CREDIBLE LOSS RATIO CLAIMS RESERVES:

THE BENKTANDER, NEUHAUS AND MACK METHODS REVISITED

Hürlimann Werner
IRMG and Aon Re (Switzerland) Ltd.
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Agenda

1. Basic identity
2. The credible loss ratio IBNR method
3. A numerical example
4. A digression: are the A.M. Best Loss Development Factors "best" factors?

1. Basic identity.

ultimate claims = ***paid claims*** + ***outstanding claims*** + ***IBNR claims***

2. The credible loss ratio IBNR method.

loss triangle of paid claims statistics:

underwriting period	development period 1	2	3	4	5	6
1	3'789'045	2'860'826	506'651	151'996	65'141	24'203
2	3'582'774	2'687'080	1'250'163	535'784	880'143	-
3	4'221'853	3'166'390	2'249'388	207'853	-	-
4	4'074'429	2'949'557	1'162'885	-	-	-
5	1'227'618	3'906'617	-	-	-	-
6	6'839'930	-	-	-	-	-

S_{ik} : ***paid claims*** for underwriting period i reported in period $i+k-1$, $1 \le i,k \le n$

V_i : ***actuarial premium*** for underwriting period i (or measure of exposure)

$$m_k = \frac{\sum_{i=1}^{n-k+1} S_{ik}}{\sum_{i=1}^{n-k+1} V_i}, \quad k = 1,\dots,n \quad : \quad \textbf{loss ratio}$$

(amount of claims per unit of actuarial premium required in the reporting period k)

C_{in-i+1} : ***accumulated paid claims*** for underwriting period i reported in the latest period of development n

Consider

$U_i^{BC} = V_i \cdot \sum_{k=1}^{n} m_k$: ***burning cost*** of ***ultimate claims*** required for the underwriting period i

$$p_i = \frac{V_i \cdot \sum_{k=1}^{n-i+1} m_k}{U_i^{BC}} = \frac{\sum_{k=1}^{n-i+1} m_k}{\sum_{k=1}^{n} m_k} \quad : \quad \textit{loss ratio payout}$$

(proportion of ultimate claims, which is expected to be paid for the underwriting period i)

First estimate of the ultimate claims (grossing up the latest accumulated paid claims amount with the loss ratio payout):

$$U_i^{ind} = \frac{C_{in-i+1}}{p_i}, \quad i = 1,\ldots,n \quad : \quad \textit{individual ultimate claims}$$

(based solely on the individual latest claims experience of an underwriting year)

$$R_i^{ind} = q_i \cdot U_i^{ind}, \quad i = 1,\ldots,n \quad : \quad \textit{individual loss ratio IBNR claims reserve}$$

($q_i = 1 - p_i$ the proportion of ultimate claims, which is expected to be paid in the future for the underwriting period i)

Second estimate of the ultimate claims (burning cost estimate):

$$R_i^{coll} = q_i \cdot U_i^{BC}, \quad i = 1,\ldots,n \quad : \quad \textit{collective loss ratio IBNR claims reserve}$$

(based solely on the portfolio claims experience of all underwriting periods)

Two ***extreme positions*** suggest credibility mixture:

$$R_i^{c} = Z_i \cdot R_i^{ind} + (1 - Z_i) \cdot R_i^{coll}, \quad i = 1,\ldots,n \quad : \quad \textit{credible loss ratio IBNR claims reserve}$$

Z_i : ***credibility weight*** of individual loss ratio reserve.

Problem of the ***estimation of the credibility weight***. Three proposals:

1) $Z_i^{GB} = p_i$: ***Benktander loss ratio IBNR claims reserve***

(the credibility weight should increase similarly as the accumulated claims develop)

2) $Z_i^{WN} = \sum_{k=1}^{n-i+1} m_k$: ***Neuhaus loss ratio IBNR claims reserve***

3) An ***optimal credibility weight*** (minimum mean squared error of credible reserve):

$$Z_i^* = \frac{p_i}{q_i} \cdot \frac{Cov[C_{in-i+1}, R_i] + p_i q_i \cdot Var[U_i^{BC}]}{Var[C_{in-i+1}] + p_i^2 \cdot Var[U_i^{BC}]}$$

PRACTICAL METHOD

Consider ***model for the loss ratio payout*** (Mack(2000)):

$$E\left[\frac{C_{in-i+1}}{U_i} \middle| U_i\right] = p_i, \quad Var\left[\frac{C_{in-i+1}}{U_i} \middle| U_i\right] = p_i q_i \beta_i^2(U_i), \quad i = 1,\ldots,n.$$

The factor q_i ensures that $Var\left[\frac{C_{in-i+1}}{U_i} \middle| U_i\right] = 0$ when $i = 1$ and that $Var\left[\frac{C_{in-i+1}}{U_i} \middle| U_i\right] \to 0$ in case of very small values p_i.

Theorem. Under the assumption of the loss ratio payout model, the optimal credibility weights Z_i^* which minimize the mean squared error $mse(R_i^c) = E\left[(R_i^c - R_i)^2\right]$ and the variance $Var[R_i^c]$ are given by

$$Z_i^* = \frac{p_i}{p_i + t_i^*}, \text{ with}$$

$$t_i^* = \frac{f - 1 + q_i + \sqrt{(f+1)^2 + q_i^2}}{2}, \quad i = 1,\ldots,n.$$

Comparison:

1) Benktander method: $t_i^{GB} = q_i, \quad i = 1,\ldots,n$

2) Neuhaus method: $t_i^{WN} = q_i + \frac{1 - \sum_{k=1}^{n} m_k}{\sum_{k=1}^{n} m_k}, \quad i = 1,\ldots,n$

3) Optimal method: $f = 1$ and $t_i^* = \frac{1}{2}\left(q_i + \sqrt{q_i^2 + 4}\right), \quad i = 1,\ldots,n$

Note **monotone decreasing credibility weights** in the underwriting periods and

$$Z_i^* \le \tfrac{1}{2}, \quad i = 1,\ldots,n,$$

3. A numerical example.

Consider loss triangle of paid claims displayed at the beginning of Section 2.

Table 3.1: parameters of credible loss ratio method

underwriting period	parameters					
	V	m	p	q	t	Z*
1	8'000'000	0.40230	1	0	0.27713	0.78301
2	9'000'000	0.33129	0.99687	0.00313	0.33405	0.74901
3	10'000'000	0.13971	0.93925	0.06075	0.37734	0.71340
4	10'000'000	0.03317	0.90488	0.09512	0.26679	0.77230
5	10'000'000	0.05560	0.76012	0.23988	0.16771	0.81925
6	12'000'000	0.00303	0.41685	0.58315	0.47409	0.46788

Table 3.2: credible loss ratio IBNR reserves

underwriting period	IBNR method				
	collective	individual	Neuhaus	Benktander	optimal
all periods	10'600'143	12'714'477	11'219'478	11'241'879	11'340'434
2	27'228	28'101	28'067	28'098	27'882
3	586'303	636'809	632'085	633'741	622'334
4	918'019	860'619	867'892	866'079	873'689
5	2'315'070	1'620'276	1'805'379	1'786'943	1'745'863
6	6'753'523	9'568'672	7'886'055	7'927'018	8'070'666

Table 3.3: credible loss ratio ultimate claims

underwriting period	IBNR method				
	collective	individual	Neuhaus	Benktander	optimal
all periods	56'940'469	59'054'803	57'559'804	57'582'205	57'680'760
1	7'397'862	7'397'862	7'397'862	7'397'862	7'397'862
2	8'963'172	8'964'045	8'964'011	8'964'042	8'963'826
3	10'431'787	10'482'293	10'477'569	10'479'225	10'467'818
4	9'104'890	9'047'490	9'054'763	9'052'950	9'060'560
5	7'449'305	6'754'511	6'939'614	6'921'178	6'880'098
6	13'593'453	16'408'602	14'725'985	14'766'948	14'910'596

Table 3.4: mean squared standard errors (ratio to minimal error)

underwriting period	IBNR method				
	collective	individual	Neuhaus	Benktander	optimal
2	1.007012	1.000787	1.000567	1.000768	1
3	1.109791	1.017720	1.008041	1.011004	1
4	1.254679	1.022139	1.004355	1.007506	1
5	1.931076	1.045325	1.010179	1.004850	1
6	1.347837	1.449922	1.006833	1.004137	1

The Neuhaus and Benktander loss ratio reserves are quite close to the optimal credible reserve. In the present situation, the Neuhaus reserve is closer to the optimal one than the Benktander reserve for all underwriting years. Through application of a credible loss ratio reserving method, the reduction in mean squared error is substantial. In absence of sufficient information to estimate the optimal credibility weights, the three simple credible methods are highly recommended for actuarial practice.

4. A digression: are the A.M. Best Loss Development Factors "best" factors?

Compare ***inverse of the loss ratio payout factors*** obtained from the ratio $\frac{U_i}{C_{in-i+1}}$

Example: A.M. Best Table of paid claims for General Liability claims made policies.

Table 4.1: loss triangle of paid claims for General Liability claims made policies

underwriting	development year									
period	1	2	3	4	5	6	7	8	9	10
1994	394'645	991'758	1'601'166	2'048'315	2'286'923	2'464'913	2'578'083	2'632'786	2'678'889	2'691'701
1995	353'192	969'542	1'592'009	1'974'155	2'272'665	2'438'559	2'566'985	2'671'750	2'741'844	
1996	388'218	1'078'557	1'646'205	2'171'862	2'443'991	2'709'179	2'827'232	2'934'048		
1997	389'758	1'082'588	1'874'868	2'427'603	2'867'146	3'125'564	3'277'475			
1998	449'549	1'312'383	2'190'150	2'989'522	3'671'202	4'103'170				
1999	391'147	1'398'570	2'506'347	3'448'315	4'134'317					
2000	683'853	1'720'774	3'136'015	4'112'141						
2001	586'225	1'962'547	3'283'757							
2002	810'359	2'149'450								
2003	648'230									

Table 4.2: inverse of loss ratio payout factors

	A.M. Best	**optimal**	Benktander	Neuhaus	collective	individual
1994	1.000	**1.000**	1.000	1.000	1.000	1.000
1995	1.010	**1.005**	1.005	1.005	1.005	1.005
1996	1.030	**1.027**	1.027	1.027	1.027	1.027
1997	1.066	**1.062**	1.062	1.062	1.061	1.062
1998	1.114	**1.112**	1.113	1.113	1.112	1.113
1999	1.226	**1.220**	1.221	1.221	1.219	1.222
2000	1.471	**1.440**	1.439	1.439	1.441	1.438
2001	1.986	**1.920**	1.914	1.915	1.927	1.903
2002	3.475	**3.344**	3.328	3.331	3.366	3.245
2003	9.903	**9.669**	9.652	9.655	9.696	9.285

A.M. Best factors systematically overestimate (slightly) the optimal and nearly optimal Benktander and Neuhaus factors.