Simplified estimation of structure parameters in hierarchical credibility

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Agenda

- Hierarchical credibility
- Pseudo-estimators of structure parameters
- New unbiased estimators
- Numerical example
- Conclusions

Hierarchical credibility structure

Sector level j

$$\Downarrow$$

Cell level k

$$\Downarrow$$

Exposure unit t

Application

Sector:	Car brand	(E.g. $j =$ Aston Martin)
		$\Downarrow$
Cell:	Car model	(E.g. $k =$ Aston Martin DB5)
		$\Downarrow$
Unit:	Individual car	(E.g. $t =$ Switzerland LU 6789)

Structure parameters (Variance components)

Y_{jkt} observed risk premium, claim frequency or severity

w_{jkt} exposure weight $\quad\quad$ $\text{Var}(Y_{jkt}) = b + a + \frac{\sigma^2}{w_{jkt}}$

Sector: $b =$ sector level variance

$$\Downarrow$$

Cell: $a =$ cell level variance

$$\Downarrow$$

Unit: $\sigma^2 =$ unit level variance

Credibility factors

- Cell level: $z_{jk} \doteq \frac{w_{jk\cdot}}{w_{jk\cdot} + \sigma^2/a}$

- Sector level: $q_j \doteq \frac{z_{j\cdot}}{z_{j\cdot} + a/b}$

- Used in credibility estimators (details omitted here)

The problem:

Estimate the structure parameters a, b and σ^2

Parameter	Standard	New
σ^2	Unbiased	= Unbiased
a	Pseudo	Unbiased
b	Pseudo	Unbiased

Usual unbiased estimator of σ^2

$$\hat{\sigma}^2 = \frac{1}{\sum_j \sum_k (T_{jk} - 1)} \sum_{j=1}^{J} \sum_{k=1}^{K_j} \sum_{t=1}^{T_{jk}} w_{jkt} (Y_{jkt} - \overline{Y}_{jk\cdot}^{w})^2$$

where

$$\overline{Y}_{jk\cdot}^{w} = \frac{\sum_t w_{jkt} Y_{jkt}}{\sum_t w_{jkt}}$$

Pseudo-estimator of a

$$a = E\left[\frac{1}{\sum_j (K_j - 1)} \sum_{j=1}^{J} \sum_{k=1}^{K_j} z_{jk} (\overline{Y}_{jk\cdot}^{w} - \overline{Y}_{j\cdot\cdot}^{zw})^2\right]$$

where

$$z_{jk} \doteq \frac{w_{jk\cdot}}{w_{jk\cdot} + \sigma^2/a}$$

$$\overline{Y}_{j\cdot\cdot}^{zw} = \frac{\sum_k z_{jk} \overline{Y}_{jk\cdot}^{w}}{\sum_k z_{jk}}$$

Omit $E[\cdot]$ and solve iteratively for a.

Unbiased estimator of a

$$\widehat{a} = \frac{\sum_j \sum_k w_{jk\cdot}(\overline{Y}^{w}_{jk\cdot} - \overline{Y}^{ww}_{j\cdot\cdot})^2 - \widehat{\sigma}^2 \sum_j (K_j - 1)}{w_{\cdot\cdot\cdot} - \sum_j (\sum_k w^2_{jk\cdot})/w_{j\cdot\cdot}}$$

where

$$\overline{Y}^{ww}_{j\cdot\cdot} = \frac{\sum_k w_{jk\cdot} \overline{Y}^{w}_{jk\cdot}}{\sum_k w_{jk\cdot}}$$

Cf. Bühlmann-Straub estimator (non-hierarchical)

Pseudo-estimator of b

$$b \;=\; E\left[\frac{1}{J-1}\sum_{j=1}^{J} q_j(\overline{Y}_{j\cdot\cdot}^{zw} - \overline{Y}_{\cdot\cdot\cdot}^{qzw})^2\right]$$

where

$$q_j \doteq \frac{z_{j\cdot}}{z_{j\cdot} + a/b}$$

$$\overline{Y}_{\cdot\cdot\cdot}^{qzw} = \frac{\sum_j q_j \overline{Y}_{j\cdot\cdot}^{zw}}{\sum_j q_j}$$

Omit $E[\cdot]$ and solve iteratively for b.

Unbiased type estimator of b

$$\widehat{b} = \frac{\sum_j z_{j\cdot}(\overline{Y}^{zw}_{j\cdot\cdot} - \overline{Y}^{zzw}_{\cdot\cdot\cdot})^2 - \widehat{a}(J-1)}{z_{\cdot\cdot} - \sum_j z^2_{j\cdot}/z_{\cdot\cdot}}$$

where

$$\overline{Y}^{zzw}_{\cdot\cdot\cdot} = \frac{\sum_j z_{j\cdot}\overline{Y}^{zw}_{j\cdot\cdot}}{\sum_j z_{j\cdot}}$$

$z_{j\cdot}$ assumed known here (depend on a and σ^2)

Comparison

Dubey & Gisler (1981) found in the non-hierarchical Bühlmann-Straub case

- "neither of the two estimators is universally better than the other" (in terms of variance)
- *Pseudo-estimator* has one and only one positive solution iff *unbiased estimator* is strictly positive
- Else *pseudo-estimators* converges to zero

Numerical example

Artificial population from Dannenburg, Kaas & Goovaerts (1996), Table 4.1

(Equal weight case in parenthesis.)

	True value	*Pseudo-est.*	*Unbiased est.*
a	1.000	1.152 (1.093)	1.209 (1.093)
b	25.000	25.309 (25.259)	25.300 (25.259)

Conclusion – Pseudo- vs. New Unbiased

- Both are reasonable

- Presumably none is univerally best in MSE

- Unbiased is easier to compute

- Pseudo-estimator is conceptually more difficult

References

Dannenburg, D.R., Kaas, R. & Goovaerts, M.J. (1996): *Practical actuarial credibility models.* IAE, Amsterdam.

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