{(\varepsilon^d C) r^{d'} + (\varepsilon^s \frac{P^0}{\kappa^s}) r^s}{\varepsilon^d C + \varepsilon^s \frac{P^0}{\kappa^s}} + \frac{P^0 - \kappa^s C}{\varepsilon^d C + \varepsilon^s \frac{P^0}{\kappa^s}}$$

$$P^m = \frac{\varepsilon^s P^0 + \varepsilon^d (\kappa^s C)}{\varepsilon^s + \varepsilon^d + \varepsilon^s \varepsilon^d (\frac{r^s}{\kappa^s} - \frac{r^d}{\kappa^d})}$$

$$\lambda^m = r^m \frac{P^m}{\kappa^s} = \frac{(\varepsilon^d \frac{r^d}{\kappa^d} - 1)\kappa^s C + (\varepsilon^s \frac{r^s}{\kappa^s} + 1) P^0}{\varepsilon^s + \varepsilon^d + \varepsilon^s \varepsilon^d (\frac{r^s}{\kappa^s} - \frac{r^d}{\kappa^d})}$$

Feedback of Profits into Capital

$$C_{t+1} = C_t + \rho(\lambda_t^m + r^f C_t)$$

$$C_{t+1} = C_t + \rho(\lambda_t^m + \tilde{\varepsilon}_t + r^f C_t) + \tilde{\vartheta}_{t+1}$$

$$C_{t+1} = C_t + \rho(\alpha\lambda_t^m + (1-\alpha)\lambda_{t-1}^m + \tilde{\varepsilon}_{t,t} + \tilde{\varepsilon}_{t-1,t} + r^f C_t) + \tilde{\vartheta}_{t+1}$$

$$P_t^0 = \gamma^t P^0$$

$$Def: \quad \bar{C}_t = \frac{C_t}{\gamma^t} \qquad st'zed \quad process$$

$$Theorem: \quad \left\{\bar{C}_t - \mu\right\} \quad is \quad AR(2) \quad process$$

Simplified Model

$$Assume: \qquad Var(\tilde{\varepsilon}_{t,s}) = Var(\tilde{\vartheta}_u) = 0$$

$$\bar{C}_{t+1} - \frac{1}{\gamma}(1+\rho r^f + \rho a\alpha)\bar{C}_t - \frac{1}{\gamma^2}\rho a(1-\alpha)\bar{C}_{t-1} = (\frac{\alpha}{\gamma} + \frac{(1-\alpha)}{\gamma^2})\rho b P^0$$

$$a = \frac{\varepsilon^d r^{d'} - \kappa^s}{\varepsilon^s + \varepsilon^d + \varepsilon^s\varepsilon^d(\frac{r^s}{\kappa^s} - \frac{r^d}{\kappa^d})} \qquad b = \frac{1 + \varepsilon^s \frac{r^s}{\kappa^s}}{\varepsilon^s + \varepsilon^d + \varepsilon^s\varepsilon^d(\frac{r^s}{\kappa^s} - \frac{r^d}{\kappa^d})}$$

Linear difference equation with cst coefficients

Simplified Model (cont‘d)

$equilibrium\ \ pt\ \ of\ \ difference\ \ equation$

$$\bar{C}^* = \frac{\left(\frac{\alpha}{\gamma} + \frac{(1-\alpha)}{\gamma^2}\right)\rho b}{1 - \frac{1}{\gamma}(1 + \rho r^f + \rho a\alpha) - \frac{1}{\gamma^2}\rho a(1-\alpha)} P^0$$

$characteristic\ \ equation$

$$\lambda^2 - \frac{1}{\gamma}(1 + \rho r^f + \rho a\alpha)\lambda - \frac{1}{\gamma^2}\rho a(1-\alpha) = 0$$

$stability \quad \Leftrightarrow \quad \left|\lambda_i\right| \prec 1 \ \ for\ \ all\ \ char.\ roots$

$instability:\ reporting\ \ lags\ \ \&\ \ capital\ \ in/outflows$

Simplified Model (cont'd)

market price corresp. to equilibrium pt

$$r^* = \frac{\frac{1}{r^s} + \frac{\varepsilon^s}{\kappa^s}}{\rho \frac{\frac{\alpha}{\gamma} + \frac{(1-\alpha)}{\gamma^2}}{(1 + \frac{1}{\gamma} - \frac{\rho}{\gamma} r^f)} + \frac{\varepsilon^s}{\kappa^s}} r^s$$

$$r^* \succ r^s \quad \Leftrightarrow \quad \gamma - 1 \succ \rho(r^f + r^s(\alpha + \frac{1-\alpha}{\gamma}))$$

Summary of Findings

Underwriting cycles are caused by:

- Supply side factors (retained earnings, capital flows)
- Demand side factors (fluctuations in base demand growth)
- External factors (changes in the value of financial assets, adverse loss developments)
- Interaction of the above factors