

Recent Developments in Claims Reserving

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September 6, 2005



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Overview

1. Basics

2. Stochastics of Chain Ladder

3. Parameter Estimation for Bornhuetter/Ferguson (*new*)

4. Paid or Incurred Data?

The basic problem of claims reserving

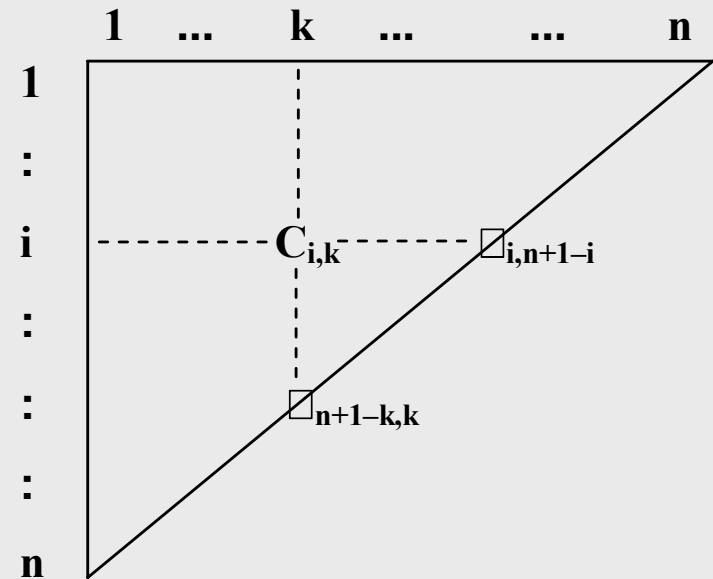
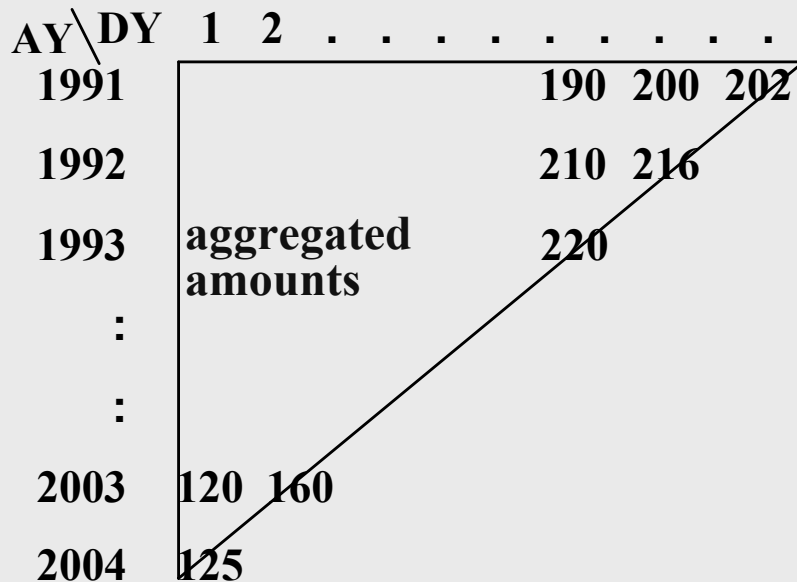
Normally, the ultimate claims amount of an accident year is not yet known at the end of that year.

Especially in Liability insurance,

the claims settlement may last several years due to

- bodily injuries and/or long lasting court trials**
- the possible time lag between occurrence and manifestation.**

This leads to the so-called run-off triangle



$C_{i,k}$ = total amount of payments
for claims of accident year (AY) i
after k years of development (DY)

Goal of claims reserving:

to complete the run-off triangle to a square
(for accounting and tariff calculation purposes)

Basic idea:

to translate the development pattern of older AYs
to the younger ones.

By far the most popular claims reserving methods are

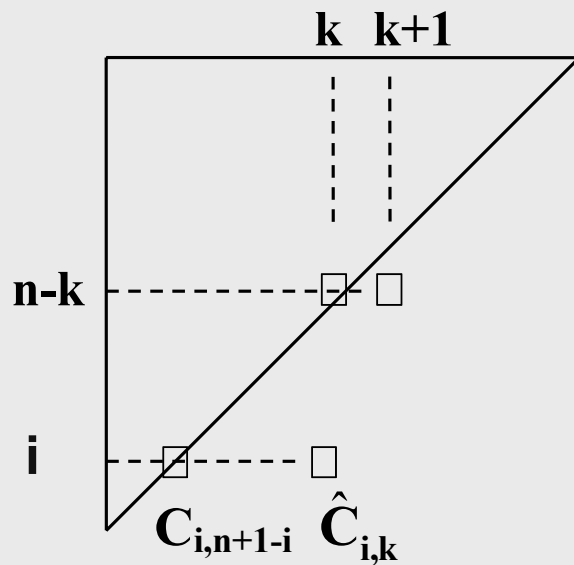
- the Chain Ladder (CL) method and
 - the Bornhuetter/Ferguson (B/F) method,
- each applied to the paid as well as to the incurred triangle.

The Chain Ladder method

Mean observed increase $k \rightarrow k+1$: $\hat{f}_{k+1} = \sum_{j=1}^{n-k} C_{j,k} \cdot \frac{C_{j,k+1}}{C_{j,k}} / \sum_{j=1}^{n-k} C_{j,k}$

$$= \sum_{j=1}^{n-k} C_{j,k+1} / \sum_{j=1}^{n-k} C_{j,k}$$

$$\hat{C}_{i,k+1} = \hat{C}_{i,k} * \hat{f}_{k+1}$$



Start with $C_{i,n+1-i}$

i.e. $\hat{C}_{i,n} = C_{i,n+1-i} \cdot \hat{f}_{n+2-i} \cdot \dots \cdot \hat{f}_n$

This method has been heavily criticized:

First by Jim STANARD (PCAS 1985):

In the stochastic model $\frac{\mathbf{E}(C_{i,k})}{\mathbf{E}(C_{i,k-1})} = \mathbf{f}_k$,

the estimators $\hat{\mathbf{f}}_k$ are biased,

$$\text{e.g. } \hat{\mathbf{f}}_n = \frac{C_{1,n}}{C_{1,n-1}} \quad \text{and} \quad \mathbf{E}(\hat{\mathbf{f}}_n) = \mathbf{E}\left(\frac{C_{1,n}}{C_{1,n-1}}\right) \neq \frac{\mathbf{E}(C_{1,n})}{\mathbf{E}(C_{1,n-1})} = \mathbf{f}_n.$$

Then by Erwin STRAUB in his book

„Non-Life Insurance Mathematics“ (1988):

„The product $\hat{C}_{i,n} = C_{i,n+1-i} \cdot \hat{f}_{n+2-i} \cdot \dots \cdot \hat{f}_n$ is estimated factorwise.

This may lead to a serious bias since

$$E(\hat{f}_{n+2-i} \cdot \dots \cdot \hat{f}_n) = E(\hat{f}_{n+2-i}) \cdot \dots \cdot E(\hat{f}_n)$$

holds only if the \hat{f}_k are uncorrelated

which is typically not the case in applications.“

$$\hat{f}_k = \frac{\sum_{i=1}^{n-k+1} C_{i,k}}{\sum_i C_{i,k-1}} \quad \longleftrightarrow \quad \frac{\sum_i C_{i,k+1}}{\sum_{i=1}^{n-k} C_{i,k}} = \hat{f}_{k+1}$$

**The decisive idea to deal with these critiques
is to use conditional expectations.**

This fits exactly the stepwise procedure of CL:

Estimate $C_{i,k}$ only if $C_{i,k-1}$ has already been estimated.

Then in $\frac{C_{i,k}}{C_{i,k-1}}$ only the numerator is a random variable,

i.e. the model is $E\left(\frac{C_{i,k}}{C_{i,k-1}} \middle| C_{i,k-1}\right) = f_k$ instead of $\frac{E(C_{i,k})}{E(C_{i,k-1})} = f_k$.

This model immediately solves STANARD's problem:

$$\mathbf{E}(\hat{\mathbf{f}}_n) = \mathbf{E} \left(\frac{\mathbf{C}_{1,n}}{\mathbf{C}_{1,n-1}} \right) = \mathbf{E} \left(\mathbf{E} \left(\frac{\mathbf{C}_{1,n}}{\mathbf{C}_{1,n-1}} \middle| \mathbf{C}_{1,n-1} \right) \right) = \mathbf{E}(\mathbf{f}_n) = \mathbf{f}_n ;$$

similarly, the other $\hat{\mathbf{f}}_k$ are shown to be unbiased.

This model
$$\mathbf{E} \left(\frac{\mathbf{C}_{i,k}}{\mathbf{C}_{i,k-1}} \middle| \mathbf{C}_{i,k-1} \right) = \mathbf{f}_k$$

can be rewritten as
$$\mathbf{E}(\mathbf{C}_{i,k} | \mathbf{C}_{i,k-1}) = \mathbf{C}_{i,k-1} \cdot \mathbf{f}_k$$

and thus explains why
$$\hat{\mathbf{C}}_{i,n} = \mathbf{C}_{i,n+1-i} \cdot \hat{\mathbf{f}}_{n+2-i} \cdot \dots \cdot \hat{\mathbf{f}}_n$$

uses $\mathbf{C}_{i,n+1-i}$ **as starting value and not** $\hat{\mathbf{E}}(\mathbf{C}_{i,n+1-i})$.

From the above model, we further deduce

$$\begin{aligned} \mathbf{E} \left(\frac{C_{i,k}}{C_{i,k-1}} \cdot \frac{C_{i,k+1}}{C_{i,k}} \right) &= \mathbf{E} \left(\mathbf{E} \left(\frac{C_{i,k}}{C_{i,k-1}} \cdot \frac{C_{i,k+1}}{C_{i,k}} \middle| C_{i,k-1}, C_{i,k} \right) \right) \\ &= \mathbf{E} \left(\frac{C_{i,k}}{C_{i,k-1}} \cdot \underbrace{\mathbf{E} \left(\frac{C_{i,k+1}}{C_{i,k}} \middle| C_{i,k-1}, C_{i,k} \right)}_{\mathbf{f}_{k+1}} \right) \\ &= \mathbf{f}_k \cdot \mathbf{f}_{k+1} = \mathbf{E} \left(\frac{C_{i,k}}{C_{i,k-1}} \right) \cdot \mathbf{E} \left(\frac{C_{i,k+1}}{C_{i,k}} \right) \end{aligned}$$

which shows that STRAUB's critique does not hold here.

Conclusion 1

In the stochastic model
$$\mathbf{E} \left(\frac{C_{i,k}}{C_{i,k-1}} \middle| C_{i,k-1} \right) = \mathbf{f}_k$$

or more precisely
$$\mathbf{E} \left(\frac{C_{i,k}}{C_{i,k-1}} \middle| C_{i,1}, C_{i,2}, \dots, C_{i,k-1} \right) = \mathbf{f}_k$$

both of the points criticized do not hold anymore!

Thus, the CL method has a sound stochastic foundation.

What is the appropriate model for the variance?

$$\hat{\mathbf{f}}_k = \sum_{i=1}^{n+1-k} \frac{C_{i,k-1}}{\sum_{j=1}^{n+1-k} C_{j,k-1}} \cdot \frac{C_{i,k}}{C_{i,k-1}} \text{ is a weighted average.}$$

A linear estimator has minimum overall variance iff the weights are inversely proportional to the variance of the summands.

$$\rightarrow \text{Var} \left(\frac{C_{i,k}}{C_{i,k-1}} \middle| C_{i,k-1} \right) \sim \frac{1}{C_{i,k-1}}, \text{ i.e. } = \frac{\sigma_k^2}{C_{i,k-1}}$$

$$\text{where } \hat{\sigma}_k^2 = \frac{1}{n-k} \sum_{i=1}^{n+1-k} C_{i,k-1} \left(\frac{C_{i,k}}{C_{i,k-1}} - \hat{\mathbf{f}}_k \right)^2$$

With this stochastic model the prediction error can be calculated:

$$\left(\text{s.e.}(\hat{C}_{i,n})\right)^2 = \widehat{\text{Var}}(C_{i,n} | n+1-i) + \widehat{\text{Var}}(\hat{C}_{i,n} | n+1-i)$$

process variance estimation variance



Recursion

$$\left(\text{s.e.}(\hat{C}_{i,k+1})\right)^2 = \left(\text{s.e.}(\hat{C}_{i,k})\right)^2 \hat{f}_{k+1}^2 + \hat{C}_{i,k}^2 \left\{ \frac{\hat{\sigma}_{k+1}^2}{\hat{C}_{i,k}} + \frac{\hat{\sigma}_{k+1}^2}{\sum_{j=1}^{n-k} C_{j,k}} \right\}$$

starting with $\left(\text{s.e.}(\hat{C}_{i,n+1-i})\right)^2 = 0$.

(Thomas MACK, AB 1993)

Extension of the model to the whole portfolio

Claims reserving is usually done at homogeneous subportfolios.

As these might be correlated, we need a model for the aggregation of the prediction errors of several subportfolios:

$$\begin{aligned} \left(\text{s.e.}(\hat{C}_{in} + \hat{D}_{in}) \right)^2 &= \widehat{\text{Var}}(C_{in} + D_{in} | n+1-i) + \widehat{\text{Var}}(\hat{C}_{in} + \hat{D}_{in} | n+1-i) \\ &= \widehat{\text{Var}}(C_{in} | n+1-i) + 2 \widehat{\text{Cov}}(C_{in}, D_{in} | n+1-i) + \widehat{\text{Var}}(D_{in} | n+1-i) + \\ &+ \widehat{\text{Var}}(\hat{C}_{in} | n+1-i) + 2 \widehat{\text{Cov}}(\hat{C}_{in}, \hat{D}_{in} | n+1-i) + \widehat{\text{Var}}(\hat{D}_{in} | n+1-i) \end{aligned}$$

Like the variances, also the covariances can be calculated recursively

under the model assumption
$$\text{Cov}\left(\frac{C_{i,k}}{C_{i,k-1}}, \frac{D_{i,k}}{D_{i,k-1}} \middle| \mathbf{k} - \mathbf{1}\right) = \frac{\rho_k}{\sqrt{C_{i,k-1} D_{i,k-1}}}$$

i.e. a fixed coefficient of correlation between $\frac{C_{ik}}{C_{i,k-1}}, \frac{D_{ik}}{D_{i,k-1}}, 1 \leq i \leq n$,

(straightforward generalisation of the variance assumption above).

Moreover, the estimation of the development factors f_k, \dots

can be improved using the data of all subportfolios simultaneously.

(C. BRAUN, AB 2004; E. KREMER, AC 2005; PRÖHL/SCHMIDT, AC 2005)

Conclusion 2

The Chain Ladder method has a sound stochastic foundation which allows the calculation of the prediction error for the whole portfolio of the insurance company.

Now let us look at the other standard method due to BORNHUETTER and FERGUSON (PCAS 1972).

Notation for Bornhuetter/Ferguson

v_i = premium volume of acc. year i , $1 \leq i \leq n$

$C_{i,k}$ = cum. loss amount of acc. year i after dev. year k , $1 \leq k \leq n$

$U_i = C_{i,n}$ = ult. loss amount (i.e. including tail)

$C_{i,n+1-i}$ = currently known loss amount (paid or incurred)

$S_{i,k} = C_{i,k} - C_{i,k-1}$ = incremental loss amount, $C_{i,0} := 0$

$R_i = C_{i,n} - C_{i,n+1-i} = S_{i,n+2-i} + \dots + S_{i,n}$ = (true) loss reserve

B/F aims at avoiding a major weakness of CL:

The CL reserve $\hat{R}_i^{\text{CL}} = C_{i,n+1-i} (\hat{f}_{n+2-i} \cdot \dots \cdot \hat{f}_n - 1)$

heavily depends on $C_{i,n+1-i}$,

e.g. yields a nonsense reserve $\hat{R}_i^{\text{CL}} = 0$ if $C_{i,n+1-i} = 0$.

The B/F reserve is $\hat{R}_i^{\text{BF}} := (1 - \hat{b}_{n+1-i}) \hat{U}_i$

with $\hat{U}_i = v_i \hat{q}_i$, $\hat{q}_i =$ estimated ultimate loss ratio (ignoring C_{i*})

$b_k \in [0; 1]$ expected known loss part after development year k

i.e. $(b_1, b_2, \dots, b_n) =$ cumul. devel. pattern ($b_n = 1$ here).

By its definition, the B/F reserve does not depend on $C_{i,n+1-i}$.

The usual practice to determine \hat{q}_i and \hat{b}_k :

- the \hat{b}_k are derived from CL:

$$b_n = 1, \hat{b}_{n-1} = \hat{f}_n^{-1}, \hat{b}_{n-2} = (\hat{f}_{n-1} \hat{f}_n)^{-1}, \dots, \hat{b}_1 = (\hat{f}_2 \cdot \dots \cdot \hat{f}_n)^{-1}$$

- the \hat{q}_i are “somehow” based on pricing and market information.

With this practice, B/F looks like a manipulated CL.

Proposition 1

The application of the CL-pattern to B/F

contradicts to the independence between $C_{i,n+1-i}$ and \hat{R}_i^{BF} .

How to correctly estimate the B/F pattern?

$$\hat{R}_i = (1 - \hat{b}_{n+1-i}) \hat{U}_i \Rightarrow \hat{b}_{n+1-i} = 1 - \frac{\hat{R}_i}{\hat{U}_i} = \frac{\hat{U}_i - \hat{R}_i}{\hat{U}_i} \approx \frac{C_{i,n+1-i}}{\hat{U}_i}$$

This suggests $\hat{b}_k = \frac{\sum_{i=1}^{n+1-k} C_{ik}}{\sum_{i=1}^{n+1-k} \hat{U}_i}$ (weighted avg. of C_{ik}/\hat{U}_i).

This may lead to inversions $\hat{b}_k > \hat{b}_{k+1}$.

Therefore the increments $\hat{\beta}_k := \frac{\sum_{i=1}^{n+1-k} S_{ik}}{\sum_{i=1}^{n+1-k} \hat{U}_i}$ should be used

$$\text{and } \hat{b}_k = \frac{\hat{\beta}_1 + \dots + \hat{\beta}_k}{\hat{\beta}_1 + \dots + \hat{\beta}_n}.$$

How to estimate q_i in a consistent way (without relying on C_{i*})?

The ult. loss ratio q_i depends strongly on the rate level contained in v_i .

In order to compare the rate levels, we consider

$$\hat{m}_k := \frac{\sum_{i=1}^{n+1-k} S_{ik}}{\sum_{i=1}^{n+1-k} v_i} = \text{average incremental loss ratio in DY } k .$$

(Then $\hat{m}_1 + \dots + \hat{m}_n$ estimates the average ultimate loss ratio.)

If an AY i has a below average rate level, v_i is smaller as it should.

Then, most of its individual incr. loss ratios $\frac{S_{i1}}{v_i}, \frac{S_{i2}}{v_i}, \dots, \frac{S_{i,n+1-i}}{v_i}$

will be higher than the corresponding $\hat{m}_1, \hat{m}_2, \dots, \hat{m}_{n+1-i}$.

This can be put together in the single weighted average

$$r_i := \sum_{k=1}^{n+1-i} \frac{\hat{m}_k}{\sum \hat{m}_k} \cdot \boxed{\frac{S_{ik} / v_i}{\hat{m}_k}} = \sum_{k=1}^{n+1-i} S_{ik} / \sum_{k=1}^{n+1-i} (v_i \hat{m}_k) = \frac{C_{i,n+1-i} / v_i}{\sum_{k=1}^{n+1-i} \hat{m}_k}$$

which will be a little bit greater (or less) than 1.00

indicating the factor by which the premium v_i has to be multiplied in order to adjust it to the average rate level of the accident years considered.

Proposition 2

**If r_i and $\hat{m}_1, \dots, \hat{m}_n$ are plausible,
the prior estimate of the ULR q_i is $(\hat{m}_1 + \dots + \hat{m}_n) r_i$.**

Final assessment of r_i

Of course, the paid triangle and the incurred triangle will not (but should) produce an identical r_i .

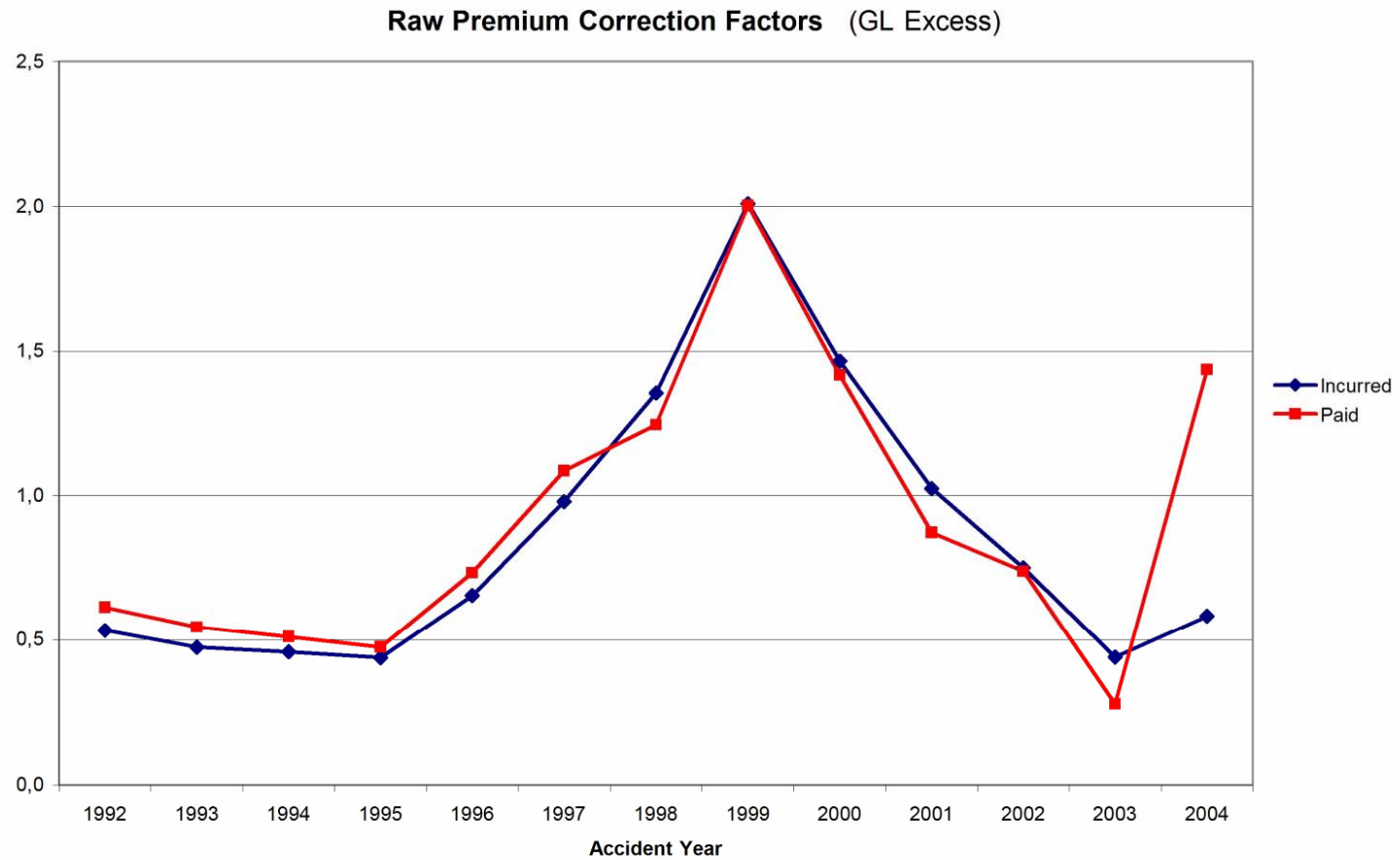
Therefore it is advisable to use the average:

$$\bar{r}_i = (r_i^{\text{paid}} + r_i^{\text{inc}}) / 2 .$$

The resulting yearly change ratios $(\bar{r}_i - \bar{r}_{i-1}) / \bar{r}_{i-1}$ should be checked against the rate level changes derived from pricing data, especially as long as r_i is not credible ($i = n, n-1$) .

Based on this, the final r_i^* is selected.

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Adjustment of \hat{m}_k for the varying rate levels

The ILRs \hat{m}_k are based on unadjusted premium volumes v_i .

Therefore, the ULR at average premium level has to be adjusted to

$$\hat{\tilde{m}}_1 + \dots + \hat{\tilde{m}}_n \quad \text{with} \quad \hat{\tilde{m}}_k := \frac{\sum_{i=1}^{n+1-k} S_{ik}}{\sum_{i=1}^{n+1-k} (v_i r_i^*)} .$$

Based on $\hat{\tilde{m}}_k$, the final ILRs \hat{m}_k^* are selected

in such a way that their development is smooth.

Moreover, we may add a tail ratio \hat{m}_{n+1}^* .

Then $\hat{m}^* = \hat{m}_1^* + \dots + \hat{m}_{n(+1)}^*$ is the prior ULR at avg. premium level.

This finally leads to $\hat{q}_i = r_i^* \hat{m}^*$ and $\hat{U}_i = v_i r_i^* \hat{m}^*$.

Revision of $\hat{\beta}_k$ and \hat{b}_k

For the pattern $\hat{\beta}_k$ and \hat{b}_k we have the following simplification:

$$\hat{\beta}_k = \frac{\sum_{i=1}^{n+1-k} S_{ik}}{\sum_{i=1}^{n+1-k} \hat{U}_i} = \frac{\sum_{i=1}^{n+1-k} S_{ik}}{\sum_{i=1}^{n+1-k} v_i r_i^* \hat{m}^*} = \frac{\hat{m}_k}{\hat{m}^*}$$

Therefore it is consequent to select $\hat{\beta}_k^* = \frac{\hat{m}_k^*}{\hat{m}^*}$,

i.e. we have $\hat{b}_k^* = \frac{\hat{m}_1^* + \dots + \hat{m}_k^*}{\hat{m}_1^* + \dots + \hat{m}_{n(+1)}^*}$.

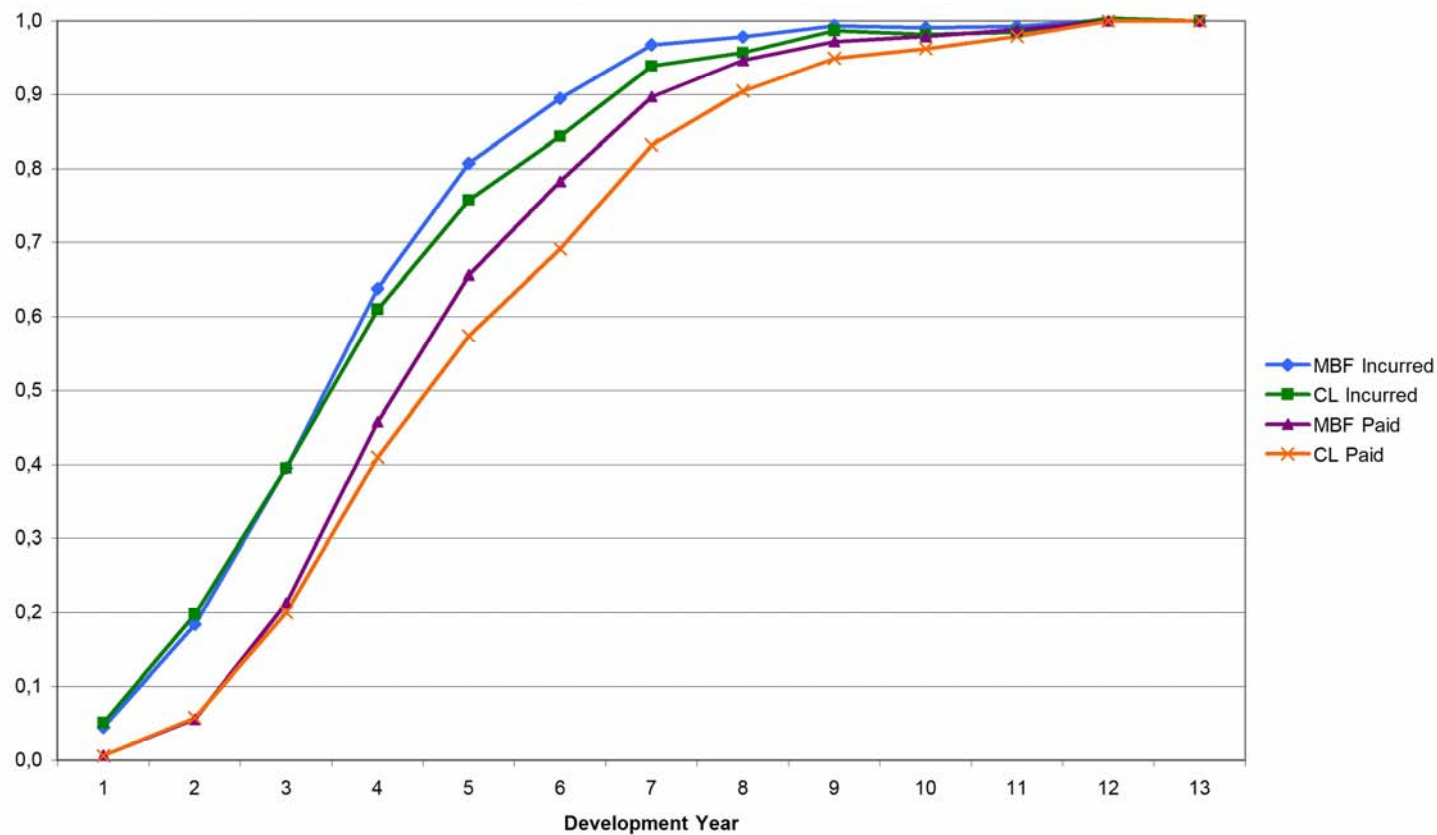
This is the genuine B/F development pattern

which is different from the CL pattern.

Recent Developments in Claims Reserving

Parameter Estimation for Bornhuetter / Ferguson

Raw Development Pattern (GL Excess)



Order of calculation for B/F

$$\hat{m}_k = \sum_{i=1}^{n+1-k} S_{ik} / \sum_{i=1}^{n+1-k} v_i \quad \text{raw incremental loss ratio}$$

$$r_i = \sum_{k=1}^{n+1-i} S_{ik} / \sum_{k=1}^{n+1-i} (v_i \hat{m}_k) \quad \text{raw premium correction factor (PCF)}$$

r_i^* = PCF adjusted for credibility, paid or incurred and rate change

$$\hat{m}_k^* \approx \sum_{i=1}^{n+1-k} S_{ik} / \sum_{i=1}^{n+1-k} (v_i r_i^*) \quad \text{adjusted and smoothed ILR}$$

$$\hat{q}_i = (\hat{m}_1^* + \dots + \hat{m}_n^*) r_i^* \quad (\text{plus tail ratio } \hat{m}_{n+1}^* \text{ if necessary})$$

$$\hat{U}_i = v_i \hat{q}_i = v_i r_i^* (\hat{m}_1^* + \dots + \hat{m}_{n(+1)}^*)$$

$$\hat{b}_k = \frac{\hat{m}_1^* + \dots + \hat{m}_k^*}{\hat{m}_1^* + \dots + \hat{m}_{n(+1)}^*}$$

$$\hat{R}_i = (1 - \hat{b}_{n+1-i}) \hat{U}_i = (\hat{m}_{n+2-i}^* + \dots + \hat{m}_{n(+1)}^*) v_i r_i^*$$

The parameter estimation for B/F

thus consists of

- selecting a development pattern of ILRs \hat{m}_k^*
- and an accident year pattern of PCFs r_i^* (same for paid + inc.)
- and the calculation of $\hat{R}_i = \left(\hat{m}_{n+2-i}^* + \dots + \hat{m}_{n(+1)}^* \right) v_i r_i^*$

and is therefore as simple as it was before.

But thus, B/F is an own method and not a manipulated CL!

(Thomas MACK, submitted to CAS Forum 2006)

Stochastic models for B/F

This way of estimating the B/F parameters can be embedded into the classical credibility IBNR model by DE VYLDER (IME 1982) and MACK (IME 1990) and thus is capable of the calculation of the prediction error (see chapter 3.2.3 of my book “Schadenversicherungsmathematik”).

Alternatively, it can be viewed as being cross-classified

$$\text{i.e. } E(S_{ik}/v_i) = r_i m_k$$

and thus be treated with GLMs like in motor rating.

The problem of the two data bases

Separate application of CL to the paid triangle $\{C_{ik}\}$ as well as to the incurred¹⁾ triangle $\{D_{ik}\}$ inevitably yields different estimates $\hat{C}_{in} \neq \hat{D}_{in}$, $2 \leq i \leq n$, for the ultimate amounts.

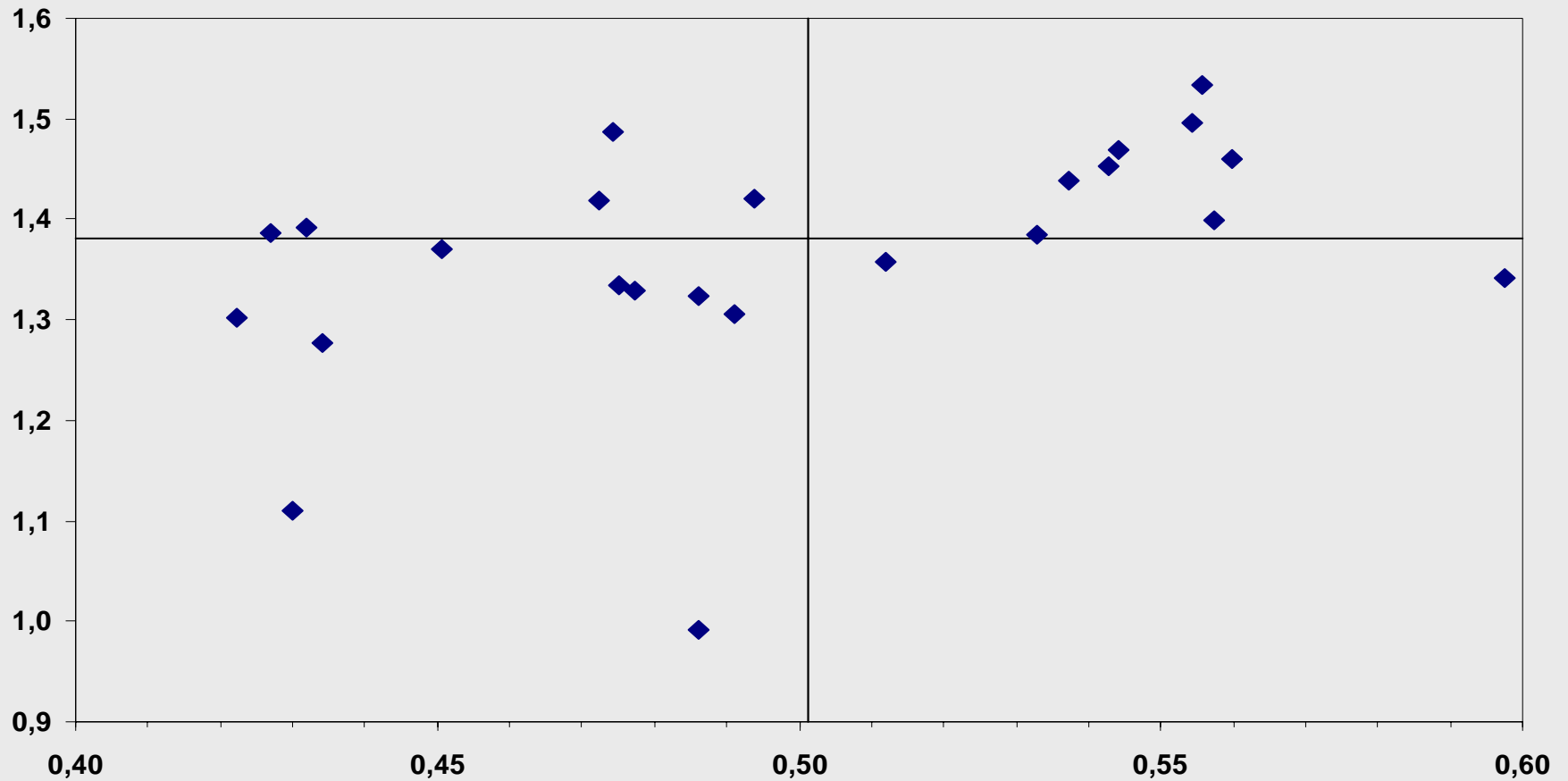
The same is true for B/F, too.

Theorem (CL):
$$\frac{\hat{C}_{in} / \hat{D}_{in}}{\sum_{j=1}^n \hat{C}_{jn} / \sum_{j=1}^n \hat{D}_{jn}} = \frac{C_{i,n+1-i} / D_{i,n+1-i}}{\sum_{j=1}^n \hat{C}_{j,n+1-i} / \sum_{j=1}^n \hat{D}_{j,n+1-i}}$$

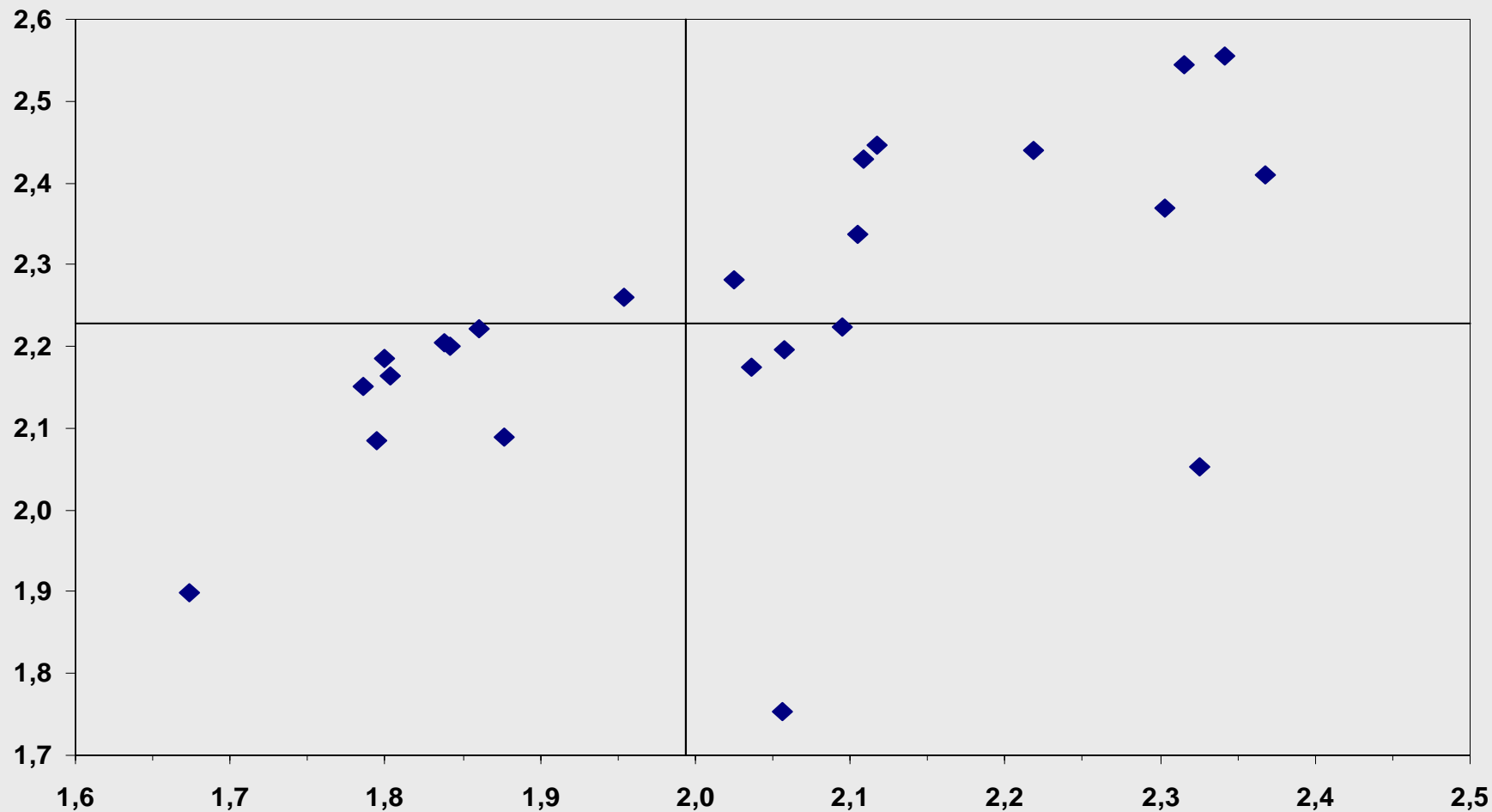
¹⁾ incurred = paid + individual case reserves

- **Separate CL predictions for paid and incurred do not change the relation between the P/I-ratio of any AY and its corresponding average over all AYs.**
- **Separate CL predictions lead to unrealistic results!**
- **In order to achieve $\hat{C}_{in} = \hat{D}_{in}$ (i.e. paid ult. = incurred ult.) with a below-average current P/I-ratio, the future P- development has to be above average and/or the future I-development below average (analogously in case of an above-average P/I-ratio).**
- **This must be observable in the past development, too!**

Dependence of incurred devel. factors $D_{i,2}/D_{i,1}$ on preceding P/I-ratios $C_{i,1}/D_{i,1}$



Dependence of paid devel. factors $C_{i,2}/C_{i,1}$ on preceding I/P-ratios $D_{i,1}/C_{i,1}$



Indeed, the individual paid dev. factors $k \rightarrow k+1$ increase linearly in $(I/P)_k$,
the individual incurred dev. factors $k \rightarrow k+1$ increase linearly in $(P/I)_k$

Therefore, the appropriate model is (“Munich Chain Ladder”)

$$\mathbf{E} \left(\frac{C_{i,k+1}}{C_{ik}} \middle| C_{ik}, D_{ik} \right) = \mathbf{f}_{k+1} + \mathbf{a}_{k+1} \left(\frac{D_{ik}}{C_{ik}} - \mathbf{E} \left(\frac{D_{ik}}{C_{ik}} \middle| C_{ik} \right) \right)$$
$$\mathbf{E} \left(\frac{D_{i,k+1}}{D_{ik}} \middle| C_{ik}, D_{ik} \right) = \mathbf{g}_{k+1} + \mathbf{b}_{k+1} \left(\frac{C_{ik}}{D_{ik}} - \mathbf{E} \left(\frac{C_{ik}}{D_{ik}} \middle| D_{ik} \right) \right)$$

This is a refinement of the separate models

$$\mathbf{E} \left(\frac{C_{i,k+1}}{C_{ik}} \middle| C_{ik} \right) = \mathbf{f}_{k+1}, \quad \mathbf{E} \left(\frac{D_{i,k+1}}{D_{ik}} \middle| D_{ik} \right) = \mathbf{g}_{k+1}$$

The analogous theorem for B/F is

$$\frac{\hat{\mathbf{D}}_{\text{in}} - \hat{\mathbf{C}}_{\text{in}}}{\mathbf{v}_i \mathbf{r}_i} - \frac{\hat{\mathbf{D}}_{+,n} - \hat{\mathbf{C}}_{+,n}}{\sum \mathbf{v}_i \mathbf{r}_i} = \frac{\mathbf{D}_{i,n+1-i} - \mathbf{C}_{i,n+1-i}}{\mathbf{v}_i \mathbf{r}_i} - \frac{\hat{\mathbf{D}}_{+,n+1-i} - \hat{\mathbf{C}}_{+,n+1-i}}{\sum \mathbf{v}_i \mathbf{r}_i}$$

which leads to replacing

$$\mathbf{E}_C \left(\frac{\mathbf{S}_{i,k+1}}{\mathbf{v}_i \mathbf{r}_i} \right) = \mathbf{m}_{k+1} \text{ with } \mathbf{E}_{C,D} \left(\frac{\mathbf{S}_{i,k+1}}{\mathbf{v}_i \mathbf{r}_i} \right) = \mathbf{m}_{k+1} + \mathbf{h}_{k+1} \left(\frac{\mathbf{D}_{ik} - \mathbf{C}_{ik}}{\mathbf{v}_i \mathbf{r}_i} - \mathbf{q}_k \right).$$

All these results are due to Gerhard QUARG (BDGVFM 2003).

Paper on prediction error for Munich CL is forthcoming.

Paper on Munich B/F is in preparation.

Conclusions:

- 1. CL is a sound method for a single triangle
but must be modified if applied to both, paid and incurred data.**
- 2. The current way of applying B/F must be modified
in order to have an own method already for a single triangle,
but then it is also capable to cope with the P/I problem.**
- 3. There exists a clear methodology to calculate the prediction error
for a single triangle.**
- 4. There is some work to be done for the calculation of the prediction
error for a portfolio of several paid and incurred triangles,
but the essential ideas are established.**

Thank you very much for your attention.

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Münchener Rück
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