Stochastic Surrender With Asymmetric Information

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Outline and Objectives.

1. Introduction.

2. No arbitrage hypothesis and enlargement of filtration.
   - Impact of the No Arbitrage (NA) hypothesis on the surrender time τ (and vice versa).
3. Stochastic surrender with asymmetry of information.
   - Surrender time = random time with Hazard process.
4. Risk neutral valuation with asymmetry of information.
   - General formulas.
5. Conclusion.

Introduction.

Introduction.

- The financial market :
  - 1 locally risk free asset (S^0_t) strictly positive : $S^0_t=\exp(D_t)$.
  - s risky assets (S^i_t for i=1,..,s).

- Hypothesis : No arbitrage opportunity in the financial market.

  ➔ There exists a measure Q equivalent to P such that $S^i_t \backslash S^0_t$ are (local)-martingales with respect to ☞.

- ☞$=(F_t)_{t\geq 0}$: filtration generated by the prices of these (s+1) financial assets.

Introduction.

- The insurer's information.
  - The information coming from the financial market : ☞.
  - The information related to the surrender time τ : ☟ = filtration generated by $H_t = 1\{\tau \leq t\}$.

➔ Insurer's information :

☝ where ☝ = $(G_t)_{t\geq 0}$ where $G_t = F_t$ v ♦(H_t).

Rem : We do not assume the independence between F_t and ♦(H_t)

NA Hypothesis and Enlargement of Filtration.

NA Hypothesis and Enlargement of Filtration.

- Risk neutral valuation with respect to ☝ implies the NA hypothesis holds on the financial market with respect to ☝.

- Is this reasonable ?
  - Yes, means the surrender decision does not carry any information that can induce arbitrages.

- Problem :
  - Discounted financial prices should be (☝,Q)-local martingales.

  But (☞,Q)-local martingales are not necessarily (☝,Q)-local martingales.

NA Hypothesis and Enlargement of Filtration.

- Sufficient condition for NA under ☝ :
  - Each (☞,Q)-local martingale is a (☝,Q)-local martingale.
  - Equivalent to the (H) hypothesis in probability :
    - « For every t, F_∞ and G_t are conditionally independent with respect to F_t »

- Does NA under ☝ implies the (H) hypothesis ?
  - If the financial market is complete under ☞ : Yes.
  - If incomplete : ?

NA Hypothesis and Enlargement of Filtration.

- In our setting, (H) hypothesis is equivalent to :

$$Q\left(\tau \leq t \middle| F_\infty\right) = Q\left(\tau \leq t \middle| F_t\right)$$

- Ex :
  - If τ is a ☞-stopping time, then (H) holds (☞ = ☝,📪no enlargement of filtration).
  - If τ is independent of ☞, then (H) holds.

    Ex : mortality, surrender with a priori fixed probabilities.

Rem : (H) is not invariant for an equivalent change of probablity measure.

Surrender with Asymmetry of Information

Surrender time in the academic literature.

- Surrender time is a ☞-optimal stopping time (Bacinello, Grosen and Jorgensen,…).
  - Surrender the first time the surrender value is higher or equal to the « fair value » of the insurance contract.
- Advantage :
  - Endogenous definition of the surrender time.
    - The surrender time depends intrinsically on the specificities of the insurance contract.
    - The surrender time depends on the evolution of the financial market.

Surrender time in the academic literature.

- Drawback :
  - The surrender decision is only based on the financial information ☞.
    - F_t is sufficient to know if someone has surrendered before t.
    - Idiosyncratic information is not relevant.
      - ➔ For a given contract, all the policyholders surrender at the same time (0-1 situation).
    - If ☞ is a brownian filtration, the surrender time is predictable.
      - ➔ no surprise.
    - If the financial market is complete, the insurance market is also complete.
  - These drawbacks come from the assumption : τ is a ☞-stopping time

    ➔ Perfect symmetry of information assumption.

- τ should'nt be a ☞-stopping time.

  ➔ introduce an asymmetry of information

Surrender time with asymmetry of information.

- a ☞-hazard process $\hat{\Gamma}_t$ is defined by :

$$Q(\tau \le t \mid F_t) = 1 - \exp(-\hat{\Gamma}_t) \qquad \forall\ t$$

- Remarks :
  - We have $\hat{\Gamma}_t$= -ln(1-Q($\tau \le$ t | F_t)). So if Q($\tau \le$ t | F_t) = 1 ➔ $\hat{\Gamma}_t$ is not defined.
  - If τ is ☞-stopping time ➔ Q($\tau \le$ t | F_t) = 0 ou 1 ➔ no ☞-hazard process.
  - Essentially, every random time not ☞-measurable admits a ☞-hazard process.
- (H)-hypothesis : Q($\tau \le$ t | F_t) is increasing.

Surrender time with asymmetry of information.

- If τ is measurable with respect to a filtration independent of ☞.
  - The policyholder takes his decision only on idiosyncratic informations not observed by the insurer.
  - $Q(\tau \leq t \mid F_t) = Q(\tau \leq t)$ ➔ a priori fixed probabilities of surrender.

- If τ is measurable with respect to a bigger filtration than ☞.
  - The policyholder takes his decision on the financial information + idiosyncratic informations not observed by the insurer.
  - τ is not independent of the financial market.
  - $Q(\tau \leq t \mid F_t)$ is a stochastic process ☞-adapted ➔ the surrender probabilities depend on the financial market.
  - Rem : τ can still be an optimal stopping time.

➔ Reconciles endogenous and exogenous models of τ under the Asymmetry of information assumption.

Risk Neutral Valuation Formula with Asymmetry of Information.

The Insurance Contract.

- Notation : $H_t = 1_{\{\tau \le t\}}$
- Insurer Payments : 3 building blocks
  - A) Term T of the contract if no surrender :

    $g(T,\omega)1_{\{\tau > T\}}$ (with g(T, ω) is F_T-Mesurable).

  - B) Cumulated payments up to surrender:

    $$C(T,\omega)1_{\{\tau>T\}} + C(\tau-,\omega)1_{\{t_0<\tau\le T\}} = \int_{t_0}^{T} (1-H_u)\,dC(u,\omega)$$

  - C) the surrender value :

    $$R(\tau,\omega)1_{\{t_0<\tau\le T\}} = \int_{t_0}^{T} R(u,\omega)\,dH_u$$

- No mortality.

The Insurance Contract.

- Policyholder Payments :
  - Premiums paid at fixed dates t_i avec i =0,…,N-1.

$$\sum_{i=0}^{N-1} P(t_i,\omega) 1_{\{\tau > t_i\}}$$

Risk Neutral Valuation.

- General Risk neutral valuation formulas :
  - Present value of the insurer payments :

$$L_t = E^Q\left[e^{-(D_T - D_t)} g(T,\omega) 1_{\{\tau > T\}} + \int_t^T e^{-(D_u - D_t)} (1 - H_u) dC(u,\omega) + \int_t^T e^{-(D_u - D_t)} R(u,\omega) dH_u \middle| \mathrm{G}_t \right]$$

  - Present value of the policyholder payments :

$$A_t = E^Q\left[\sum_{i=0}^{N-1} e^{-(D_{t_i} - D_t)} P(t_i,\omega) 1_{\{\tau > t_i\}} \middle| G_t \right]$$

  - Insurance contract's « Fair value » : $V_t = A_t - L_t$

Risk Neutral Valuation Formula.

- If τ admits an increasing (Q,☞)-hazard process $\hat{\Gamma}_t$:
  - Present value of the insurer's payments :

$$L_t = 1_{\{\tau > t\}} E^Q \left[\begin{array}{l} e^{-(D_T - D_t)} e^{-(\hat{\Gamma}_T - \hat{\Gamma}_t)} g(T,\omega) \\ + \int_t^T e^{-(D_u - D_t)} e^{-(\hat{\Gamma}_u - \hat{\Gamma}_t)} dC(u,\omega) \\ + \int_t^T e^{-(D_u - D_t)} e^{\hat{\Gamma}_t} R(u,\omega) d(1 - e^{-\hat{\Gamma}_u}) \end{array} \middle| F_t \right]$$

  - Present value of the policyholder's payments :

$$A_t = 1_{\{\tau > t\}} E^Q \left[\sum_{i=0}^{N-1} e^{-(D_{t_i} - D_t)} e^{-\left(\hat{\Gamma}_{t_i} - \hat{\Gamma}_t\right)} P\left(t_i, \omega\right) \middle| F_t \right]$$

  ➔ Equivalent to no surrender but modified yield curve : $(D + \hat{\Gamma})_t$

- Rem : no continuity assumption needed on $\hat{\Gamma}$.

Conclusions.

Conclusions.

1. Equivalence between NA and (H) hypothesis (under market completeness).
2. Asymmetry of information assumption :
   - Has more realistic implications.
     - ☞ is no more sufficient to know if someone has surrendered or not.
     - Surrender decision depends on idiosyncratic elements.
     - Surrender decision can depend on the financial market.
     - The surrender is not predictable.
     - Incompleteness of insurance market with respect to surrender risk.
   - Imply the existence of an ☞-hazard process.
     - Weak assumption on the proba of a surrender risk.
     - No continuity of the hazard process + Intensity does not have to exist.
     - Pricing equivalent to pricing with a modified stochastic yield curve.
     - For the insurer, under Q, the surrender time is the time of the first jump of a cox process.

Conclusion.

- Reconcile exogenously and endogenously defined surrender times under the asymmetry of information assumption.
  - Endogenous surrender : Out of reach ?
  - Exogenous surrender : the only one useful in practice ?
    - Need more econometric research.
- Material not cover in this presentation :
  - Change of measure for the enlarged filtration.
  - Application to unit-linked contract.