A Dynamic Claims Reserving Model
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Chain Ladder

	Development period J			
Accident Year I	1	2	3	4
1				
2	Accumulated Amount			Projection
3				
4				

A Dynamic Claims Reserving Model

Drawing on Norberg's theory, a Marked Poisson Process (MPP) is defined as follows:

A *claim* is a set $C=(T,J,Y_J,Y_{J+1}, \ldots,Y_D,G)$, where

- T : time of occurrence
- J : reporting delay-period 1,2,3
- Y_k: incurred amount in period $k \geq J$
- G: Policy and claim information

- $Z=(J,Y_J,Y_{J+1}, \ldots,Y_D,G)$ is the 'Mark'.

Stochastic Claims Reserve at time D

$$\text{IBNS} = \sum_{\substack{k > D-i+1 \\ l : Tl < \mathrm{D}}} \mathrm{Y}_{\mathrm{k,\ l}}$$

$$\text{IBNR} = \sum_{\substack{k \geq j > D-i+1 \\ l : Tl < \mathrm{D}}} \mathrm{Y}_{\mathrm{k,\ l}}$$

Our main purpose is to
determine the conditional distribution of the stochastic reserves RBNS and IBNR given the observation.

The method:

1: Describe and estimate the claim-distribution
2: Determine the conditional distributions of the claims given the observation.
3: Simulate the future
4: Calculate IBNR and RBNS Net and Gross

Process distribution

$\{(T_l, Z_l)\}_{l=1,\ldots,N}$ a MPP with intensity $w(t)$ and position-dependent distribution P_t of Z if

- t‘s are generated by a Poisson process with intensity $w(t)=w_i\sigma_m$ and
- $P_t = P_t(Z)$ are mutually independent and also independent of the Poisson process.

Decomposing the process:

The (i,m,j,g)-component process: $Z_{i,m,j,g} = (j, Y_j, Y_{j+1}, \ldots Y_D, g)$

Main result, Norberg: The Z's independent MPP and

$$w_{i,m,j,g}(t) = w_i \sigma_m P_t\{J=j, G=g\}$$

- The dist. of the $Z_{i,m,j,g}$ is the conditional distribution of Z given J=j and G=g.
- The $N_{i,m,j,g}$ are independent and Poisson distributed and independent of the marks $Z_{i,m,j,g}$.

Further assumptions conc. J and G

- Business Mix: $P_t\{G=g\}$ constant in (i,m) i.e.
  $P_t\{G=g\} = c\,(e_{i,m,g}/e_{i,m}) f_{I,M,G}(i,m,g)$

- Chain Ladder: $P_t\{J=j|G=g\}$ independent on i, i.e.
  $P_t\{J=j|G=g\} = f'_{J,G}(j,g,t-[t])$

<u>Implication:</u>

$N_{i,m,j,g} \sim Po[e_{i,m,g} c w_i \sigma_m f_{I,M,G}(i,m,g) f_{M,J,G}(j,m,g)]$

Further assumptions conc. $Y_j, \ldots, Y_D$

The distribution of Y_k given $Y_j, \ldots, Y_{k-1}$ only depends on

- t through (i,m) -that's why we need m!
- $Y_j, \ldots, Y_{k-1}$ through $h(Y_j, .., Y_{k-1})$ – the dynamic element.

<u>Implication:</u>

$P_t(Y_D, Y_{D-1}, \ldots, Y_j, j, g) =$
$P(Y_D | h(Y_{D-1}, \ldots, Y_j), i, m, j, g) *$
$P(Y_{D-1} | h(Y_{D-2}, \ldots, Y_j), i, m, j, g) * \ldots * P(Y_j, i, m, j, g).$

Example.

$h(Y_j,..,Y_{k-1})=Y_j+..+Y_{k-1}=S_{k-1}$ i.e. the incurred amount at the beginning of the period k.

<u>Focus:</u> Distribution of Y_k given S_{k-1}

Split the analyses into $S_{k-1}=0$ and $S_{k-1}>0$

<u>$S_{k-1}=0$</u>: Split further into:

$P(Y_k=0)$, $P(0<Y_k<L)$, $P(L\leq Y_k)$

Let $p_{>0} = P(Y_k > 0)$,$p_{>L} = P(Y_k > L \mid Y_k > 0)$

$Y_k=0$ with probability $1-p_{>0}$
$L\leq Y_k$ with probability $p_{>L}p_{>0}$
$0<Y_k<L$ with probability $(1- p_{>L})p_{>0}$

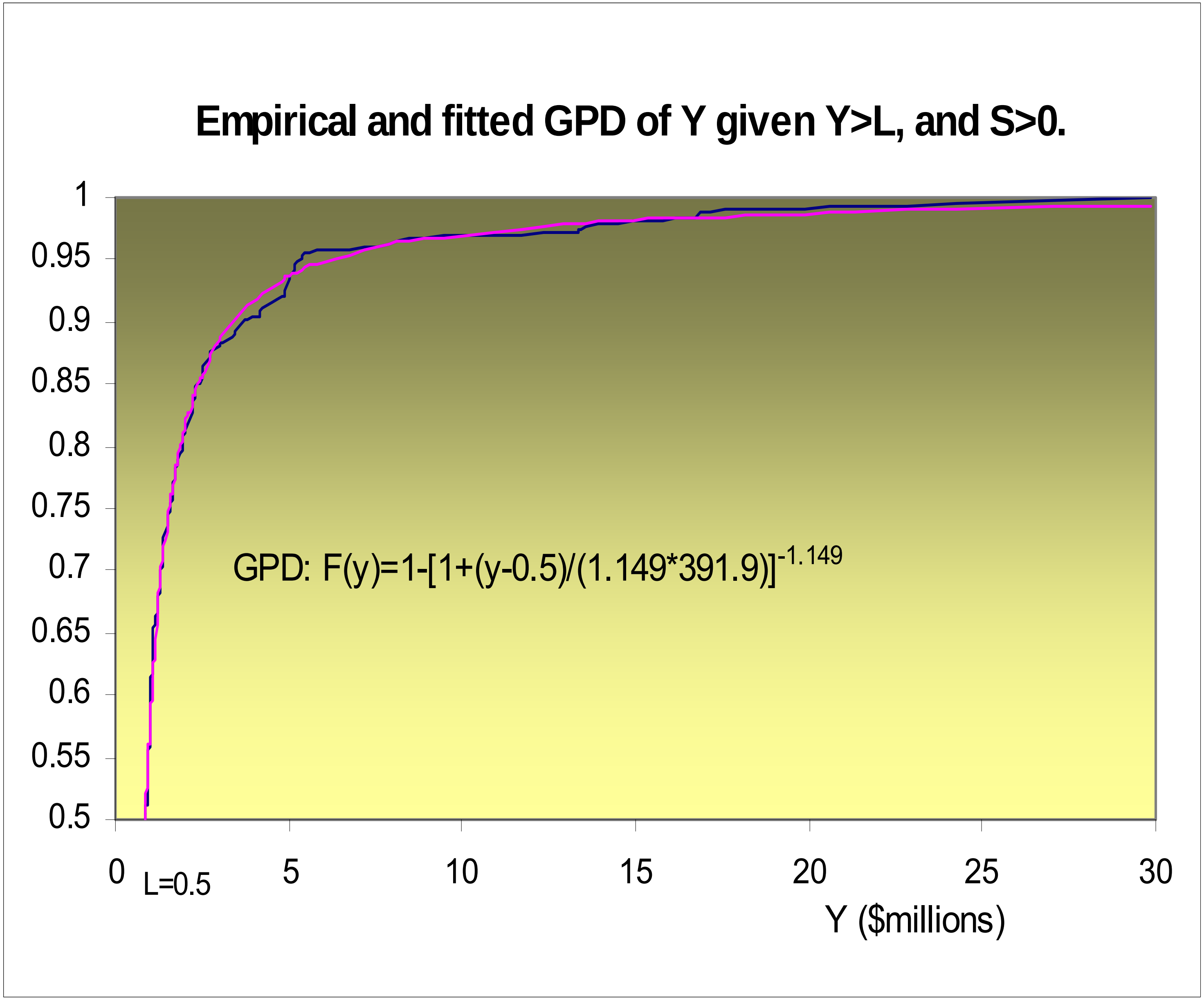
Model $p_{>0}$ and $p_{>L}$
GLM: link=logit, dist=Binomial,
covariates i,m,j,k,g

S_{k-1}=0, conditional distributions of Y_k

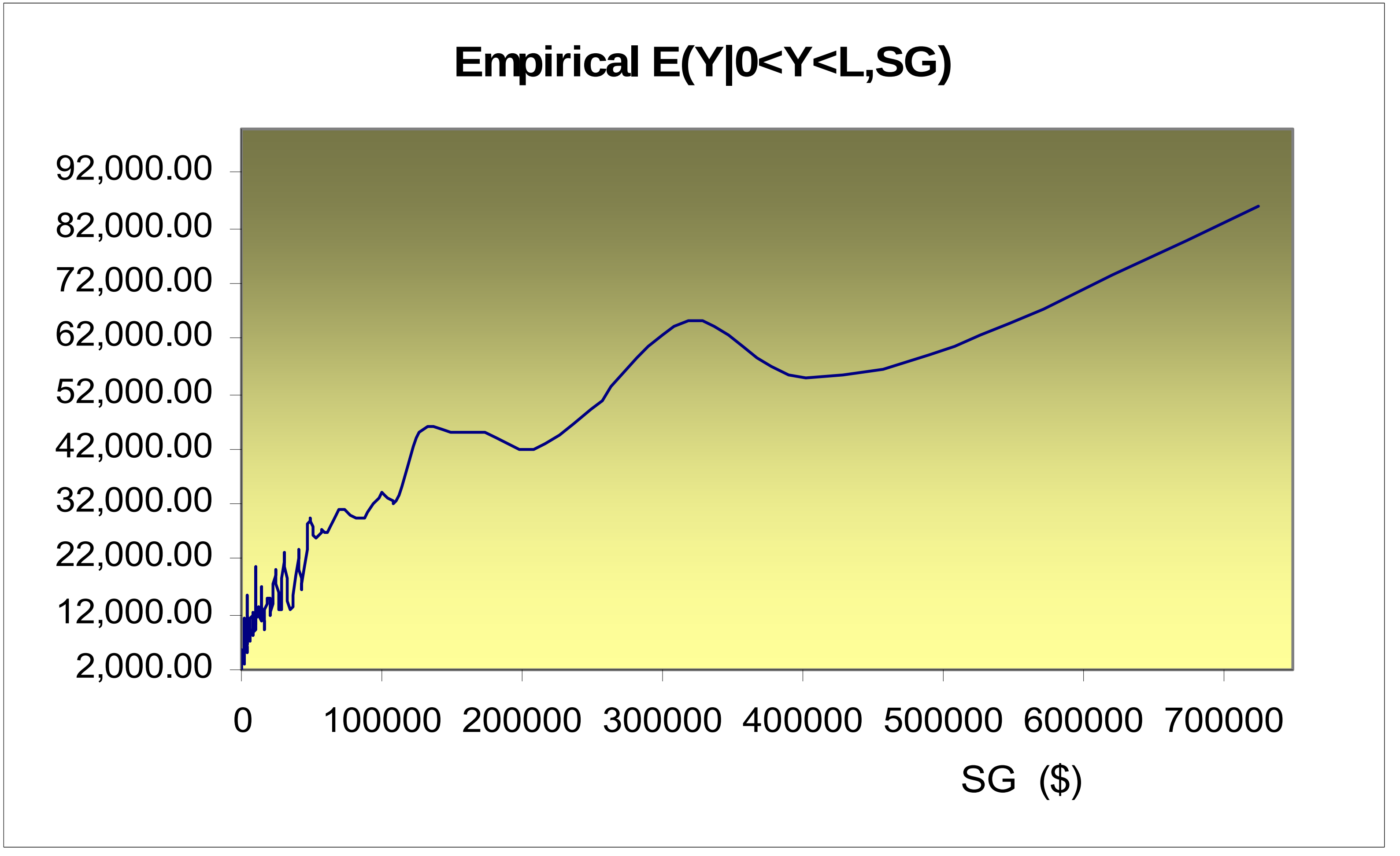
$Y_k=0$:	$P(Y_k=0)=1$
$L \leq Y_k$:	Generalised Pareto Distribution
$0<Y_k<L$:	GLM: link=log, dist=Gamma covariates i,m,j,k,g

<u>$S_{k-1}>0$:</u> A bit more complicated: Split into 5:

$Y_k=0$, $0<Y_k<L$, $L \leq Y_k$, $Y_k=-S_{k-1}$ and $-S_{k-1}<Y_k<0$


Empirical and fitted GPD of Y given Y>L, and S>0.
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
GPD: F(y)=1-[1+(y-0.5)/(1.149*391.9)]^-1.149
0
L=0.5
5
10
15
20
25
30
Y ($millions)

Correlation between Y and S

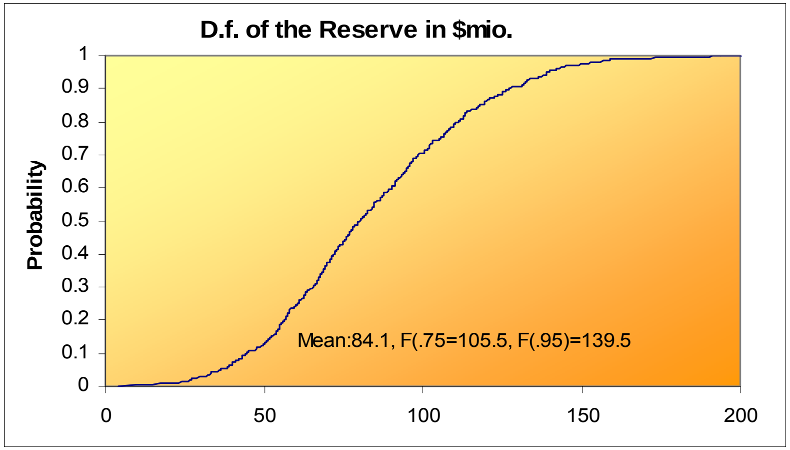


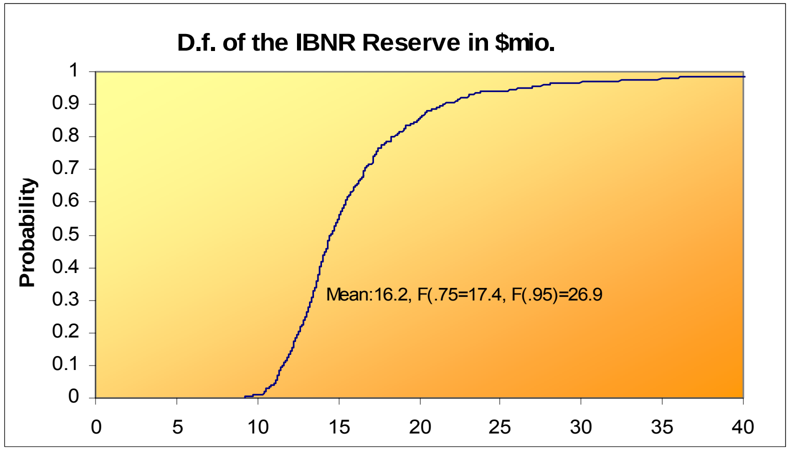
Reserve Distribution

Simulate number of IBNR claims per cell (i,m,j,g)

Simulate future Y's per claim according to the estimated parameters and summarise

Repeat 1000 times.





Bootstrap the total uncertainty

Since the observation is random, the estimated parameters, as functions of the observation, are also random.

1: Sample M claims with replacement.
2: Fit the model based on M
3: Simulate Y's given the original claims.
4: repeat 1-3 1000 times.

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