RISK BASED CAPITAL MODELLING FOR P&C INSURERS AND FINANCIAL SENSITIVITY

Laura Ballotta
Cass Business School
L.Ballotta@city.ac.uk

Nino Savelli
Catholic University of Milan
nino.savelli@unicatt.it

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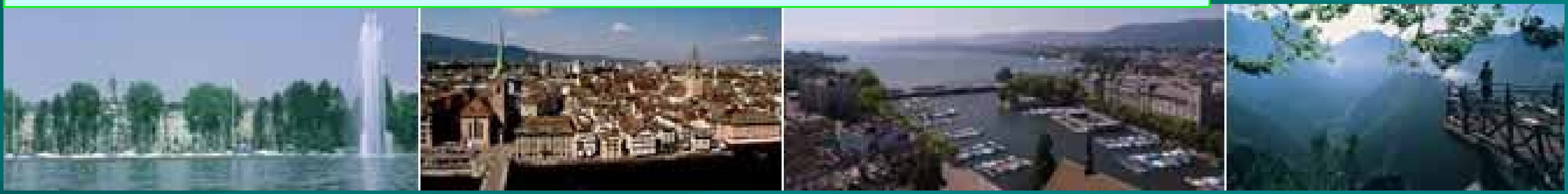


Motivation: Insurance Risk Management and Solvency (1)

- **MAIN PILLARS OF THE INSURANCE MANAGEMENT:**

  the strategic triangle of competing forces:
  - market share
  - return for stockholders' capital
  - financial strength/stability
- **RISK-BASED CAPITAL REQUIREMENTS:**

  to assess the risk capital of the company according to its own real risk profile. Simulation models may be used for defining New Rules for Capital Adequacy (see e.g. IAA Solvency Working Party)
- **INTERNAL RISK MODELS (IRM):**

  to be used
  - for solvency purposes (e.g. Pillar 1&2 of Solvency II)
  - to define the most appropriate management's strategies

  IRM could allow for a more comprehensive representation of the business of an individual firm than the standard formula, with capital requirements more significantly aligned to the effective risk of the company

Motivation: Insurance Risk Management and Solvency (2)

- **A NEW APPROACH FOR THE SUPERVISORY AUTHORITIES:**
  - Stress testing in order to assess the solvency profile of the Insurer
  - Validation and approval of the Insurer IRM on the basis of
    - Prudential requirements
    - Comparability & consistency requirements (with respect to the supervisor's view of key minimum performance criteria)
  - Indication of the appropriate course of action to follow in case of an excessive risk of insolvency over the short term

- **THE AIM OF THIS PAPER:**
  - to propose a possible simple risk model for a P&C insurer incorporating
    - the underwriting risk
    - the financial risk
  - to analyse specifically the impact of different asset allocation strategies on the risk profile of the insurer

Agenda

- General framework of the model
  - The Insurance sub-model
  - The Investment sub-model
  - The asset allocation rule
- The numerical experiment
  - The impact of various asset allocations on the Insurer's Risk Based Capital
- Final Comments

General framework of the model

- **Company:**
  General Insurer with only **1 line of casualty** insurance
- **Time Horizon: 3** years
- **Aggregate Claim amount:**
  Compound Mixed Poisson Process
- **Number of Claims:**
  Negative Binomial distribution
- **Claim Size:**
  LogNormal distribution
- **Dynamic Ins. Portfolio:** Volume of premiums increases every year according to **real growth** and **claim inflation**
- **Reinsurance:**
  reinsurance cover is **ignored**
- **Investment Portfolio:**
  1 category of assets for **Equities** and other **5** categories for **Gov.Bonds**, differentiated according to time to maturity (1, 2, 3, 5 and 10 years)
- **Investment Return:**
  - **Geometric Brownian motion** for **equities**
  - **CIR process** for **interest rates**
- **Asset Allocation rule:**
  constant proportion
- **Monte Carlo approach:**
  100,000 simulations
- **Risks not included:**
  - **Claim Reserve risk**
  - **Credit** and **Operational risk**
  - **ALM risk**

Risk Reserve process (U_t)

$$\tilde{U}_t = (1+\tilde{j}_t)\cdot\tilde{U}_{t-1} + (\pi_t - \tilde{X}_t - E_t) + \tilde{j}_t \cdot \tilde{L}_{t-1} - T\tilde{X}_t - \tilde{D}_t$$

- **U_t** = Risk Reserve at time t
- **π_t = Gross premiums at year t**
- **X_t** = Aggregate claims amount year t
- **E_t** = general and acquisition expenses year t

- **j_t** = Investment return rate of year t
- **L_{t-1}** = Loss Reserve at time t-1
- **TX_t** = Taxation amount year t
- **D_t** = Dividends year t

Gross Premiums, Safety Loading and Loss Reserve

- **Gross Premiums:**

$$\pi_t = (1+i)*(1+g)*\pi_{t-1}$$

i = claim inflation rate (constant)

g = real growth rate (constant) – assumed not related to the market level of the premiums

- Furthermore:

$$\pi_t = (1+\varphi)*\mathrm{E}(X_t) + c*\pi_t$$

where:

- φ = **safety loading** coefficient

- c = expenses loading coefficient

- **Loss Reserve:**

$$L_t = \delta * \pi_t$$

with coefficient δ constant

- The **safety loading coefficient** φ is computed according to the standard deviation principle:

$$(1-tx)\cdot\left[\varphi\cdot \mathrm{E}(\tilde{X}_1)+L_0\cdot \mathrm{E}(\tilde{j}_1)\right]=b\cdot\sqrt{Var(\tilde{X}_1)+L_0^2\cdot Var(\tilde{j}_1)}$$

in other words, **the insurer is asking for an expected profit from the insurance business equal to *b* (0.35) for each unit of insurance risk** (measured in terms of standard deviation)

Aggregate Claims Amount (X_t)

$$\tilde{X}_t = \sum_{i=1}^{\tilde{k}_t} \tilde{Z}_{i,t}$$

only 1 LoB is considered

CAT claims not considered

- k_t = **Claim Number** of year *t*
  here assumed to be **Negative Binomial** distributed, i.e.
  - **k** follows a Poisson distribution with a stochastic parameter n*q,
  - **q** is a multiplicative random **structure variable** with mean 1 and distributed as a Gamma(h,h), which captures short-term fluctuations (we ignore systematic changes)
  - **n** is the expected number of claims increases with the real growth rate, i.e. $n_t = n_0 * (1+g)^t$
- $Z_{i,t}$ = **Claim Size** for the i-th claim of year *t* (independent of **k**)
  here assumed to be **LogNormal** distributed, with values increasing every year according to the deterministic claim inflation (**i**) only.
  The claim sizes Z_t **are assumed to be i.i.d.** random variables
- **X_t are time independent variables.** In the real world, though, long-term cycles are present and then significant auto-correlation might be observed (especially for the case of medium/long-term analyses).

The Investment Model (1)

- The insurer invests
  - α% of the available resources in an equity index, *S*, and
  - $(1-\alpha)$% in a portfolio of zero coupon bonds, *P*, with different redemption dates
    - $\beta^{(\tau)}$ % is invested in the bond with time to maturity τ
  - the asset allocation and the asset mix are constant over time

$$dS_t = \mu S_t dt + \sigma S_t dW_t; \qquad dr_t = \kappa(\theta - r_t)dt + v\sqrt{r_t}\,dZ_t$$

$$dP_t(t+i) = a(t;t+i)P_t(t+i)dt + b(t;t+i)P_t(t+i)dZ_t$$

$$i \in N := \{1,2,3,5,10\}$$

$$dW_t dZ_t = \rho dt \Rightarrow Z_t = \rho W_t + \sqrt{1-\rho^2}\,Y_t$$

$$\frac{dA_t}{A_t} = \left[\alpha\mu + (1-\alpha)a(t;t+i)\right]dt + \left[\alpha\sigma + (1-\alpha)\rho\Sigma(t;t+i)\right]dW_t + (1-\alpha)\sqrt{1-\rho^2}\,\Sigma(t;t+i)\,dY_t$$

$$a(t;t+i) = r_t + \lambda(t,r); \qquad \Sigma(t;t+i) = \sum_{i\in N}\beta^{(i)}b(t;t+i)$$

The Investment Model (2)

- We also assume that every year the insurer invests the cashflows, *F*, originated by the "pure" insurance business in the financial portfolio *A*
  - The cashflows arise from consideration of
    - the overall annual rate of growth of the premium (which depends on g and i)
    - the amount of claims deferred from the previous year and paid in the current year, C_t^d
    - the amount of claims occurred in the current year and settled during the same period, C_t^c

$$F_t = (1-c)\pi_t - \left(\tilde{C}^c{}_t + C^d{}_t\right) = \pi_t\left[(1-c) + \delta\left(1 - \frac{1}{(1+i)(1+g)}\right)\right] - \tilde{X}_t$$

$$A_t = \alpha\left[A_{t-1} + F_{t-1}\right]\frac{S_t}{S_{t-1}} + (1-\alpha)\left[A_{t-1} + F_{t-1}\right]\sum_{i\in N}\beta^{(i)}\frac{P_t(t+i)}{P_{t-1}(t+i)}$$

The Asset Allocation Rule

- Asset allocation

		Std. Insurer	Insurer A	Insurer B	Insurer C
Equity	α	15%	30%	50%	100%
Bond port.	$1-\alpha$	85%	70%	50%	0%

- Asset mix (bond portfolio)

Maturity (years)	τ	1	2	3	5	10
Weight	$\beta(\tau)$	40%	25%	15%	10%	10%

Parameters of the Insurance Model

Initial expected number of claims	20000
Variance structure variable	0.02
Skewness structure variable	+ 0.28
Initial expected claim size ($)	6000
Variability coefficient of claim size	7

NOTE:
Parameters as **Var(q)**, **E(Z)** and **CV(Z)**
are derived from the IAA Solvency Working Party Report (2004)

Claim Inflation	5%
Real Growth rate	5%
Expenses Loading coefficient	25%
Safety Loading coefficient (φ)	Depending on asset allocation
Loss Reserve ratio	120%
Taxation rate	0%
Dividends rate	0%

Initial Risk Premium (mill $)	120.0
Initial Gross Premiums (mill $) π_0	Depending on asset allocation

Parameters of the Investment Model

Equity Index		
Expected rate of return	μ	10%
Volatility	σ	20%

Interest rate (CIR)		
Long run mean	θ	4%
Speed	κ	0.1
Diffusion	υ	4.7%
Market price of interest rate risk	λ	-0.005
Correlation	ρ	-0.2
Current short rate	r_0	4.5%

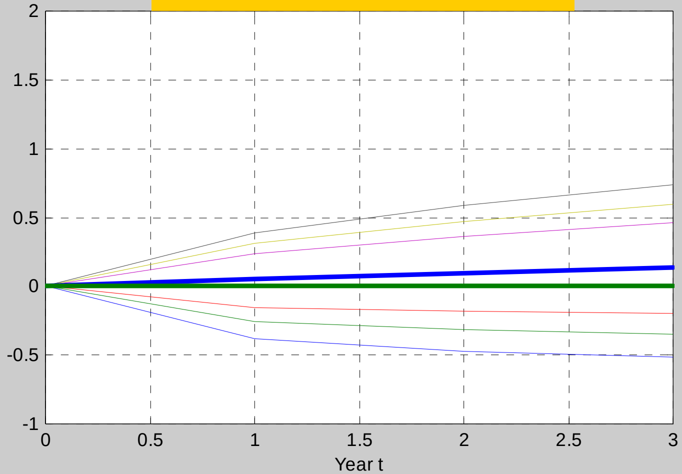
Zero Yield Curve		
1 year	$r(0,1)$	4.47%
2 years	$r(0,2)$	4.38%
3 years	$r(0,3)$	4.39%
5 years	$r(0,5)$	4.43%
10 years	$r(0,10)$	4.49%

Source: Bank of England (31/12/2004)

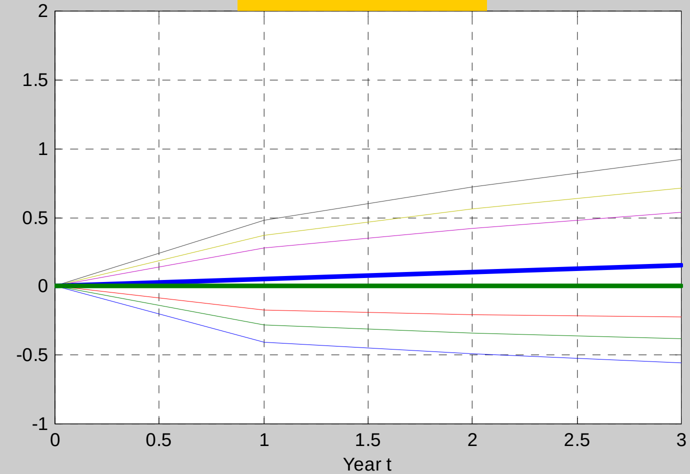
The percentiles of the capital ratio $\tilde{U}_t/\pi_t$

(0.1%; 1%; 25%; 50%; 75%; 99%;99.9%)

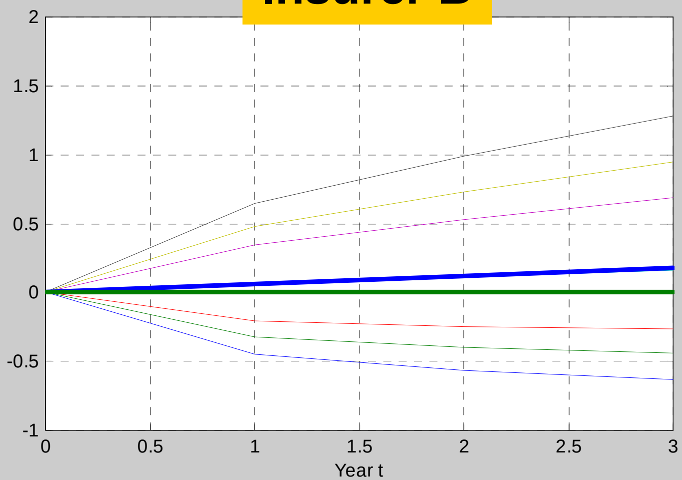
Standard Insurer



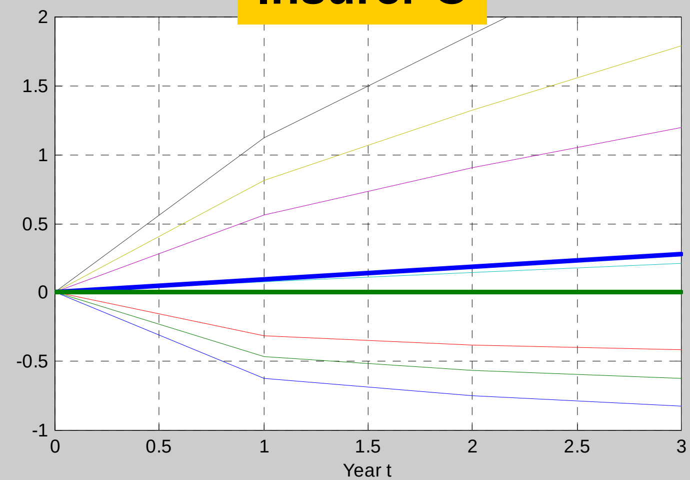
Insurer A



Insurer B



Insurer C



100,000 simulations

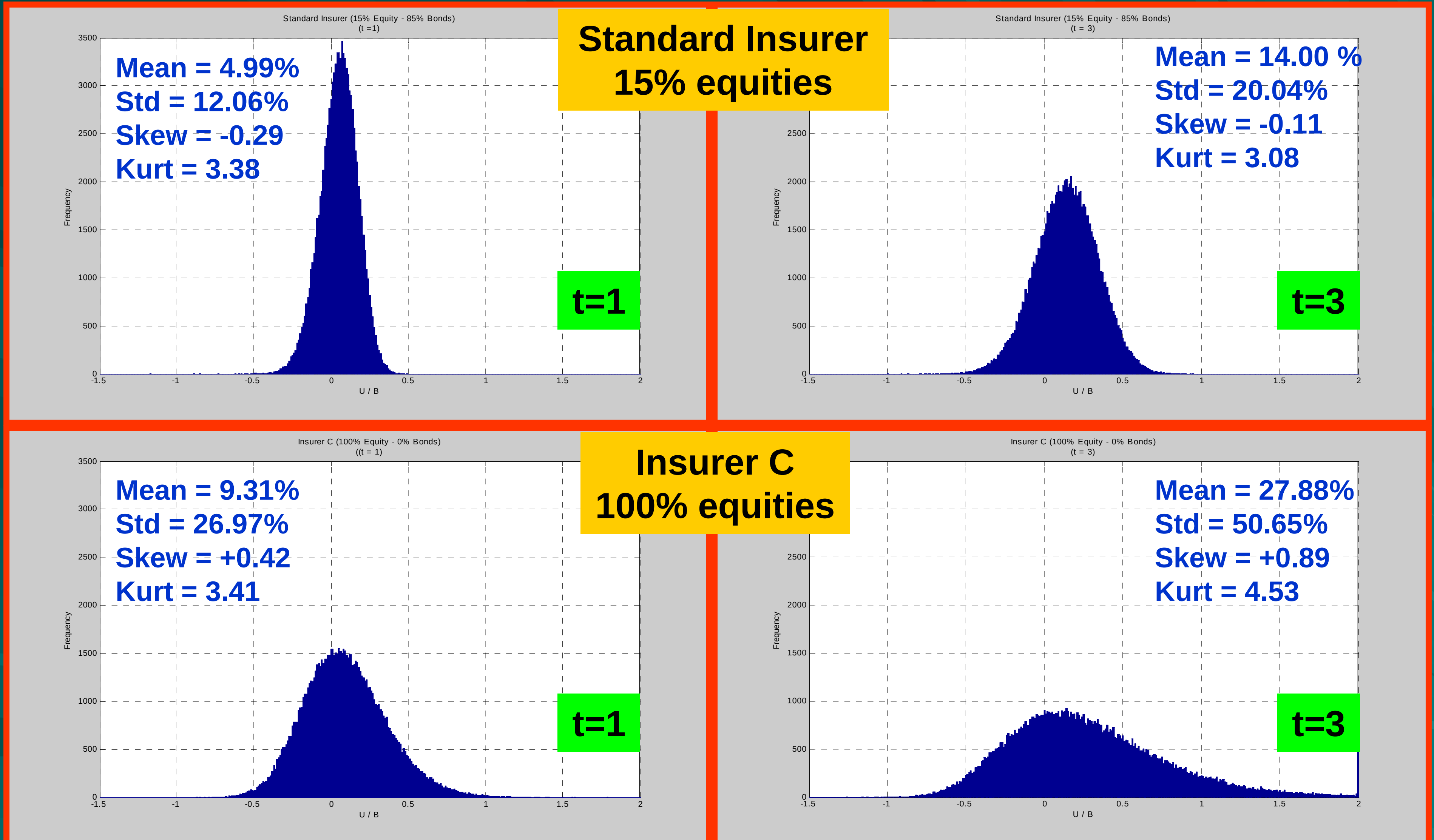
Moments of capital ratio $\tilde{U}_t/\pi_t$

	Standard Insurer (15% Equities - 85% Bonds)			Insurer A (30% Equities - 70% Bonds)		
	T=1	T=2	T=3	T=1	T=2	T=3
Mean (%)	4.99	9.62	14.00	5.51	10.58	15.37
Std (%)	12.06	16.70	20.04	13.74	19.12	23.14
Skew	- 0.29	-0.17	-0.11	-0.12	-0.01	0.07
Kurt	3.39	3.15	3.08	3.36	3.17	3.21

	Insurer B (50% Equities - 50% Bonds)			Insurer C (100% Equities - 0% Bonds)		
Mean (%)	6.37	12.35	18.21	9.31	18.63	27.88
Std (%)	16.84	23.80	29.28	26.97	39.76	50.65
Skew	0.13	0.273	0.36	0.42	0.703	0.89
Kurt	3.26	3.35	3.43	3.41	4.03	4.53

100,000 simulations

Prob. Distrib. of the capital ratio u_t



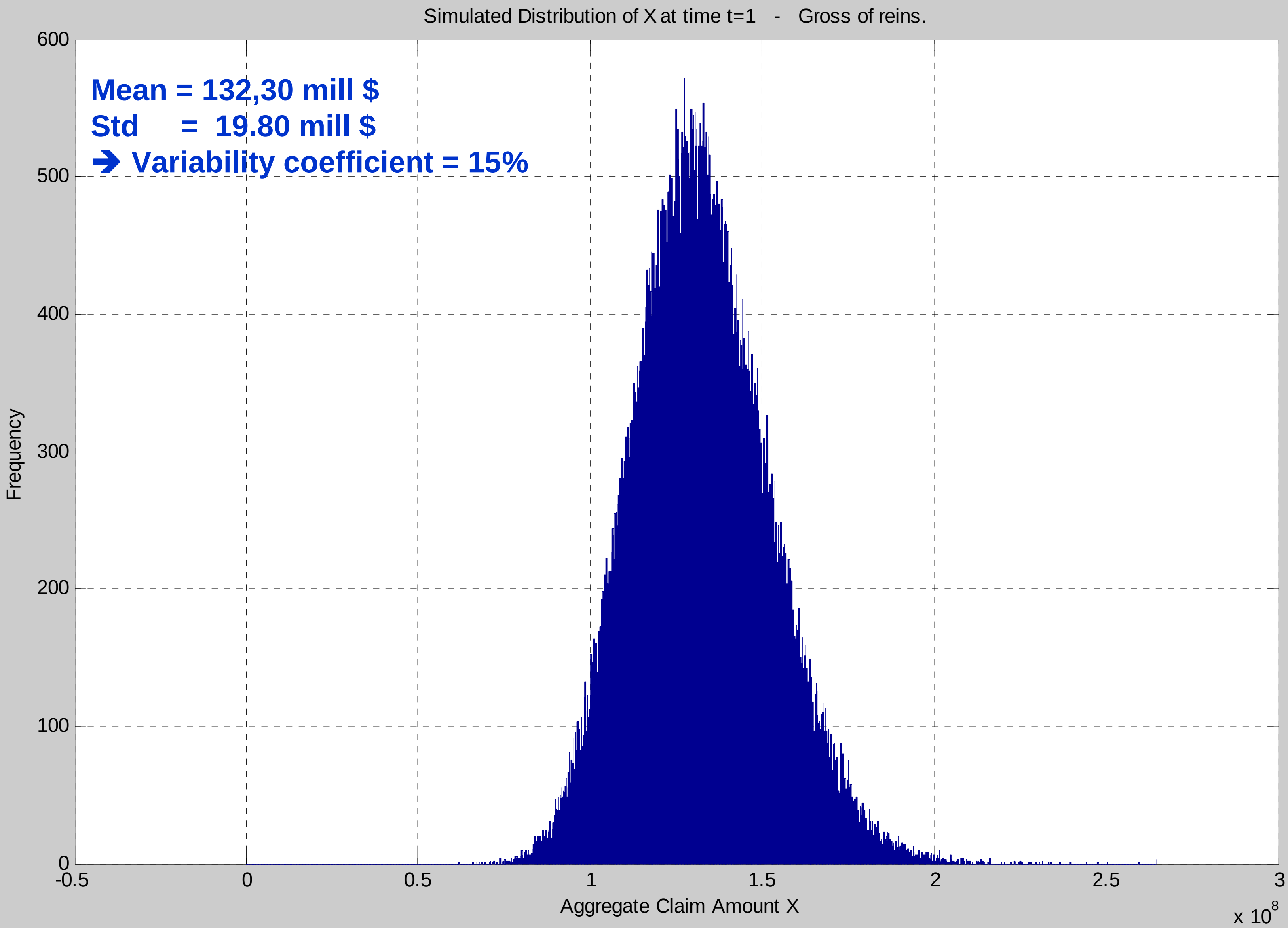
The Risk Based Capital

	Standard Insurer (15% Equities - 85% Bonds)			Insurer A (30% Equities - 70% Bonds)		
Safety loading coefficient and initial gross premium	$\varphi = -1.42\% \rightarrow \pi_0 = 157.73$			$\varphi = -2.08\% \rightarrow \pi_0 = 156.67$		
RBC ratio (per unit of initial Gross Premiums π_0)	T=1	T=2	T=3	T=1	T=2	T=3
CL: 99.0 %	26.75	34.71	40.14	28.93	37.27	42.89
CL: 99.9 %	40.13	52.28	60.28	42.34	53.19	63.30

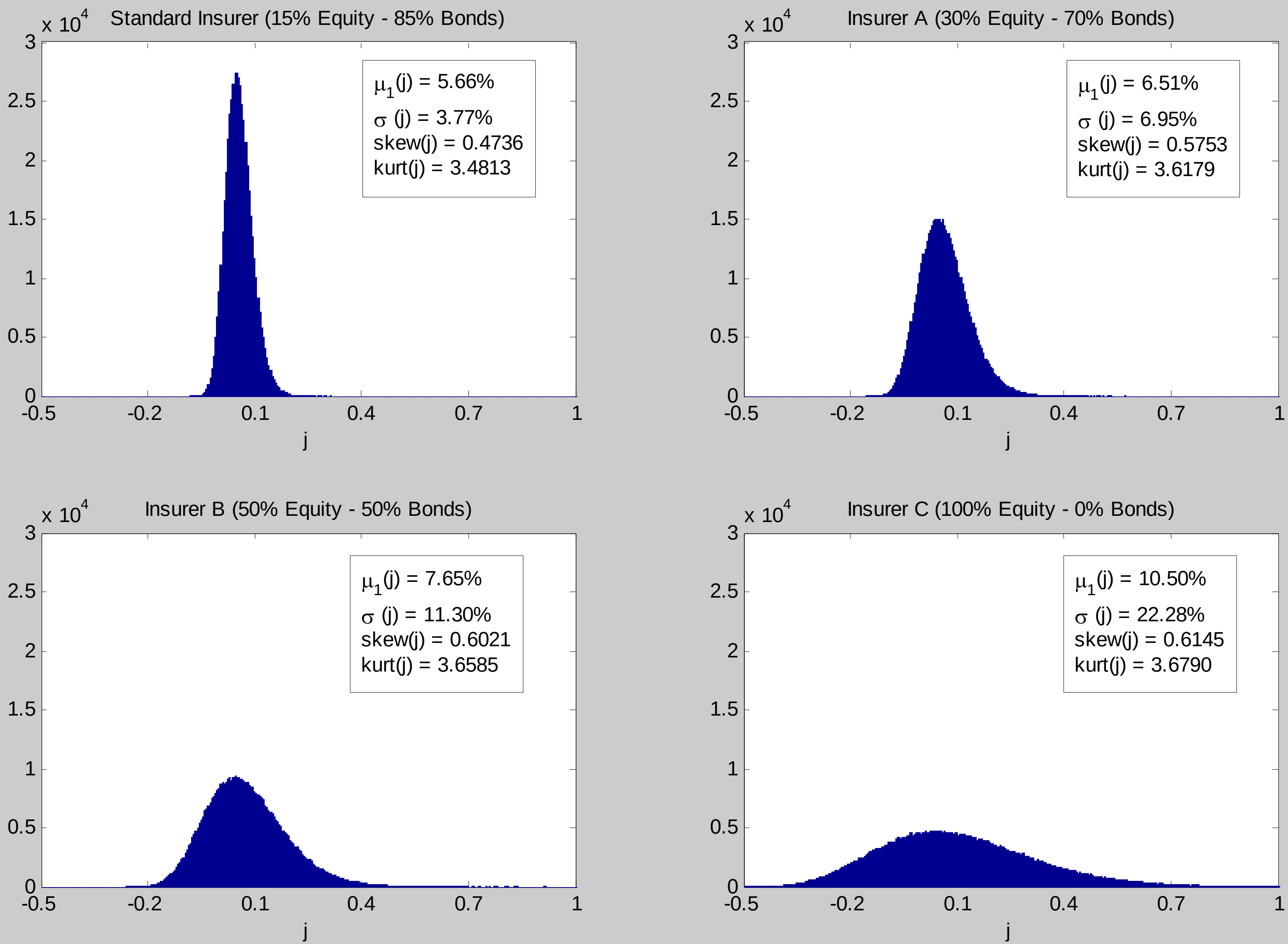
	Insurer B (50% Equities - 50% Bonds)			Insurer C (100% Equities - 0% Bonds)		
	$\varphi = -2.59\% \rightarrow \pi_0 = 155.86$			$\varphi = -2.92\% \rightarrow \pi_0 = 155.33$		
CL: 99.0 %	32.95	42.17	48.24	46.69	57.00	62.41
CL: 99.9 %	46.59	60.39	69.24	62.43	75.58	82.77

100,000 simulations

The claim distribution X_1

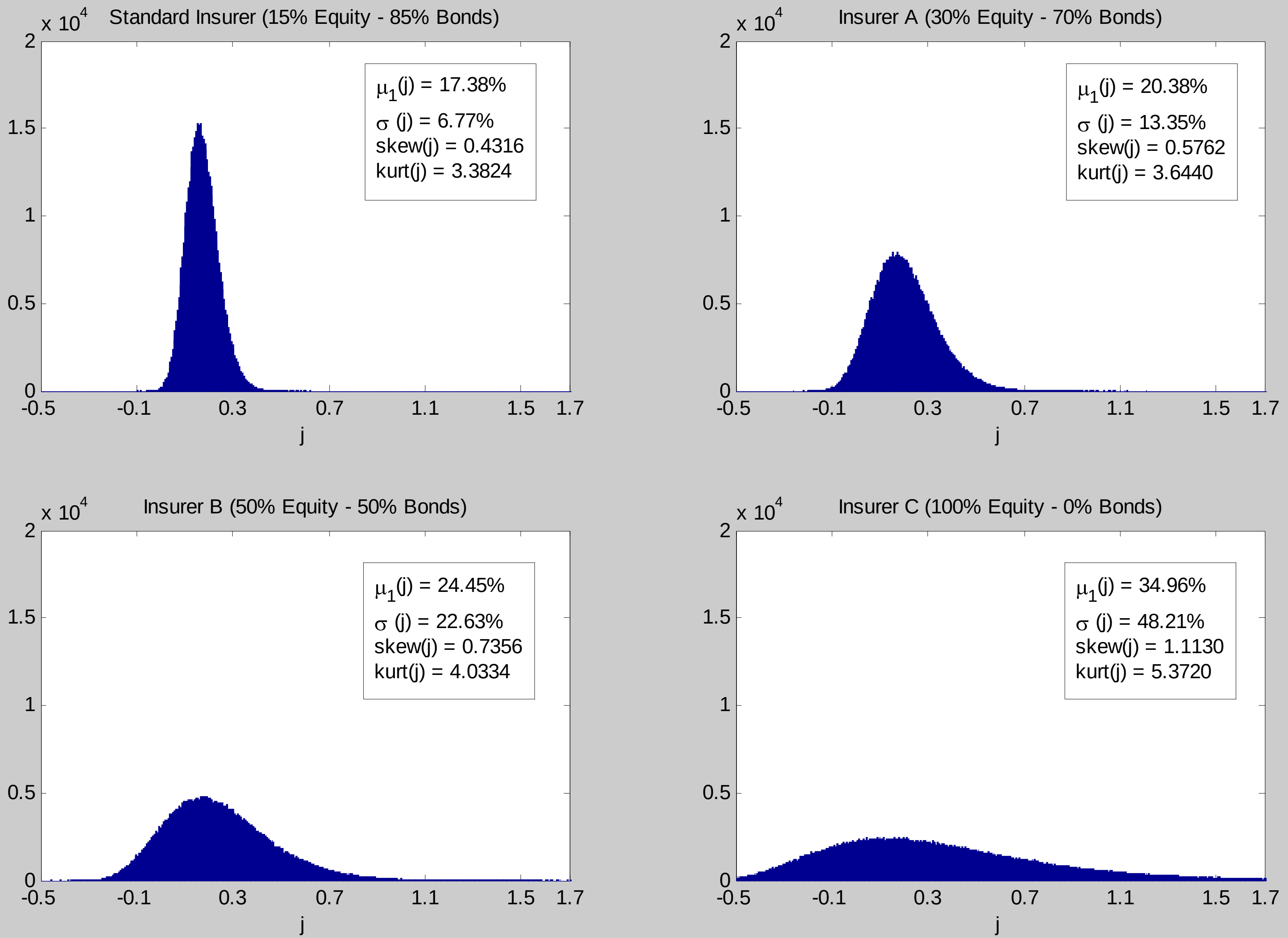


Investment returns distribution *j* (0,1)



500,000 simulations

Investment returns distribution *j* (0,3)



500,000 simulations

Final Comments

- The introduction in the general framework of the financial risk originates
  - Higher instability over time of the capital ratio
  - Stronger capital requirement for solvency purposes
- Different asset allocation affects the safety loading coefficient
  - A higher percentage in equity, α, means a lower safety loading, φ
  - However, the corresponding increase in the risk based capital means that a higher contribution has to come from shareholders
- Work in progress
  - Multiline insurer with underwriting cycle
    - Correlation between LoBs
  - Reinsurance
  - Link between safety loading and claim growth rate
  - Alternative rules for the calculation of the safety loading, φ