Dependence structures for a reinsurance portfolio exposed to natural catastrophe risk

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This paper analyses the impact of various dependence structures on the annual aggregate loss distribution for a reinsurance portfolio exposed to natural catastrophe risks. We consider dependence structures based on (a) standard copulas (Gaussian copula, t-copula), (b) empirical copulas implicit in the output of natural catastrophe models (CAT models), and (c) so-called event-induced dependencies, a natural mapping of regional marginal distributions onto losses associated with the individual events of CAT models. We show how to calibrate the standard copulas. Our results show that, for Atlantic and European wind risk, a Gaussian copula model gives loss distributions that are very similar to those obtained using both event-induced dependencies and empirical copulas. We do not see, however, any evidence of tail dependence in the simulated events that would warrant the use of t-copulas.

Keywords: portfolio model, natural catastrophe, dependence, copulas, reinsurance.

Introduction

Portfolio modelling forms an integral part of risk management in a reinsurance company. It allows the quantification of the overall underwriting risk at different probabilities of occurrence and is typically used to assess the overall capital needed to support the underwriting risk. It is also used to allocate capital to the different lines of business in order to assess their profitability.

A key element of any portfolio modelling is the description of dependence between the individual reinsurance transactions within the portfolio. Dependence plays an essential role in determining the tails of the portfolio loss distribution and consequently the estimation of capital needs. This paper presents the results of a detailed analysis of dependence that accompanied the development of PartnerRe's portfolio model of natural catastrophe risks. It illustrates the extensive sensitivity analysis that allowed us to gain confidence in the robustness of the model.

Natural catastrophe risks offer an ideal playground to study the effect of dependence on a portfolio model. On one hand they pose a significant accumulation risk for a reinsurer as they may simultaneously affect a large number of individual transactions in different lines of business. On the other hand there exist well-established CAT models that simulate catastrophic events and the resulting loss to particular reinsurance treaty. These CAT models give a natural way to aggregate loss at the portfolio level for each simulated event. In contrast to many other lines of business catastrophe reinsurance benefits from the extensive knowledge of the physical drivers of risks, and ultimately of correlation within the portfolio.

The rest of the paper is organized as follows. We first present a brief description of the portfolio model. Then we describe the procedure we used to parameterise the various copulas we considered. At the end we compare the loss distribution for the different dependence structures and discuss the presence of tail dependence.

Model description

The portfolio model is based on pricing information recorded for each individual reinsurance treaty in our internal databases. This information is available for each region where the treaty has exposure and for each peril it is exposed to. For example a treaty having exposure to windstorm in Europe has pricing data for all the countries that may be affected by this peril. Overall we have pricing data for more than 300 combinations of perils and regions.

Within a given zone and for the same peril we assume that all treaties are fully correlated and use comonotonic addition to aggregate the losses[1]. This procedure produces for each zone a marginal distribution, decomposed into a severity distribution and a frequency of occurrence of events in the zone.

[1] Embrechts P., McNeil A., and Strausmann D., Correlation and Dependence in Risk Management: Properties and Pitfalls, ed. M.A.H. Dempster, Cambridge University Press, Cambridge, pp. 176-223 (2002)

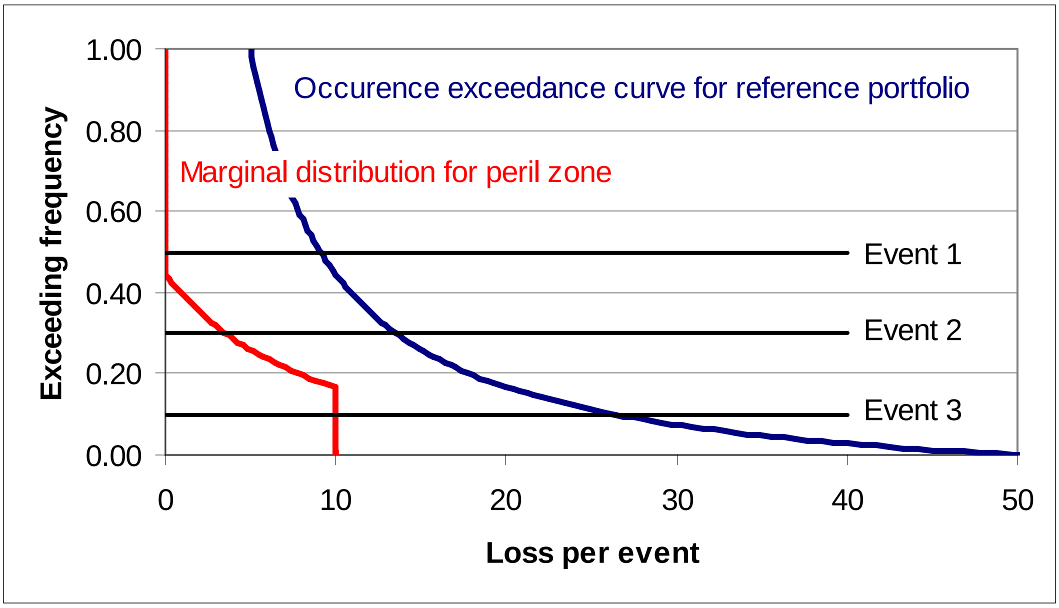

Figure 1: Illustration of mapping procedure for event-induced dependence structure. Each event loss on the occurrence exceedance curve for a reference run of a CAT model is mapped onto the loss corresponding to the same exceeding frequency for the marginal distribution of the peril zone.

This paper compares the results obtained by imposing different dependence structures between the losses occurring in different zones. On one hand we use standard models of dependence, namely Gaussian copula, t copula, and empirical copula[2]. The dependencies are applied to distribution of annual losses within the different zones. On the other hand, we developed a new type of dependence structure, so-called event-induced dependence, which share with CAT models the natural aggregation of losses at the event level.

The event-induced dependence method essentially maps the severity distribution calculated for a particular zone onto the occurrence exceeding frequency curve that is calculated by running a CAT model on a reference portfolio concentrated in this zone. The mapping procedure has the property that if one were to derive a frequency-severity model within each zone based on the mapped event losses, one would recover the frequency-severity model for that zone that was used as input to the procedure.

To illustrate the mapping procedure, suppose we consider three particular events out of a CAT model as illustrated in Figure 1. Assume that, based on the losses they cause to a test portfolio for the peril zone, their exceeding frequencies have been found to be: Event 1: 0.5, Event 2: 0.3, and Event 3: 0.1 events per year. We see how the mapped event losses can be directly read off the exceeding frequency curve for the peril-zone[3]. The mapped losses for our portfolio would then be 0 for event 1 (compared to 9 million for the test portfolio), 3.5 million for event 2 (compared to 12.5 million for the test portfolio), and 10 million for event 3 (compared with 25 million for the test portfolio).

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[2] Embrechts P., F. Lindskog, and McNeil A., Modelling dependence with copulas and applications to Risk Management *Handbook of Heavy Tailed Distributions in Finance* Ed: S. Rachev, Elsevier, Chapter 8, 329-384.

[3] Notice that events with exceeding frequency above the entry frequency of the program are assigned a loss of zero, as they correspond to small but frequent events whose losses are too small to reach the attachment point of the layer.

The mapping procedure simply assigns a loss to both countries on the base of the corresponding marginal distributions, but preserves the dependence structure implicit in the CAT model simulations. The dependence between losses in various zones results from events that simultaneously hit different regions. For example a windstorm passing over France and Germany will cause losses to test portfolios in both countries.

Results

This section first present the parametrization of copulas using runs of CAT models and discusses the importance of tail dependence. Then we compare annual aggregate distributions using the different types of copulas to the ones derived with the event-induced dependence model. We use as case study the loss distribution due to hurricanes in the Atlantic and to windstorms in Europe. For Atlantic wind, the zones consist of three main zones on the East Coast of the US and the Caribbean islands. For Europe wind, the zones coincide with the countries.

Parametrization of copulas

The parametrization of copulas is based on a set of simulated annual losses from a CAT model. We define reference portfolios in each region affected by a given peril and run the CAT model on each region separately to arrive at the set of simulated years.

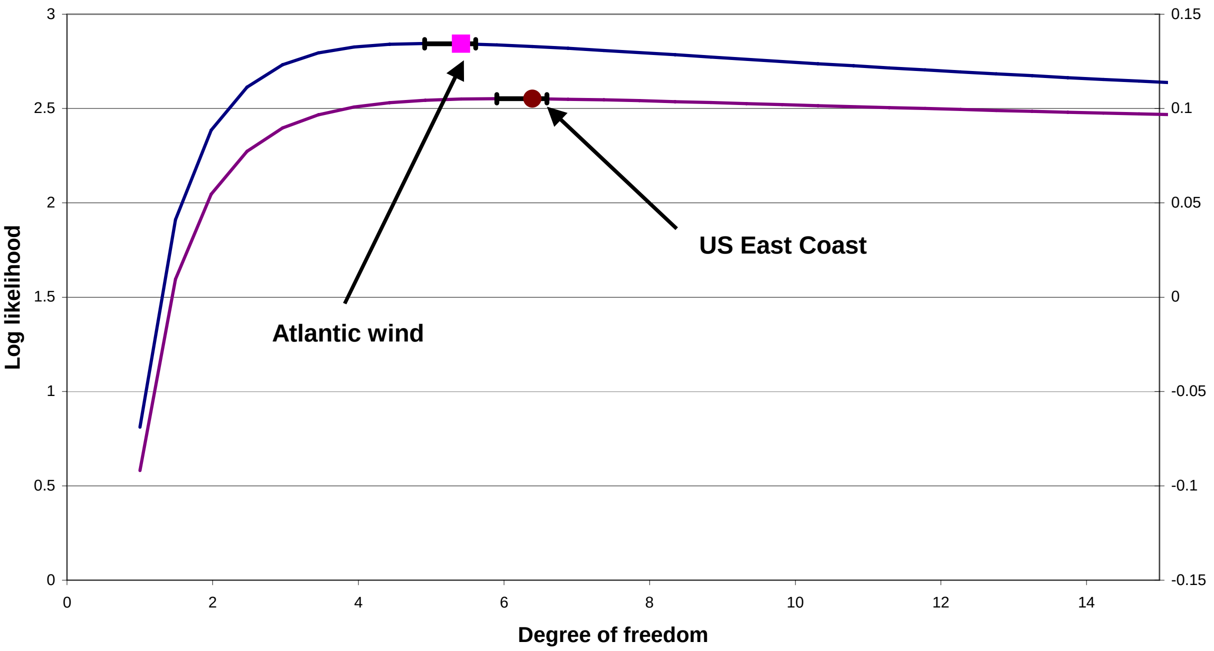

Figure 2: Log likelihood fit of degree of freedom for t-copula dependence structure. The fit is based on simulated annual losses using at cat model and for two different regions, namely the whole Atlantic hurricane region (i.e. including Caribbean) and the US East cost. The degree of freedom for the whole Atlantic is slightly lower than for the US East coast only.

The Gaussian copula is defined via Spearman’s rank correlation matrix, which is estimated with the set of simulated years. The t copula requires both a rank correlation matrix, Kendall’s Tau, as well as a degree of freedom. The degree of freedom controls the level of tail dependence present in the distribution, with a large degree of freedom corresponding to a low tail dependence. Figure 2 shows the log-likelihood fit for the degree of freedom in the case of

Atlantic wind risk[4]. The fitted degree of freedom takes intermediate values that correspond to moderate tail dependence. We use two independent CAT models to fit the degree of freedom and do not observe any great sensitivity in the fitted value.

The simulation of empirical copula applies the ranking of annual losses from the CAT model to simulated losses from the marginal distribution[5]. For each simulated annual loss *L* on the test portfolios we look at its ranking of losses *R(L)* for the corresponding zone and assign as loss the quantile on the marginal distribution for the zone $F^{-1}[R(L)/N]$, where *N* is the number of simulated years. The empirical copula approach is very similar to the event-induced dependence model but works on the rank of annual aggregate losses rather than event losses.

Tail dependence

We do not see any evidence of tail dependence in the set of annual losses simulated with CAT models, at least as measured by the upper tail dependence[6]. This measure is defined as the joint probability, *T(q)*, of having in both zones losses exceeding their respective quantiles, $F^{-1}_i(q)$ (i=1,2), at a given exceeding probability $p=1-q$ or return period:

$$T(q) = P\left[Y > F_1^{-1}(q) \middle| X > F_2^{-1}(q)\right]$$

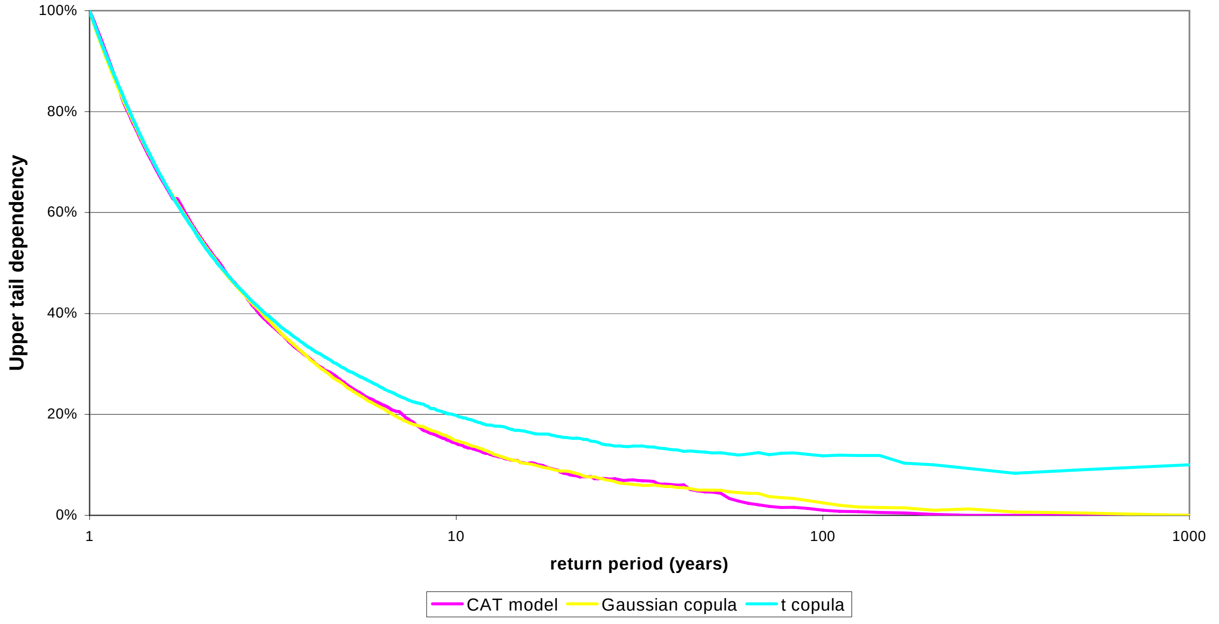

Figure 3: Comparison of upper-tail dependence for different copulas and simulated annual losses from a Cat model. We see no evidence of tail dependence, which tends to be overestimated by the t-copula.

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[4] Mashal, Roy and Zeevi, Assaf J., "Beyond Correlation: Extreme Co-movements Between Financial Assets" (October 14, 2002) http://ssrn.com/abstract=317122.

[5] Deheuvels P., La Fonction de Dépendance empirique et ses propriétés. Un test non paramétrique d'indépendance. Acad. R. Belg., Bull. Cl. Sci., 5 Série 65 (1979) 274-292.

[6] See note 1.

The presence of tail dependence would cause the measure *T(q)* to level off at a non-zero value for long return period, i.e. *q* approaching 1. Figure 3 shows the upper tail dependence between the two main zones on the US East coast as a function of return period .We compare the tail dependence present in the simulated years from a CAT model to the Gaussian and t copula using the fitted rank correlation coefficients and degree of freedom. We see that the simulated years with CAT model do not exhibit any tail dependence, which is in line with a Gaussian copula. The t copula, however, significantly overestimates the tail dependence.

We now turn to the comparison of annual aggregate loss distribution derived from different dependence structures. Figure 4 shows the annual aggregate loss distribution for the overall exposure to Atlantic hurricane using different copulas as well as the event-induced dependence. In comparison to this latter model, Gaussian and empirical copula give a good description of the aggregate distribution over the whole range of return periods. The t copula does, however, overestimates the losses in the tail of the distribution for return periods above 50 years. Similar results will be presented for Europe windstorm risk.

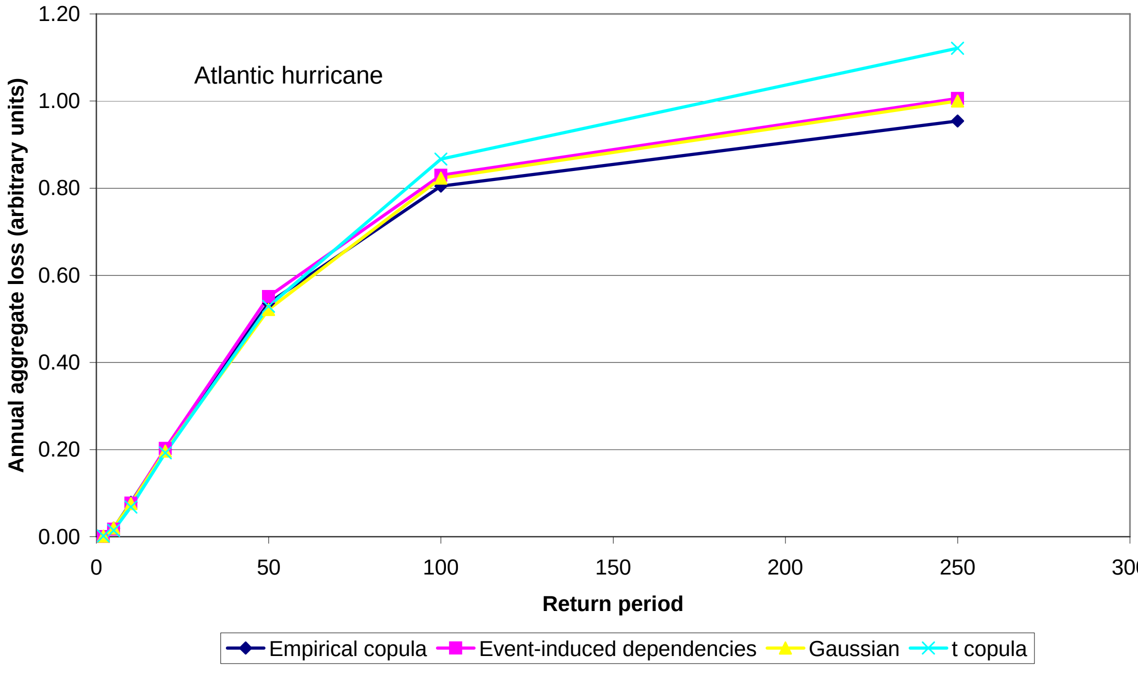

Figure 4 Annual aggregate loss distribution for Atlantic hurricane exposure with different dependence structures as a function of return period in years. We see that Gaussian and empirical copula give very similar results to the event-induced dependence. The t copula tends to overestimate the tail of the distribution.

Figure 5 shows the loss distribution for Europe windstorms as a function of return periods for the event-induced dependence and the copula models. As in the case of Atlantic hurricanes, we see that the empirical copula results in loss distributions very similar to the event-induced dependence. The Gaussian copula slightly underestimates the tail of the distribution while the t copula clearly overestimates it.

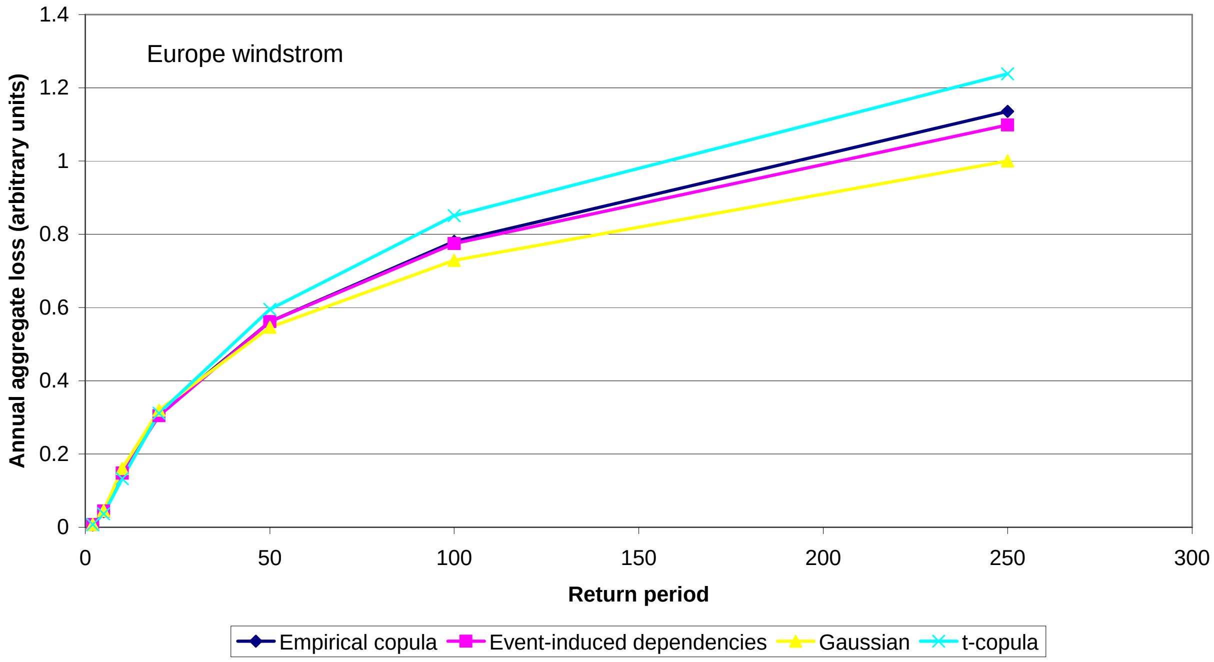

Figure 5: Annual aggregate loss distribution for Europe windstorm exposure with different dependence structures. We see that empirical copula gives very similar results as the event-induced dependence. The t copula tends to overestimate the tail of the distribution.

Conclusions

This paper presented a thorough analysis of the impact of different dependence structure for the calculation of annual aggregate loss distribution on a reinsurance portfolio exposed to natural catastrophes. We have presented a way to calibrate copula structures using simulated annual losses from CAT models. We have also introduced the event-induced dependence, which directly maps the marginal distribution of losses within a zone onto losses associated with the events of a CAT model.

We showed that empirical copula and event-induced dependence give very similar loss distributions. This result is not too surprising, as both methods are meant to reproduce the inherent dependency structures present in the CAT model simulations. The main difference lies in the fact that empirical copula are applied to the marginal distributions of annual aggregate losses while the event-induced dependence works at the event level.

We do not see any evidence of tail dependence in the simulated CAT events. A t copula model that contains tail dependence tends to systematically overestimate the losses above 50-year return periods. A simple Gaussian model, however, does a better job at describing the dependence present in CAT models, although it slightly underestimates the tail of the distribution for Europe windstorm losses.