



Sharing Risk – An Economic Perspective

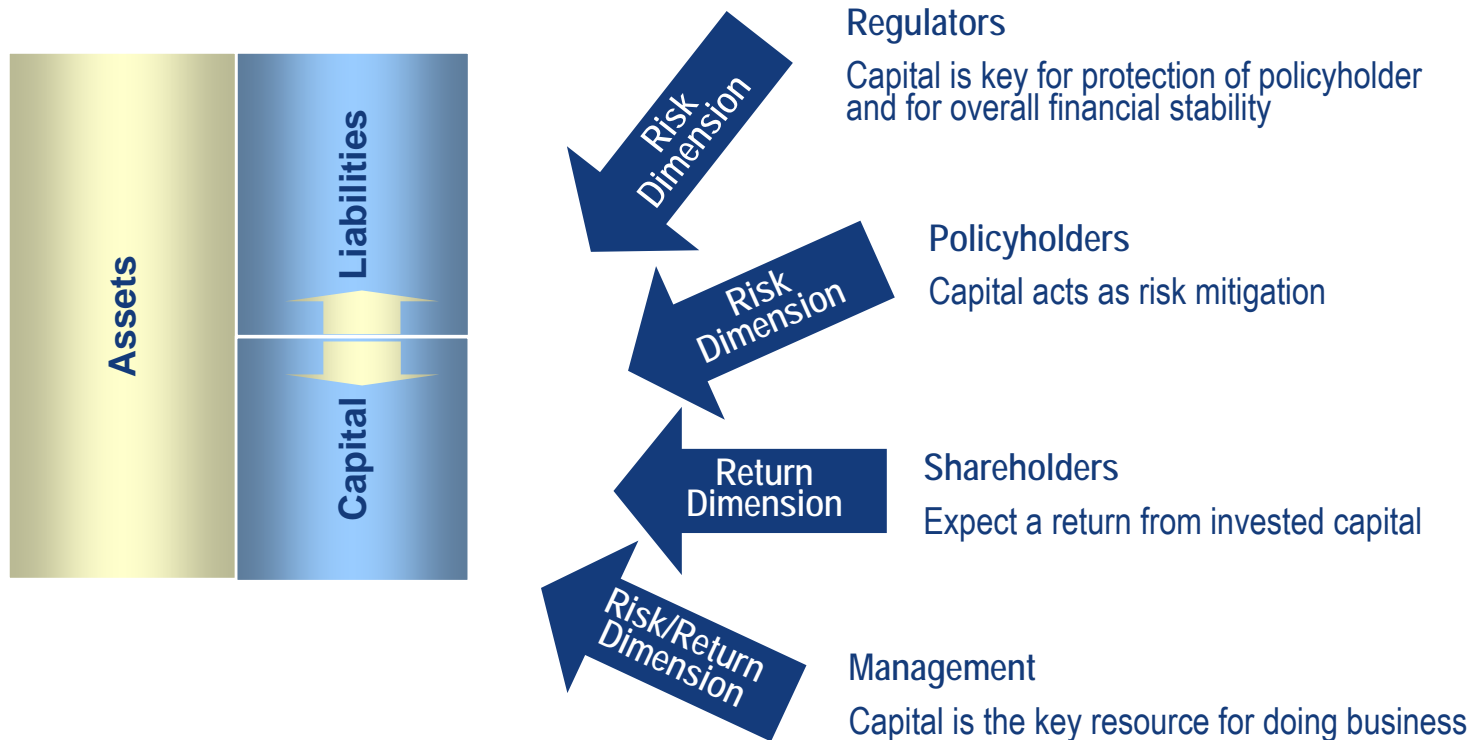
36th ASTIN Colloquium, Zurich, 5.9.2005

Andreas Kull, Global Financial Services Risk Management

 **ERNST & YOUNG**

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Capital: Shared and competing issue



Risk transfer strategies should balance risk and return dimension by taking into account

1. **Capital constraints** (regulators, policyholders)
2. **Economic bottom line**, i.e. impact of risk transfer on overall return (shareholders, management)

Sharing and Transferring of Risk

- Risk transfer should balance risk/return dimensions by taking into account
 - **Capital constraints** (regulators, policyholders)
 - **Economic bottom line**, ie. impact of risk transfer on overall return (shareholders, management)
- A sound model of risk transfer and risk sharing should
 - Allow for relevant constraints (e.g. available capital, risk tolerance)
 - Focus on economic value
 - Include effects of portfolio diversification and dependency issues
 - Allocate risk based capital consistently
- What if risk exchanging parties employ similar principles?
 - Consistently model economics of risk transfer (premium principle)
 - Take into account portfolio structure of both risk ceding and receiving party
- Key issue: Sharing risk 'optimally' in this context

What risks should be shared/transferred?

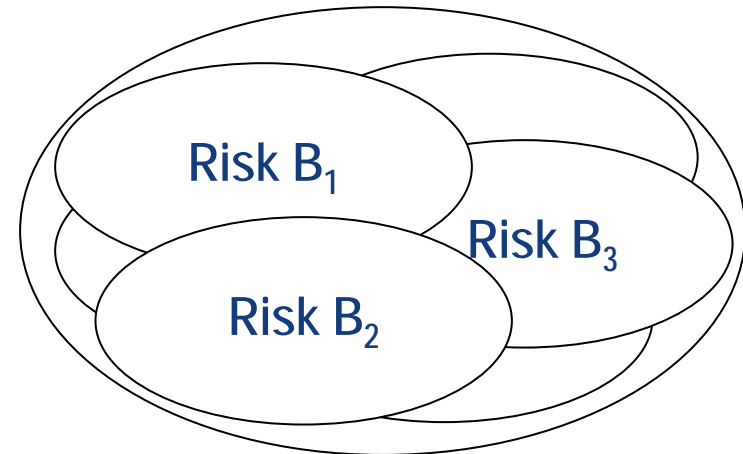
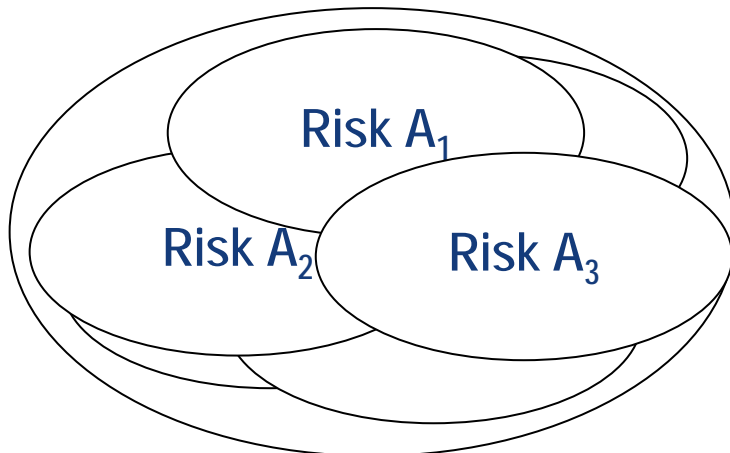
Key parameter Party A

- Cost of capital
- Portfolio structure
- Risk tolerance level

Key parameter Party B

- Cost of capital
- Portfolio structure
- Risk tolerance level

Transfer risk ...



Portfolio A

... in exchange
for premium

Portfolio B

Objective Party A

- Transfer risk
- Meet capital constraint
- Maximize overall return

What risk for
which premium?

Objective Party B

- Take over risk in exchange
for premium
- Attain a profitability hurdle

A Simple Model (I)

Net underwriting profit (risk class i)	$U_i = P_i - (L_i - R_i) - P_i^*$	P : Premium, L : Loss, P^* : Risk transfer premium, R : Loss covered by risk transfer.
Economic result (risk class i)	$Z_i = U_i - \lambda \cdot \text{RBC}[Z_i]$	RBC: Risk based capital, λ : cost of capital
Risk based capital (risk class i)	$\text{RBC}[Z_i] = -\text{E}[Z_i Z \leq F_Z^{-1}(\alpha)]$	α : Risk tolerance level, RBC: Risk based capital (defined as conditional expected shortfall relative to portfolio result)
Premium for risk transfer (risk class i) <small>quantities with * refer to risk receiving party</small>	$P_i^* = (1 - \lambda^*) \cdot \text{E}[R_i] + \text{E}[R_i R_i \leq F_{Z^*}^{-1}(\alpha^*)]$	P^* : Premium for risk transfer, R : loss covered by risk transfer, λ^* : Cost of capital, α^* : Risk tolerance level
Portfolio distribution, Total RBC	$Z = \sum Z_i, \quad \text{RBC}[Z] = \sum \text{RBC}[Z_i]$	Note additivity of RBC
Objective and constraint	$\text{E}[Z(r)] = \max!, \quad \text{RBC}[Z(r)] = \text{const}$	r : Risk transfer control parameter(s), eg XL retentions

A Simple Model (II)

- Typical constraint extreme value problem
- Solved using Lagrangian multipliers κ , i.e. by maximizing expression

$$\varphi(\vec{r}) = E[Z(\vec{r})] + \kappa \cdot \text{RBC}[Z(\vec{r})]$$

- This leads to condition

$$\begin{aligned} \nabla_{\vec{r}} [& (\lambda^* - \lambda + \kappa \cdot (1 - \lambda^*)) \cdot E[R] - && \text{Risk receiving portfolio} \\ & \lambda^* \cdot (1 - \kappa) \cdot E[R | R \leq F_Z^{-1}(\alpha^*)] + && \text{Risk ceding portfolio} \\ & (\lambda - \kappa) \cdot E[R | R \leq F_Z^{-1}(\alpha)] = 0 \end{aligned}$$

- Other conditions include concavity of $E[Z(r)]$ and convexity of $\kappa \cdot E[Z(r)]$.

A Simple Model (IV)

- Solution $\vec{r} = \vec{r}_0$
 - is a vector of optimal risk transfer control parameters (e.g. XL retentions)
 - characterizes **optimal risk transfer per risk class i**
 - **maximizes overall economic result** on portfolio level
 - for a **given risk based capital RBC_0** .
- Main parameters
 - Structure of risk transferring and receiving **portfolios** (Z and Z^*)
 - **Risk tolerance levels** (α and α^*)
 - **Capital costs** (λ and λ^*)
 - **available risk based capital** (RBC_0 , related to Lagrangian parameter κ)
- Numerical solution necessary

Example

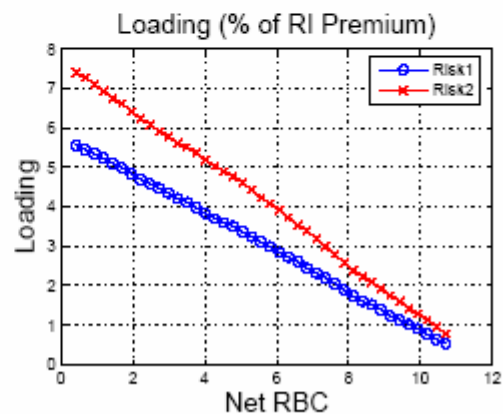
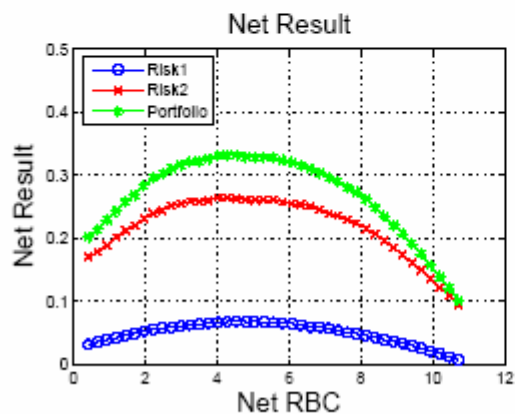
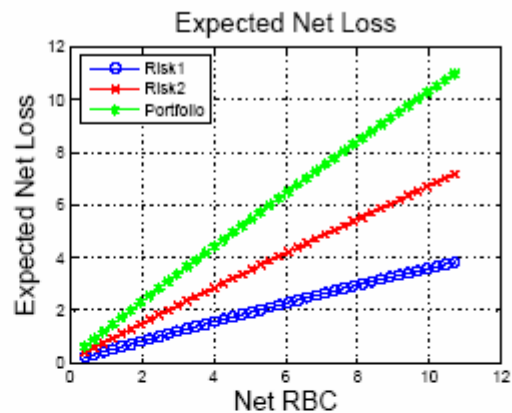
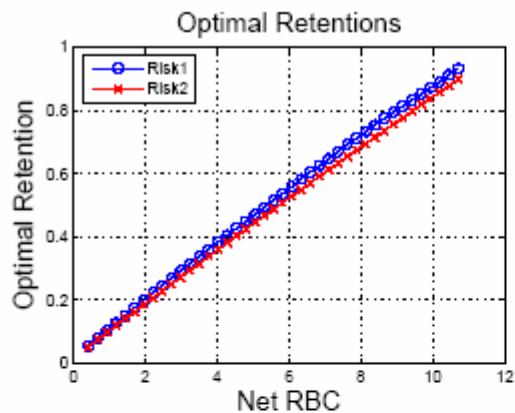
- Illustration for simple proportional and non-proportional reinsurance structures
- Portfolio structures (simple compound Poisson processes)

	Severity			Frequency
	Type	μ	σ	λ
Risk class A ₁	Normal	2.0	1.0	2
Risk class A ₂	LogNormal	2.0	1.5	4
Risk class B ₁	Normal	2.0	1.0	4
Risk class B ₂	LogNormal	2.0	1.5	8

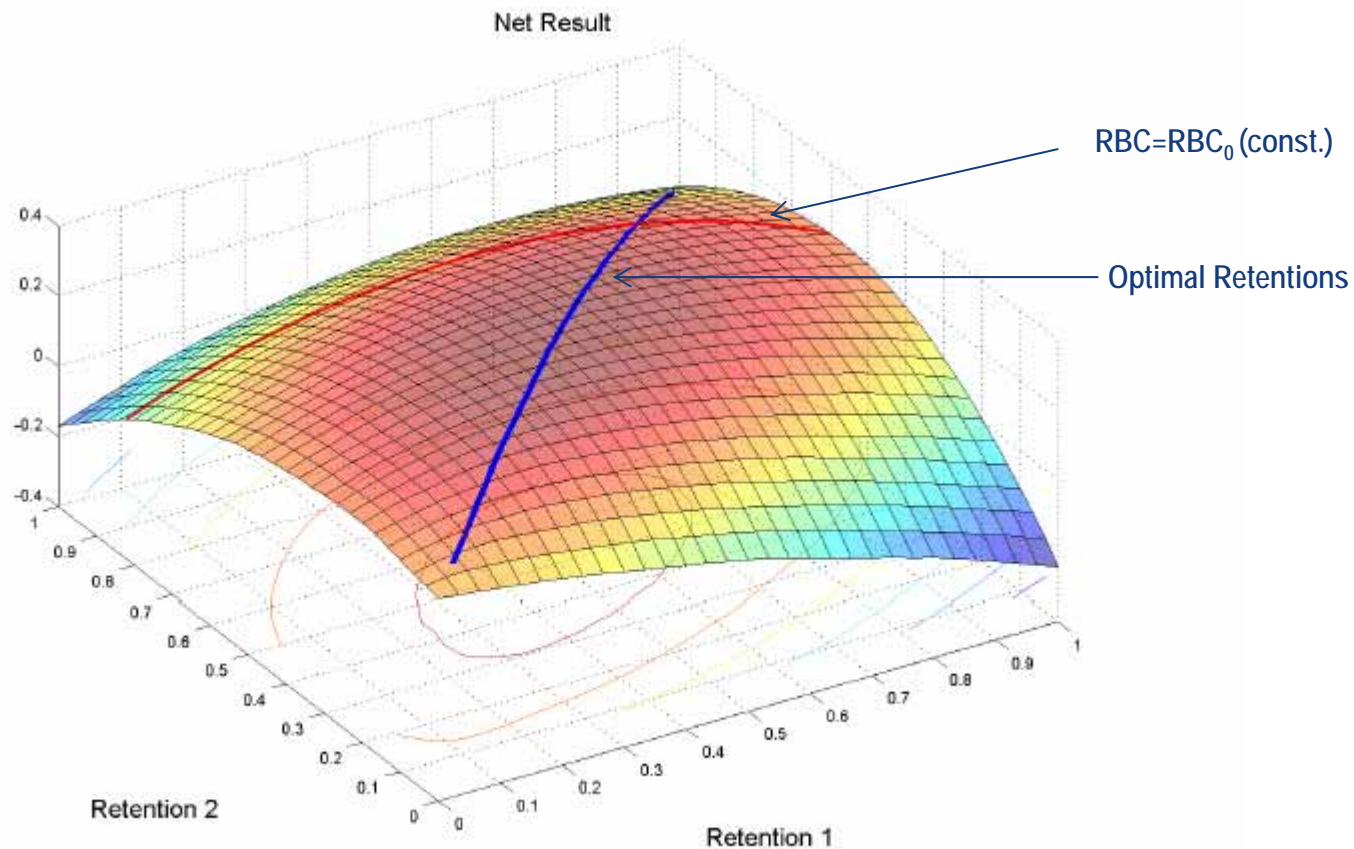
- Risk tolerance levels and capital costs

	Risk Tolerance	Cost of Capital
Portfolio A	10%	9%
Portfolio B	10%	14%

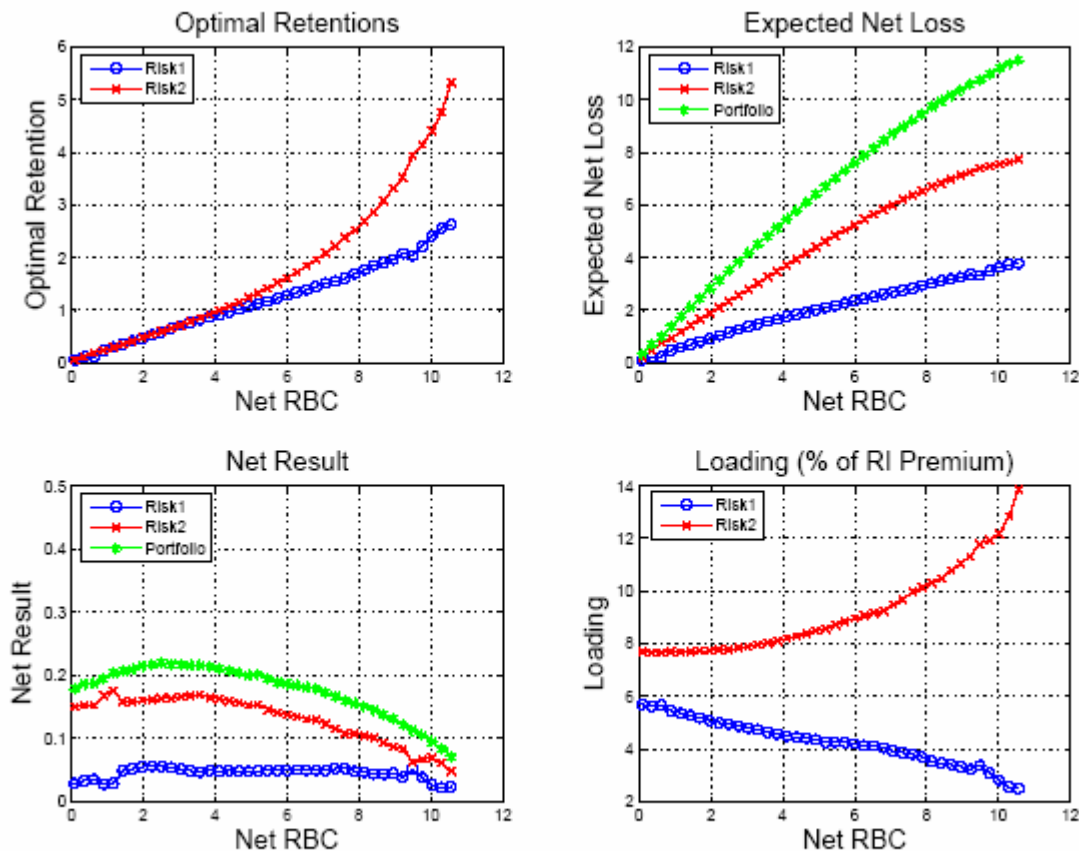
Example – Proportional Reinsurance (I)



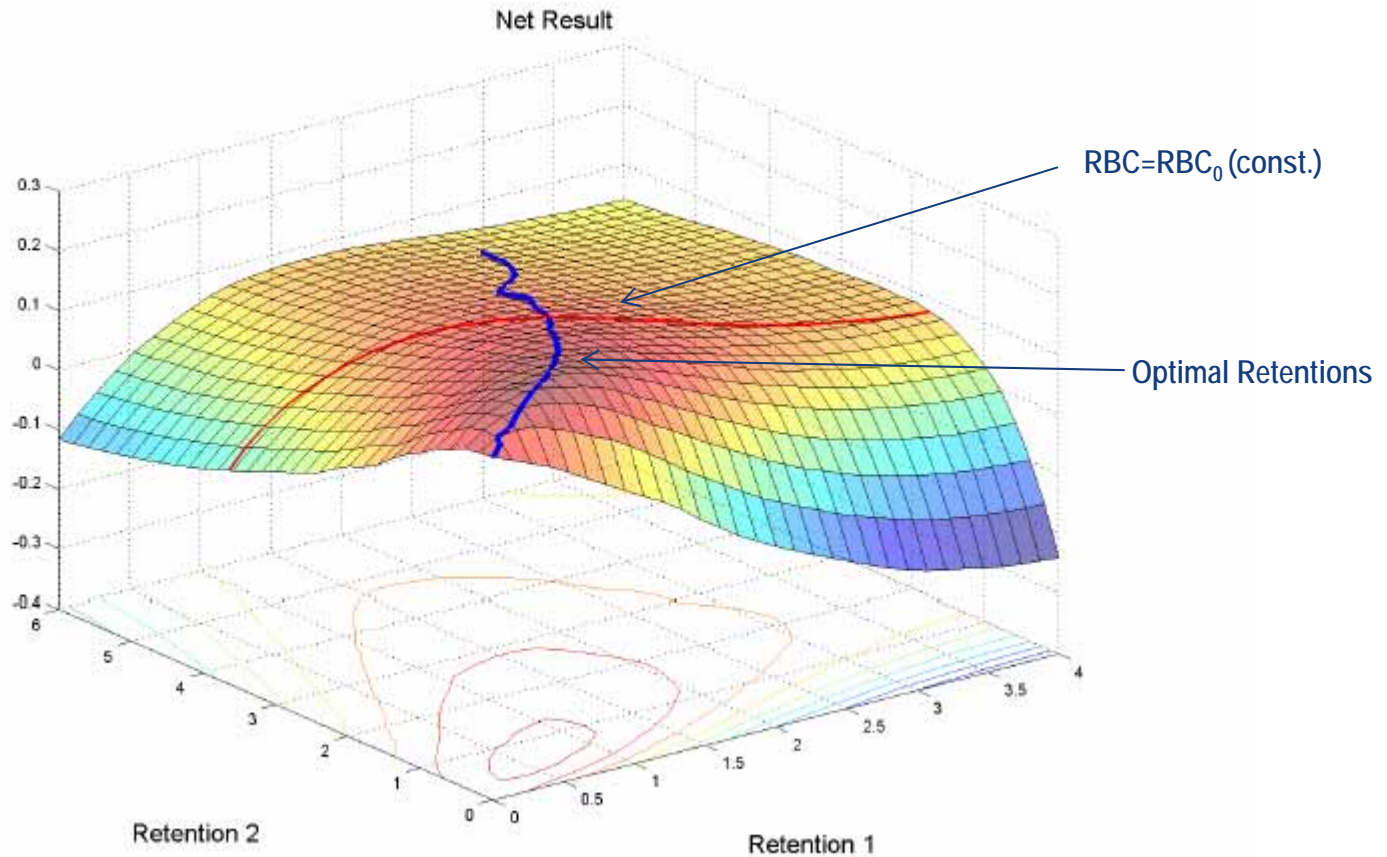
Example – Proportional Reinsurance (II)



Example – Non-Proportional Reinsurance (I)



Example – Non-Proportional Reinsurance (II)



Conclusions

- **Sharing risk and diversification** is crucial
- Workable scheme for companies that have internal models in place
- Optimal also for risk receiving party? → Extend the framework to n risk sharing parties

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