Extended Solvency Margin as a Measure of the Insolvency Risk of Non-life Insurance Companies

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Presentation Plan

- Introduction
- Basic Concepts and Definitions
- Solvency Margin - SM
- Extended Solvency Margin - ESM
- Generalizations of ESM
- Conclusions

Introduction

- Project „SOLVENCY II”
- Interpretation of the EU SM

Basic Concepts and Definitions

Risk of insolvency – a state of an insurance company, in which a solvency indicator or a set of indicators (e.g. the solvency margin, risk-based capital, the probability of ruin) arbitrarily adopted by the insurance company, laid down in legal regulations, or adopted by the authority charged with assessing the solvency of an insurance company, exceeds an acceptable limit.

Basic Concepts and Definitions

The insolvency risk indicator (*IRI*)

$$IRI(t+1) = \frac{AVAILABLE\ CAPITAL_t}{REQUIRED\ CAPITAL(t+1)}$$

The condition of solvency

$$AVAILABLE\ CAPITAL_t > REQUIRED\ CAPITAL\ (t+1)$$

The available capital - a safety buffer at the given point in time t

The required capital for the period $t+1$ - the foreseen level of risk for the activities of the insurance company measured using the capital required to protect solvency

Basic Concepts and Definitions

The scale of safety - the function linking the capital requirement with risk and with the range of insurance activities

$$\frac{b(t+1)}{v(t+1)} = \beta$$

- The foreseen premium $b(t+1)$ - a good estimate of the potential risk and scale of insurance activities
- The available capital $v(t+1)$

Basic Concepts and Definitions

The scale of safety based on Council Directive 73/239/EEC

$$\beta(d) = \begin{cases} \dfrac{d}{KG} & \text{for } d \le \dfrac{KG}{0{,}18} \\ \dfrac{1}{0{,}18} & \text{for } \dfrac{KG}{0{,}18} < d \le H \cdot P_1 \\ \dfrac{d}{0{,}18 \min\{d; H \cdot P_1\} + 0{,}16 \max\{0; d - H \cdot P_1\}} & \text{for } d > H \cdot P_1 \end{cases}$$

H denotes the coefficient dependent on the reinsurers' share in claims incurred ($0{,}5 \le H \le 1$),

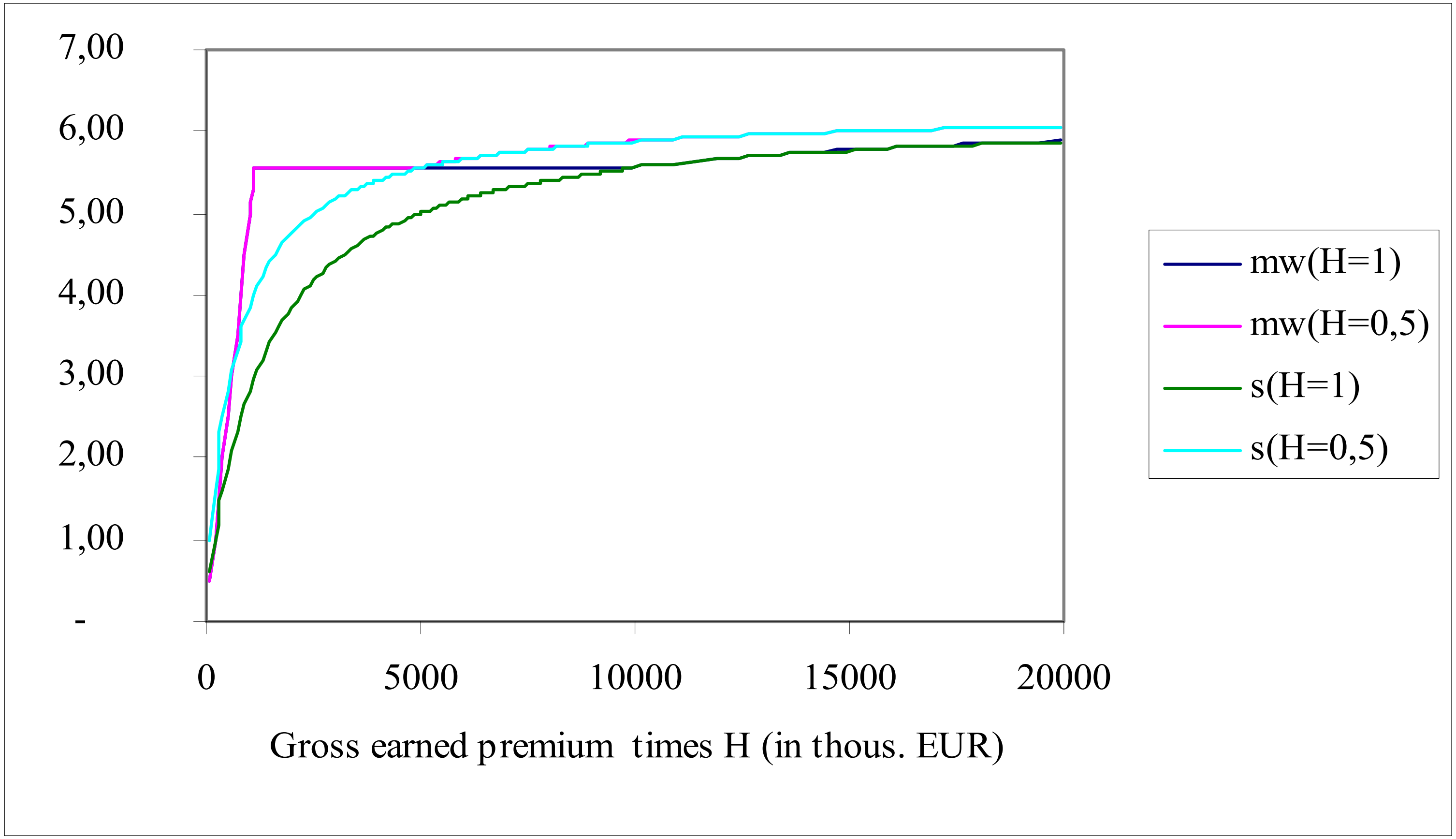
d – the forecast amount of gross earned premiums b multiplied by H,

KG – the guarantee fund, which, depending on the insurance group, may equal 200 000, 300 000, 400 000 or 1 400 000 EUR,

P_1 – the fixed (threshold value) equal 10 000 000 EUR.

Basic Concepts and Definitions

The scale of safety based on Council Directive 73/239/EEC



Basic Concepts and Definitions

Solvency Margin Model - model used to assessment risk of insolvency of insurance company in which:

- The potential risk and scale of insurance activities are measured by the foreseen premium
- The premium is estimated using production function determining the standard for insurance activity
- The scale of safety is a function of the foreseen premium

Solvency Margin

The model based on Council Directive 73/239/EEC

Solvency margin (*MW*) for non-life insurance:

$$MW(t+1) = \max\{MW1(t+1); MW2(t+1)\}$$

where

$$MW1(t+1) = \max\{0{,}18 \min\{d(t+1); H \cdot P_1\} + 0{,}16 \max\{0; d(t+1) - H \cdot P_1\}\}$$

$$MW2(t+1) = \max\{0{,}26 \min\{z(t+1); H \cdot P_2\} + 0{,}23 \max\{0; z(t+1) - H \cdot P_2\}\}$$

$$d(t+1) = d(t) \quad , \quad z(t+1) = \frac{1}{3}\sum_{i=t-2}^{t} z(i) \quad \text{or} \quad z(t+1) = \frac{1}{7}\sum_{i=t-6}^{t} z(i)$$

z - the forecast amount of gross claims inccured x multiplied by H

P_2 - the fixed (threshold value) equal 7 000 000 EUR.

Solvency Margin

The model based on Council Directive 73/239/EEC

The condition of solvency

$$SW > \max\left\{\max\left\{KG; \frac{MW}{3}\right\}, MW\right\}$$

where

SW - the available capital,

$\max\left\{\max\left\{KG; \frac{MW}{3}\right\}, MW\right\}$ - the required capital

Extended Solvency Margin

The financial result for year *t*:

$$F(t) = B(t) - X(t) - K(t) - O(t) - B_R(t) + X_R(t) + K_R(t) + I(t) - L(t),$$

where: $B(t)$ - gross earned premium in year *t*,

$X(t)$ - gross claims incurred in year *t*,

$K(t)$ - acquisition and administrative costs in year *t*,

$O(t)$ - balance of remaining costs and income on the technical account in year t,

$B_R(t)$ - reinsurance premium earned in year *t*,

$X_R(t)$ - reinsurers' share in claims incurred in year *t*,

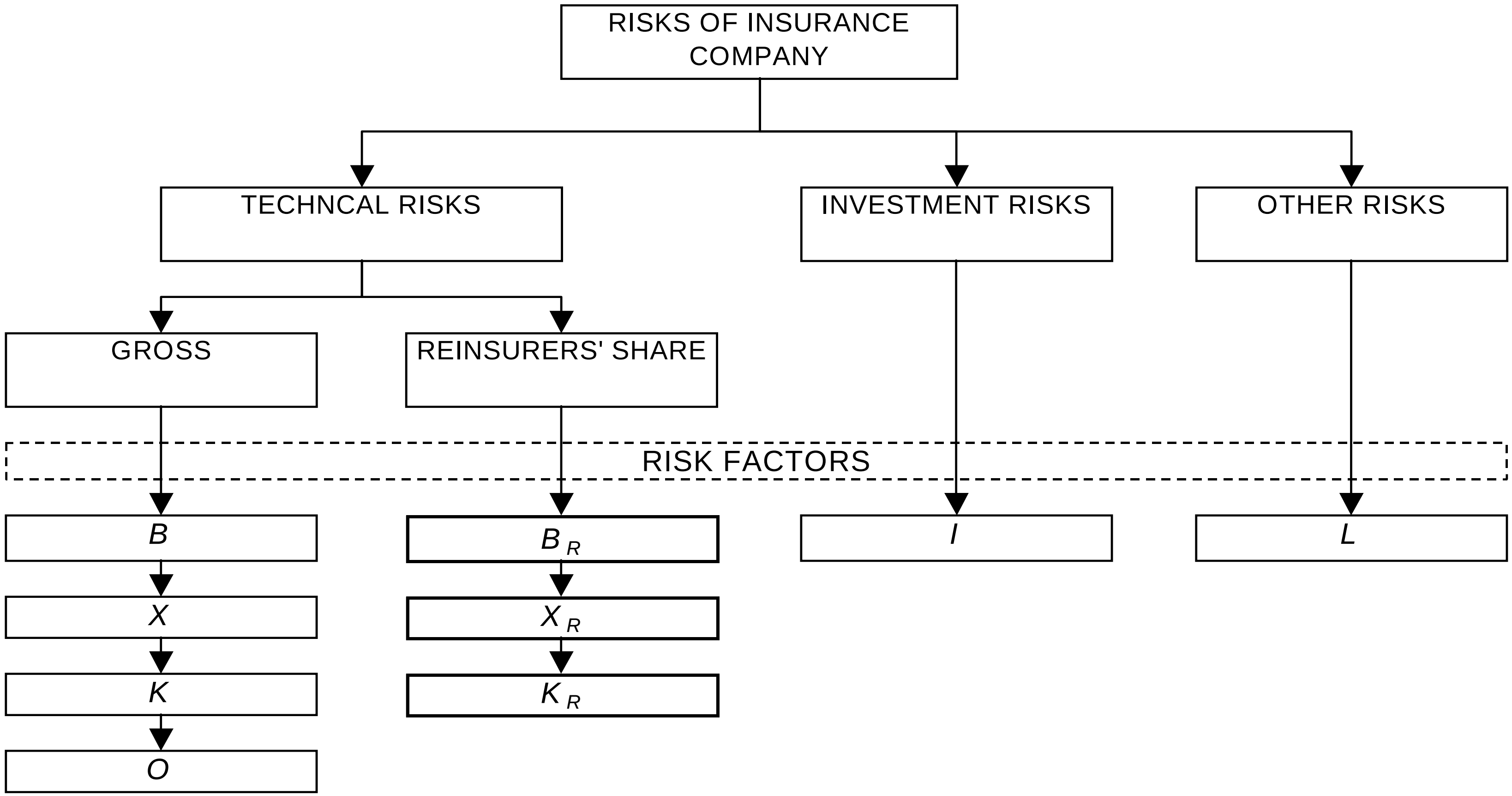
$K_R(t)$ - reinsurers' provisions and share in the profits of reinsurers in year *t*.

$I(t)$ - net income from investments, including changes in the value of assets and the cost of investment activities in year *t*,

$L(t)$ - the balance of remaining costs and income in year *t*, including extraordinary profits and losses.

Extended Solvency Margin

Taxonomy of risk in the ESM model



Extended Solvency Margin

Coefficients determining an insurance process:

$$\alpha_1 = \frac{\bar{x}(t)}{\bar{b}(t)}, \; \alpha_2 = \frac{\bar{k}(t)}{\bar{b}(t)}, \; \alpha_3 = \frac{\bar{o}(t)}{\bar{b}(t)}, \; \alpha_4 = \frac{\bar{b}_R(t)}{\bar{b}(t)}, \; \alpha_5 = \frac{\bar{x}_R(t)}{\bar{b}_R(t)}, \; \alpha_6 = \frac{\bar{k}_R(t)}{\bar{b}_R(t)},$$

$$\alpha_7 = \frac{\bar{i}(t)}{\bar{b}_N(t)}, \; \alpha_8 = \frac{\bar{l}(t)}{\bar{b}_N(t)}, \; \alpha_9 = \frac{\bar{f}(t)}{\bar{b}_N(t)}$$

$\bar{b}_N(t) = E[B_N(t)] = E[B(t) - B_R(t)] = \bar{b}(t) - \bar{b}_R(t)$ - the expected value of the earned premium net of reinsurance

Extended Solvency Margin

$\left[1, \alpha_1, \ldots, \alpha_9\right]$ - the insurance process.

$\left[1, \hat{\alpha}_1, \ldots, \hat{\alpha}_9\right]$ - the reference insurance process

The reference insurance process determines the conditions for equilibrium in a given period in time (e.g. annually), where by „equilibrium" one understands that the insurance company performs according to what is expected, as measured by its financial result.

Extended Solvency Margin

$\mathbf{y} = [y_i(t)], \quad i = 0,1,\ldots,9$ - the actual activity of the insurance company

$\hat{\mathbf{y}} = [\hat{y}_i(t)], \quad i = 0,1,\ldots,9$ - the effective insurance activity

The effective insurance activity - the hypothetical activity that satisfies the condition of equilibrium posited by the reference insurance process with at least one of its coordinates equal to the relevant coordinate of vector $\mathbf{y}$.

Extended Solvency Margin

The scale of safety $\beta\big(b(t)\big)$ - example

$$\beta\big(b(t)\big) = A\,\frac{b(t) - C}{b(t) + B}$$

where $A = \lim_{b \to \infty} \beta\big(b(t)\big) = \dfrac{6{,}25}{G} \quad \beta_{,}\big(KG \quad\big) = 1 \quad \beta_{,}\big(P_1\big) = \dfrac{5{,}55}{G}$,

G - the share of the earned premium net of reinsurance in the gross earned premium

Extended Solvency Margin

The capital requirements in relation to the respective variables (risk factors):

$$v_0(t+1)=\frac{y_0(t+1)}{\beta},\ v_i(t+1)=\frac{y_i(t+1)}{\beta\hat{\alpha}_i},\ i=1,2,3,4,$$

$$v_i(t+1)=\frac{y_i(t+1)}{\beta\hat{\alpha}_4\hat{\alpha}_i},\ i=5,6,\ v_i(t+1)=\frac{y_i(t+1)}{\beta(1-\hat{\alpha}_4)\hat{\alpha}_i},\ i=7,8$$

Extended Solvency Margin

The condition of solvency

$$SW > v'(t+1)$$

where

SW - the available capital,

$v'(t+1)$ - the required capital (ESM)

$$v'(t+1) = \max\{v_i(t+1); i = 0,1,2,3,7,8\} - \min\{v_i(t+1); i = 4,5,6\}$$

Extended Solvency Margin
Empirical example – Polish data

Tab. 1. Parameters of the reference insurance process

1	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$	$\hat{\alpha}_5$	$\hat{\alpha}_6$	$\hat{\alpha}_7$	$\hat{\alpha}_8$	$\hat{\alpha}_9$
100%	73,4%	35,4%	1,1%	62,4%	69,0%	26,3%	13,9%	2,0%	-6,8%

Tab. 3. The correspondence of assessments as to the risk of insolvency obtained on the basis of SM and ESM

	Method	Extended solvency margin			
	Set of companies	**Reference set**		**Set of joint stock companies in the period 1995-2002**	
Method	Class of risk	0	1	0	1
Solvency margin	0	59,7%	9,0%	68,9%	9,3%
	1	3,0%	28,4%	3,1%	18,7%

Generalizations of ESM

- The insurance portfolio
  - product diversity
  - geographic diversity
- The investment portfolio
- More complex form of the safety scale

$$\frac{y_0(t+1)}{v(t+1)} = \beta(y_0(t))\frac{S[\mathbf{z}'(t)\boldsymbol{\gamma}]}{1-\widetilde{P}}$$

- Different production functions

Conclusions