A STUDY OF THE BONUS-MALUS SYSTEM

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RESUME

This paper sets out to accomplish the following:

i. Based on set assumptions, we vectorially define the number of policyholders per merit class by year of enrollment and number of years elapsed. Thus calculating the distribution of class sizes for a year in which the system has reached a steady state.
ii. We calculate the class-size distribution for each risk level by introducing multiple risk levels for policyholders.
iii. We calculate insurance premiums based on a bonus-malus system and a flat-rate system, taking into consideration the premium increase/reduction rate for each class. In this way, we ascertain the characteristics of the bonus-malus system.
iv. We calculate the loss ratio for each merit class based on the distribution of policyholders at each risk level in each merit class.

We arrived at the following conclusions:

i. Compared to a flat-rate system, a bonus-malus system is better able to reflect risk levels in insurance premiums.
ii. When rate classes are subdivided, the premium differential attributable to rate class between policyholders at the same risk level is less pronounced under a bonus-malus system than under a flat-rate system. But even bonus-malus systems cannot fully absorb all differences in policyholder risk levels.
iii. Based on this paper's assumptions, we determined the relationship between premium increase/reduction rates and loss ratios for each merit class.

KEYWORDS

Bonus-malus system; rate class subdivisions.

1. INTRODUCTION

The so-called "Japanese Big Bang" financial deregulation initiative has liberalized premium rates in the current Japanese insurance industry. In the automobile insurance sector, insurers have maintained the existing "bonus-malus system", in which policyholders are assigned to merit classes, with potential annual movement up or down to other assigned classes, depending on the number of accident claims filed by the policyholder over the previous year.

Rate of premiums are increased or reduced, depending on the assigned merit class. But in addition to this system, the auto insurance industry has also begun implementing refinements such as rate class subdivisions, primarily so that rate of premiums can better reflect the general risk level of policyholders.

This paper examines the degree to which risk level is reflected in premiums under the bonus-malus system, compared to flat-rate systems. We do so by calculating, based on set assumptions, the distribution of merit classes based on policyholder risk levels under a bonus-malus system. In addition to ascertaining the characteristics of bonus-malus systems, we also attempt to analyze the effects of rate class subdivisions.

2. Definitions

This paper assumes that new policyholders enroll at the beginning of the year and that merit class changes are made once annually, at the end of the year. The following symbols are defined as follows:

- i : Merit class for the year in question
- j : Merit class for the following year
- $x_{i,t,s}$: Number of policyholders belonging to the group of policyholders enrolling s years after the system inception, and who belong to merit class i as of t years after system inception

  In addition, $x_{i,t,s}$ is expressed as $x_{i,t}$ when $x_{i,t,s}$ takes the same value for any value of s. It is expressed as $x^*_{i,t}$ when policyholders are differentiated by risk levels (* represents risk level), and as ${}^{\#}x^*_{i,t}$ when policyholders are differentiated by rate class (# represents rate class).

$$\vec{x}_{t,s} = \begin{pmatrix} x_{1,t,s} \\ \vdots \\ x_{i,t,s} \\ \vdots \\ \end{pmatrix} , \quad \vec{x}_t = \begin{pmatrix} x_{1,t} \\ \vdots \\ x_{i,t} \\ \vdots \\ \end{pmatrix} , \quad \vec{x}^*_t = \begin{pmatrix} x^*_{1,t} \\ \vdots \\ x^*_{i,t} \\ \vdots \\ \end{pmatrix} , \quad {}^{\#}\vec{x}^*_t = \begin{pmatrix} {}^{\#}x^*_{1,t} \\ \vdots \\ {}^{\#}x^*_{i,t} \\ \vdots \\ \end{pmatrix}$$

- $y_{i,u}$: Number of policyholders in merit class i as of u years after system inception

  When $u \to \infty$, $y_{i,u}$ is expressed as y_i. It is written as y^*_i when policyholders are differentiated by risk levels and as ${}^{\#}y_i$, ${}^{\#}y^*_i$ when policyholders are differentiated by rate class.

$$\vec{y}_u = \begin{pmatrix} y_{1,u} \\ \vdots \\ y_{i,u} \\ \vdots \\ \end{pmatrix} = \sum_{s=0}^{u} \vec{x}_{u-s,s} , \quad \vec{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ \end{pmatrix} , \quad \vec{y}^* = \begin{pmatrix} y^*_1 \\ \vdots \\ y^*_i \\ \vdots \\ \end{pmatrix} , \quad {}^{\#}\vec{y} = \begin{pmatrix} {}^{\#}y_1 \\ \vdots \\ {}^{\#}y_i \\ \vdots \\ \end{pmatrix} , \quad {}^{\#}\vec{y}^* = \begin{pmatrix} {}^{\#}y^*_1 \\ \vdots \\ {}^{\#}y^*_i \\ \vdots \\ \end{pmatrix}$$

z_i : Merit class i's premium rate coefficient relative to the standard premium rate

$\vec{z} = \begin{pmatrix} z_1 & \cdots & z_i & \cdots \end{pmatrix}$

$a_{i,j,t,s}$: Probability that a policyholder belonging to the group of policyholders enrolling s years after the system inception and also belonging to merit class i as of t years after system inception will be in merit class j in year $t + 1$

Here, $0 \le a_{i,j,t,s} \le 1$ for any value of i, j, t, or s; and $\sum_j a_{i,j,t,s} = 1$ any value of i, t, or s.

Additionally, $a_{i,j,t,s}$ is expressed as $a_{i,j}$ when $a_{i,j,t,s}$ takes the same value for any value of t or s.

$$A_{t,s} = \begin{pmatrix} a_{1,1,t,s} & \cdots & a_{i,1,t,s} & \cdots \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{1,j,t,s} & \cdots & a_{i,j,t,s} & \cdots \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ & \cdots & & \cdots \end{pmatrix}, \quad A = \begin{pmatrix} a_{1,1} & \cdots & a_{i,1} & \cdots \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{1,j} & \cdots & a_{i,j} & \cdots \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ & \cdots & & \cdots \end{pmatrix}$$

$p_{i,j,t,s}$: The renewal rate for policyholders belonging to the group of policyholders enrolling s years after the system inception, who also belonged to merit class i as of t years after system inception, and would be in merit class j in year t+1

Here, $0 \le p_{i,j,t,s} \le 1$ for any value of i, j, t, or s. Additionally, $p_{i,j,t,s}$ is expressed as p when $p_{i,j,t,s}$ takes the same value for any value of i, j, t, or s.

$$B_{t,s} = \begin{pmatrix} p_{1,1,t,s}a_{1,1,t,s} & \cdots & p_{i,1,t,s}a_{i,1,t,s} & \cdots \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ p_{1,j,t,s}a_{1,j,t,s} & \cdots & p_{i,j,t,s}a_{i,j,t,s} & \cdots \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ & \cdots & & \cdots \end{pmatrix}$$

Additionally, $B_{t,s}$ is expressed as B when $B_{t,s}$ is the same for any value of t or s.

$$\prod_{t=0}^{u-s} B_{t,s} = B_{u-s,s} B_{u-s-1,s} \cdots B_{0,s}$$

3. Number of Policyholders per Merit Class

3.1. Trends in Policyholder Numbers

a. Closed System

A closed system is one that is open for enrollment only at inception, subsequently barring the enrollment of new policyholders. We consider the number of policyholders in each merit class of a closed system as of u years after system inception.

Since no new enrollment occurs except at system inception,

$$\vec{x}_{t,s} = \vec{0} \quad (s \ne 0)$$

Accordingly,

$$\vec{y}_u = \sum_{s=0}^{u} \vec{x}_{u-s,s} = \vec{x}_{u,0}$$

Here, since $\vec{x}_{t+1,s} = B_{t,s}\vec{x}_{t,s}$, $\vec{y}_u$ can be expressed as follows:

$$\vec{y}_u = \vec{x}_{u,0} = B_{u-1,0}B_{u-2,0}\cdots B_{0,0}\vec{x}_{0,0} = \prod_{t=0}^{u-1} B_{t,0}\vec{x}_{0,0}$$

b. Open System

Next, we consider the number of policyholders in each merit class in an open system as of u years after system inception. An open system is one that enrolls new policyholders both at inception and thereafter.

Since $\vec{x}_{t+1,s} = B_{t,s}\vec{x}_{t,s}$

$$\vec{y}_u = \sum_{s=0}^{u} \vec{x}_{u-s,s} = \sum_{s=0}^{u-1}\prod_{\tau=0}^{u-s-1} B_{\tau,s}\vec{x}_{0,s} + \vec{x}_{0,u}$$

If we assume that $x_{i,0,s}$ takes the same value for any value of s, and that $B_{t,s}$ is the same for any value of t or s,

$$\vec{y}_u = \sum_{s=0}^{u-1}\prod_{\tau=0}^{u-s-1} B\vec{x}_0 + \vec{x}_0 = \sum_{s=0}^{u-1} B^{u-s}\vec{x}_0 + \vec{x}_0 = \sum_{s=0}^{u} B^{s}\vec{x}_0$$

Multiplying both sides of the equation by $(E - B)$, we have

$$(E-B)\vec{y}_u = (E-B)\sum_{s=0}^{u} B^{s}\vec{x}_0 = \sum_{s=0}^{u} B^{s}\vec{x}_0 - \sum_{s=1}^{u+1} B^{s}\vec{x}_0 = \left(E - B^{u+1}\right)\vec{x}_0$$

If we assume that $|E - B| \neq 0$,

$$\vec{y}_u = (E-B)^{-1}\left(E - B^{u+1}\right)\vec{x}_0 \qquad (3,1,1)$$

3.2. Calculation of Number of Policyholders

We calculate the number of policyholders in each merit class based on the following assumptions.

a. Assumptions

i. New Enrollment

We assume that $x_{i,0,s}$ takes the same value for any value of s, that $x_{6,0} = 1$ and $x_{i(i\neq 6),0} = 0$.

ii. Renewal Rate

We assume that $p_{i,j,t,s}$ takes the same value for any value of i, j, t, or s, and that $p = 0.95$.

iii. Merit Class Changes

We set the merit classes to coincide with the system in effect in Japan as of 1998. Specifically, we assumed the existence of 16 classes ranging from merit class 1, in which premiums are raised by the highest percentage, to merit class 16, in which premiums are reduced by the highest percentage. Changes in a policyholder's merit class are made as follows:

1) When an accident claim is filed during the past year

The policyholder's merit rating falls by three classes for each accident for which a claim is filed during the year, down to a minimum of class 1.

2) If no accident claims are filed during a year

The policyholder's merit rating rises by one class, up to a potential ceiling of class 16.

iv. Maturity of System

We assume that sufficient time has elapsed since system inception and that u can be considered equal to infinity.

b. When Policyholders' Risk Level is Uniform

If the number of accident claims in one year is assumed to conform to a Poisson distribution, with an average λ, regardless of merit class, enrollment year, number of years elapsed, or other factors, $a_{i,j,t,s}$ can be defined as follows for any value of t or s, in accordance with assumption iii above.

$$a_{i,j} = \begin{cases} 1-\sum_{j=2}^{16} a_{i,j} & (j=1) \\ e^{-\lambda}\dfrac{\lambda^k}{k!} & (j>1, j=i-3k, k=1,2,3\cdots) \\ e^{-\lambda} & (i<16, j=i+1 \quad \text{or} \quad i=j=16) \\ 0 & (\text{in all cases other than the above case}) \end{cases} \qquad (3,2,1)$$

Based on equation (3,2,1) and assumption ii, $B_{t,s}$ is the same for any value of t or s, and

$$B = pA \qquad (3,2,2)$$

Since $p < 1$

$$B^{\infty} = p^{\infty} A^{\infty} = 0A^{\infty} = O \qquad (3,2,3)$$

Based on assumptions i and iv and equations (3,1,1), (3,1,2), and (3,2,3),

$$\vec{y} = (E-B)^{-1}(E-B^{\infty})\vec{x}_0 = (E-pA)^{-1}\vec{x}_0 \qquad (3,2,4)$$

Here we calculate $\vec{y}$ when $\lambda = 0.05$, 0.10, and 0.20. The calculation results are given in TABLE 3,2b.

c. When Policyholder Risk Levels Vary

Consider a case in which the policyholders consist of an assortment of low risks ($\lambda = \lambda^L$), neutral risks ($\lambda = \lambda^N$), and high risks ($\lambda = \lambda^H$).

If we assume that the number of new policyholders at each risk level in each merit class is $\vec{x}_0^L$, $\vec{x}_0^N$, and $\vec{x}_0^H$, respectively, and the number of policyholders at each risk level in each merit class is $\vec{y}^L$, $\vec{y}^N$, and $\vec{y}^H$, respectively, the total number of policyholders $\vec{y}$ in each class is

$$\vec{y} = \vec{y}^L + \vec{y}^N + \vec{y}^H$$

Here,

$$\lambda^L = 0.05,\ \lambda^N = 0.10, \text{ and } \lambda^H = 0.20$$

We calculate $\vec{y}^L$, $\vec{y}^N$, $\vec{y}^H$ according to equation (3,2,4), then calculate $\vec{y}$, assuming that $x_{6,0}^L = 0.40$, $x_{6,0}^N = 0.40$, $x_{6,0}^H = 0.20$, and $x^{*}_{i(i\neq 6),0} = 0$. The calculation results are given in TABLE 3,2c.

TABLE 3,2b

i	y_i			$\frac{y_i}{\sum_{i=1}^{16} y_i}$		
	$\lambda = 0.05$	$\lambda = 0.10$	$\lambda = 0.20$	$\lambda = 0.05$	$\lambda = 0.10$	$\lambda = 0.20$
1	0.0132	0.0788	0.6282	0.07%	0.39%	3.14%
2	0.0202	0.1057	0.6753	0.10%	0.53%	3.38%
3	0.0729	0.2189	0.8658	0.36%	1.09%	4.33%
4	0.1174	0.3081	0.9779	0.59%	1.54%	4.89%
5	0.1547	0.3770	1.0304	0.77%	1.89%	5.15%
6	1.1856	1.4290	2.0398	5.93%	7.15%	10.20%
7	1.1163	1.3351	1.8152	5.58%	6.68%	9.08%
8	1.0514	1.2479	1.6102	5.26%	6.24%	8.05%
9	0.9907	1.1668	1.4242	4.95%	5.83%	7.12%
10	0.9602	1.1574	1.3333	4.80%	5.79%	6.67%
11	0.9170	1.1031	1.1818	4.58%	5.52%	5.91%
12	0.8732	1.0412	1.0319	4.37%	5.21%	5.16%
13	1.2068	1.4645	1.1968	6.03%	7.32%	5.98%
14	1.0905	1.2589	0.9309	5.45%	6.29%	4.65%
15	0.9855	1.0822	0.7240	4.93%	5.41%	3.62%
16	9.2444	6.6253	2.5343	46.22%	33.13%	12.67%
Total	20.0000	20.0000	20.0000	100.00%	100.00%	100.00%

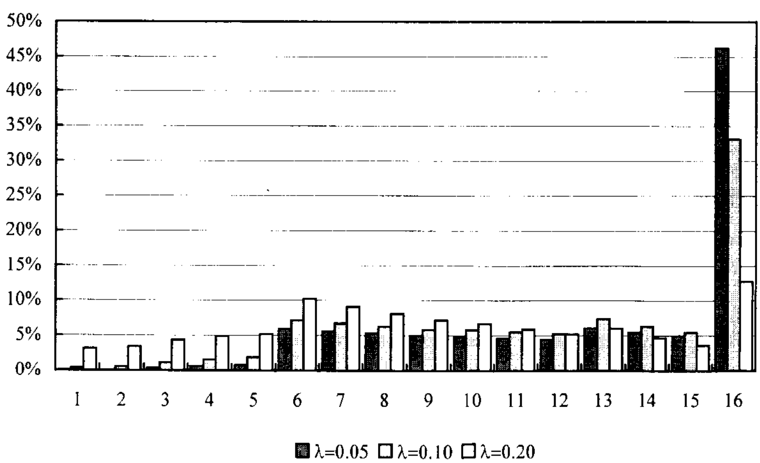

FIGURE 3,2b: $\frac{y_i}{\sum_{i=1}^{16} y_i}$

TABLE 3,2c

i	y_i^L	y_i^N	y_i^H	y_i	$\frac{y_i^L}{\sum_{i=1}^{16} y_i}$	$\frac{y_i^N}{\sum_{i=1}^{16} y_i}$	$\frac{y_i^H}{\sum_{i=1}^{16} y_i}$	$\frac{y_i}{\sum_{i=1}^{16} y_i}$
1	0.0053	0.0315	0.1256	0.1625	0.03%	0.16%	0.63%	0.81%
2	0.0081	0.0423	0.1351	0.1854	0.04%	0.21%	0.68%	0.93%
3	0.0292	0.0875	0.1732	0.2899	0.15%	0.44%	0.87%	1.45%
4	0.0470	0.1232	0.1956	0.3658	0.23%	0.62%	0.98%	1.83%
5	0.0619	0.1508	0.2061	0.4188	0.31%	0.75%	1.03%	2.09%
6	0.4742	0.5716	0.4080	1.4538	2.37%	2.86%	2.04%	7.27%
7	0.4465	0.5340	0.3630	1.3436	2.23%	2.67%	1.82%	6.72%
8	0.4206	0.4992	0.3220	1.2418	2.10%	2.50%	1.61%	6.21%
9	0.3963	0.4667	0.2848	1.1478	1.98%	2.33%	1.42%	5.74%
10	0.3841	0.4630	0.2667	1.1137	1.92%	2.31%	1.33%	5.57%
11	0.3668	0.4412	0.2364	1.0444	1.83%	2.21%	1.18%	5.22%
12	0.3493	0.4165	0.2064	0.9721	1.75%	2.08%	1.03%	4.86%
13	0.4827	0.5858	0.2394	1.3079	2.41%	2.93%	1.20%	6.54%
14	0.4362	0.5036	0.1862	1.1259	2.18%	2.52%	0.93%	5.63%
15	0.3942	0.4329	0.1448	0.9719	1.97%	2.16%	0.72%	4.86%
16	3.6978	2.6501	0.5069	6.8547	18.49%	13.25%	2.53%	34.27%
Total	8.0000	8.0000	4.0000	20.0000	40.00%	40.00%	20.00%	100.00%

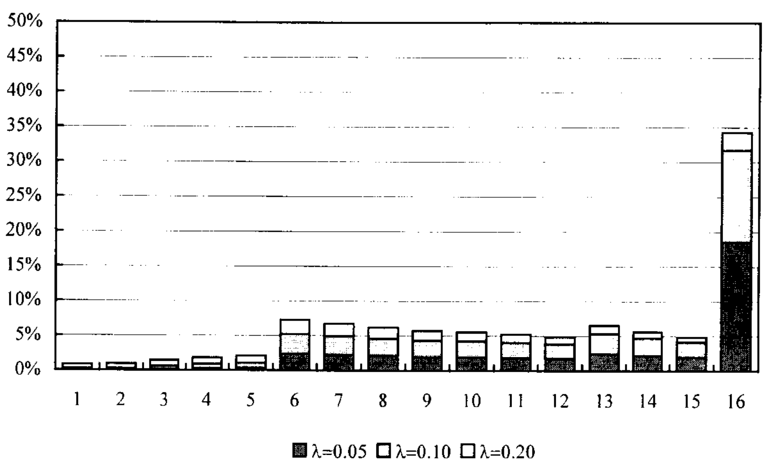

FIGURE 3,2c: $\frac{y_i^L}{\sum_{i=1}^{16} y_i}$, $\frac{y_i^N}{\sum_{i=1}^{16} y_i}$ and $\frac{y_i^H}{\sum_{i=1}^{16} y_i}$

4. Rate Calculation

We now attempt to calculate premium rates based on the assumptions made in 3.2.c. above.

4.1. Uniform Rate Class for All Policyholders

We now proceed to analyze the characteristics of bonus-malus systems, assuming that the standard premium is uniform for all policyholders and that differences in premiums arise only from increases or reductions based on merit class.

a. Premium Calculation

Designating the standard premium as π, we calculate total premiums as follows:

$$\text{Total premiums} = \pi \cdot \bar{z} \cdot \bar{y}$$

We calculate total claims paid as follows, assuming an average claim payment per accident of C, regardless of merit class, year of enrollment, years elapsed, or number of accidents.

$$\text{Total claims paid} = C \cdot \left(\lambda^L \cdot \sum_{i=1}^{16} y_i^L + \lambda^N \cdot \sum_{i=1}^{16} y_i^N + \lambda^H \cdot \sum_{i=1}^{16} y_i^H \right)$$

If we designate the expected loss ratio as r, the following holds true, according to the principle of equivalence of premium income and claims payments.

$$r \cdot \pi \cdot \bar{z} \cdot \bar{y} = C \cdot \left(\lambda^L \cdot \sum_{i=1}^{16} y_i^L + \lambda^N \cdot \sum_{i=1}^{16} y_i^N + \lambda^H \cdot \sum_{i=1}^{16} y_i^H \right)$$

$$\pi = \frac{C \cdot \left(\lambda^L \cdot \sum_{i=1}^{16} y_i^L + \lambda^N \cdot \sum_{i=1}^{16} y_i^N + \lambda^H \cdot \sum_{i=1}^{16} y_i^H \right)}{r \cdot \bar{z} \cdot \bar{y}}$$

Here, if we set C at ¥500,000, r at 60%, and the premium rate coefficient is as follows which corresponds to the system in effect in Japan as of 1998,

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
z_i	1.50	1.40	1.30	1.20	1.10	1.00	0.90	0.80	0.70	0.60	0.50	0.45	0.42	0.40	0.40	0.40

$$\pi = \frac{500000 \times 2}{0.6 \times 11.997863} = 138{,}914$$

Alternately, if we assume a flat premium regardless of merit class, the premium rate coefficient is 1 for any value of i. Hence,

$$\pi = \frac{500000 \times 2}{0.6 \times 20.000000} = 83{,}333$$

b. Calculation of Average Premium and Loss Ratio by Risk Level

Below, we compare average premiums and loss ratios for each risk level in a bonus-malus and a flat-rate system.

Total premiums for each risk level are

$$\text{Total premiums} = \pi \cdot \bar{z} \cdot \bar{y}^{*}$$

Accordingly, the average premium per policyholder is

$$\text{Average premium} = \frac{\pi \cdot \vec{z} \cdot \vec{y}^{*}}{\sum_{i=1}^{16} y_i^{*}}$$

In addition, total claims paid per risk level is

$$\text{Total claims} = C \cdot \lambda^{*} \cdot \sum_{i=1}^{16} y_i^{*}$$

Hence, the loss ratio per risk level is

$$\text{Loss ratio} = \frac{\text{Total claims}}{\text{Total premiums}} = \frac{C \cdot \lambda^{*} \cdot \sum_{i=1}^{16} y_i^{*}}{\pi \cdot \vec{z} \cdot \vec{y}^{*}}$$

The calculation results are given in TABLE 4,1b.

TABLE 4,1b

Risk Level	Bonus-malus System		Flat-rate System	
	Average Premium	Loss Ratio	Average Premium	Loss Ratio
Low	73,912	33.82%	83,333	30.00%
Neutral	81,244	61.54%	83,333	60.00%
High	106,354	94.03%	83,333	120.00%

We see from TABLE 4,1b that introducing a bonus-malus system in place of a flat-rate system permits an insurer to allow premiums to reflect risk levels.

c. Calculation of Loss Ratio by Merit Class

Below, we calculate and compare loss ratios for, claims paid per policyholder, and claims paid per policyholder (payment coefficient) for each merit class if we assume that the anticipated loss ratio multiplied by the standard premium equals 1.

Total premiums for merit class i are

$$\text{Total premiums} = \pi \cdot z_i \cdot y_i$$

Total claims paid for merit class i are

$$\text{Total claims} = C \cdot \left(\lambda^L \cdot y_i^L + \lambda^N \cdot y_i^N + \lambda^H \cdot y_i^H\right)$$

Accordingly, the loss ratio is

$$\text{Loss ratio} = \frac{\text{Total claims}}{\text{Total premiums}} = \frac{C \cdot \left(\lambda^L \cdot y_i^L + \lambda^N \cdot y_i^N + \lambda^H \cdot y_i^H\right)}{\pi \cdot z_i \cdot y_i}$$

Claims paid per policyholder is

$$\text{Claims paid per policyholder} = \frac{\text{Total claims}}{\text{Number of policyholders}} = \frac{C \cdot \left(\lambda^L \cdot y_i^L + \lambda^N \cdot y_i^N + \lambda^H \cdot y_i^H\right)}{y_i}$$

The payment coefficient is

$$\text{Payment coefficient} = \frac{\text{Claims paid per policyholder}}{\text{Standard premium} \times \text{Expected loss ratio}} = \frac{C \cdot \left(\lambda^L \cdot y_i^L + \lambda^N \cdot y_i^N + \lambda^H \cdot y_i^H\right)}{\pi \cdot 0.6 \cdot y_i}$$

The calculation results are listed in TABLE 4,1c.

TABLE 4,1c

i	Loss ratio	Claims Paid per Policyholder	Payment Coefficient	Premium Rate Coefficient
1	42.16%	87,851	1.0540	1.50
2	43.88%	85,335	1.0238	1.40
3	42.83%	77,352	0.9281	1.30
4	44.11%	73,522	0.8821	1.20
5	46.41%	70,911	0.8508	1.10
6	40.22%	55,876	0.6704	1.00
7	44.15%	55,202	0.6623	0.90
8	49.04%	54,500	0.6539	0.80
9	55.30%	53,776	0.6452	0.70
10	64.01%	53,350	0.6401	0.60
11	75.64%	52,536	0.6303	0.50
12	82.60%	51,632	0.6195	0.45
13	85.57%	49,924	0.5990	0.42
14	87.43%	48,582	0.5829	0.40
15	85.14%	47,310	0.5676	0.40
16	72.37%	40,211	0.4824	0.40

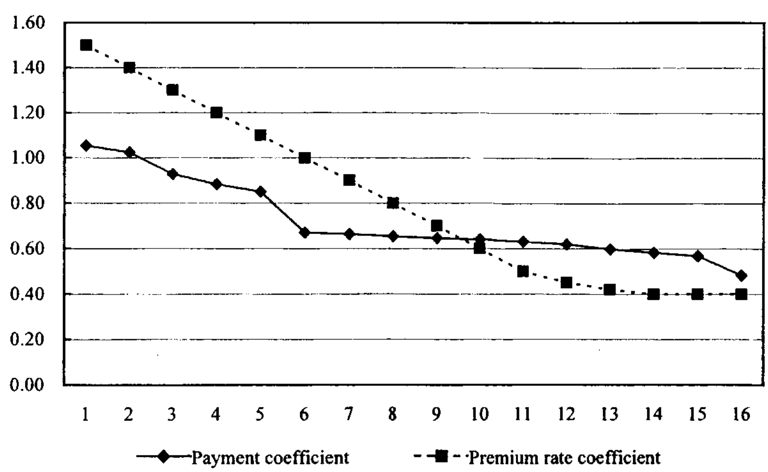

FIGURE 4,1c: Payment Coefficient and Premium Rate Coefficient

From TABLE 4,1c and FIGURE 4,1c, we see that under the assumptions made for this paper, the loss ratio for merit classes 1 through 9 is below the expected loss ratio of 60%, a favorable result.

We also see that the premium reduction rates for merit classes 10 through 16 have been set somewhat high, resulting in a higher-than-anticipated loss ratio.

4.2. Multiple Rate Classes

We now calculate premiums for a scenario featuring two rate classes, α and β, with policyholders assigned to one or the other. We also analyze the effects of classifying premium rates.

a. Assumptions

We designate the number of new policyholders in each risk cell within each merit class as ${}^{\alpha}\bar{x}_0^L$, ${}^{\alpha}\bar{x}_0^N$, ${}^{\alpha}\bar{x}_0^H$, ${}^{\beta}\bar{x}_0^L$, ${}^{\beta}\bar{x}_0^N$ and ${}^{\beta}\bar{x}_0^H$. We designate the number of policyholders in each merit class as ${}^{\alpha}\bar{y}^L$, ${}^{\alpha}\bar{y}^N$, ${}^{\alpha}\bar{y}^H$, ${}^{\beta}\bar{y}^L$, ${}^{\beta}\bar{y}^N$, and ${}^{\beta}\bar{y}^H$. The number of policy holders in each rate class within each merit class, ${}^{\alpha}\bar{y}$ and ${}^{\beta}\bar{y}$, are

$$ {}^{\alpha}\bar{y}={}^{\alpha}\bar{y}^L+{}^{\alpha}\bar{y}^N+{}^{\alpha}\bar{y}^H \quad , \quad {}^{\beta}\bar{y}={}^{\beta}\bar{y}^L+{}^{\beta}\bar{y}^N+{}^{\beta}\bar{y}^H $$

Here, we calculate ${}^{\alpha}\bar{y}$ and ${}^{\beta}\bar{y}$, assuming that

$$ {}^{\alpha}x_{6,0}^L = 0.30, \quad {}^{\alpha}x_{6,0}^N = 0.15, \quad {}^{\alpha}x_{6,0}^H = 0.05, $$

$$ {}^{\beta}x_{6,0}^L = 0.10, \quad {}^{\beta}x_{6,0}^N = 0.25, \quad {}^{\beta}x_{6,0}^H = 0.15, $$

and ${}^{\#}x_{i(i\neq 6),0}^{*} = 0.$

The calculation results are given in TABLE 4,2a-1 and 4,2a-2.

b. Premium Calculation

We now designate the standard premium as ${}^{\alpha}\pi$ and ${}^{\beta}\pi$, and calculate it in the same manner as in 4.1.a..

$$ {}^{\#}\pi = \frac{C\cdot\left(\lambda^L\cdot\sum_{i=1}^{16}{}^{\#}y_i^L+\lambda^N\cdot\sum_{i=1}^{16}{}^{\#}y_i^N+\lambda^H\cdot\sum_{i=1}^{16}{}^{\#}y_i^H\right)}{r\cdot\bar{z}\cdot{}^{\#}\bar{y}} $$

Accordingly,

$$ {}^{\alpha}\pi = \frac{500000\times 0.8}{0.6\times 5.712603} = 116{,}701 $$

$$ {}^{\beta}\pi = \frac{500000\times 1.2}{0.6\times 6.285260} = 159{,}102 $$

Alternately, if we assume a flat rate in each rate class,

$$ {}^{\alpha}\pi = \frac{500000\times 0.8}{0.6\times 10.000000} = 66{,}667 $$

$$ {}^{\beta}\pi = \frac{500000\times 1.2}{0.6\times 10.000000} = 100{,}000 $$

TABLE 4,2a-1

i	${}^{\alpha}y_i^L$	${}^{\alpha}y_i^N$	${}^{\alpha}y_i^H$	${}^{\alpha}y_i$	$\frac{{}^{\alpha}y_i^L}{\sum_{i=1}^{16}{}^{\alpha}y_i}$	$\frac{{}^{\alpha}y_i^N}{\sum_{i=1}^{16}{}^{\alpha}y_i}$	$\frac{{}^{\alpha}y_i^H}{\sum_{i=1}^{16}{}^{\alpha}y_i}$	$\frac{{}^{\alpha}y_i}{\sum_{i=1}^{16}{}^{\alpha}y_i}$
1	0.0040	0.0118	0.0314	0.0472	0.04%	0.12%	0.31%	0.47%
2	0.0061	0.0159	0.0338	0.0557	0.06%	0.16%	0.34%	0.56%
3	0.0219	0.0328	0.0433	0.0980	0.22%	0.33%	0.43%	0.98%
4	0.0352	0.0462	0.0489	0.1303	0.35%	0.46%	0.49%	1.30%
5	0.0464	0.0566	0.0515	0.1545	0.46%	0.57%	0.52%	1.54%
6	0.3557	0.2144	0.1020	0.6720	3.56%	2.14%	1.02%	6.72%
7	0.3349	0.2003	0.0908	0.6259	3.35%	2.00%	0.91%	6.26%
8	0.3154	0.1872	0.0805	0.5831	3.15%	1.87%	0.81%	5.83%
9	0.2972	0.1750	0.0712	0.5434	2.97%	1.75%	0.71%	5.43%
10	0.2881	0.1736	0.0667	0.5283	2.88%	1.74%	0.67%	5.28%
11	0.2751	0.1655	0.0591	0.4997	2.75%	1.65%	0.59%	5.00%
12	0.2620	0.1562	0.0516	0.4697	2.62%	1.56%	0.52%	4.70%
13	0.3620	0.2197	0.0598	0.6416	3.62%	2.20%	0.60%	6.42%
14	0.3272	0.1888	0.0465	0.5625	3.27%	1.89%	0.47%	5.63%
15	0.2956	0.1623	0.0362	0.4942	2.96%	1.62%	0.36%	4.94%
16	2.7733	0.9938	0.1267	3.8938	27.73%	9.94%	1.27%	38.94%
Total	6.0000	3.0000	1.0000	10.0000	60.00%	30.00%	10.00%	100.00%

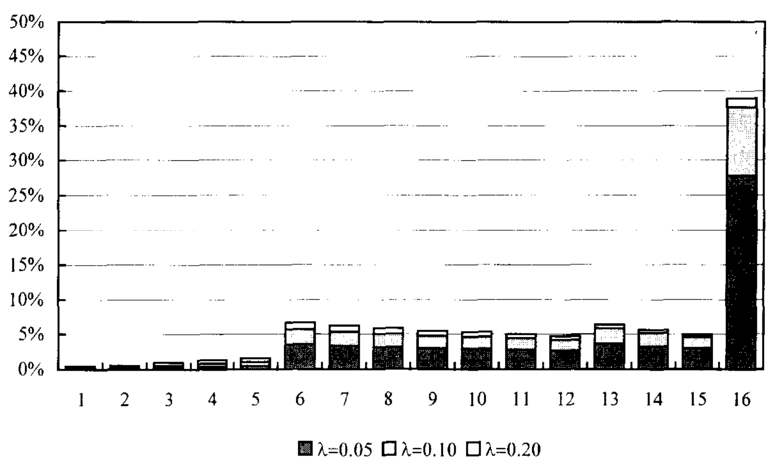

FIGURE 4,2a-1: $\frac{{}^{\alpha}y_i^L}{\sum_{i=1}^{16}{}^{\alpha}y_i}$, $\frac{{}^{\alpha}y_i^N}{\sum_{i=1}^{16}{}^{\alpha}y_i}$ and $\frac{{}^{\alpha}y_i^H}{\sum_{i=1}^{16}{}^{\alpha}y_i}$

TABLE 4,2a-2

i	${}^{\beta}y_i^L$	${}^{\beta}y_i^N$	${}^{\beta}y_i^H$	${}^{\beta}y_i$	$\frac{{}^{\beta}y_i^L}{\sum_{i=1}^{16}{}^{\beta}y_i}$	$\frac{{}^{\beta}y_i^N}{\sum_{i=1}^{16}{}^{\beta}y_i}$	$\frac{{}^{\beta}y_i^H}{\sum_{i=1}^{16}{}^{\beta}y_i}$	$\frac{{}^{\beta}y_i}{\sum_{i=1}^{16}{}^{\beta}y_i}$
1	0.0013	0.0197	0.0942	0.1153	0.01%	0.20%	0.94%	1.15%
2	0.0020	0.0264	0.1013	0.1297	0.02%	0.26%	1.01%	1.30%
3	0.0073	0.0547	0.1299	0.1919	0.07%	0.55%	1.30%	1.92%
4	0.0117	0.0770	0.1467	0.2355	0.12%	0.77%	1.47%	2.35%
5	0.0155	0.0943	0.1546	0.2643	0.15%	0.94%	1.55%	2.64%
6	0.1186	0.3573	0.3060	0.7818	1.19%	3.57%	3.06%	7.82%
7	0.1116	0.3338	0.2723	0.7177	1.12%	3.34%	2.72%	7.18%
8	0.1051	0.3120	0.2415	0.6586	1.05%	3.12%	2.42%	6.59%
9	0.0991	0.2917	0.2136	0.6044	0.99%	2.92%	2.14%	6.04%
10	0.0960	0.2893	0.2000	0.5854	0.96%	2.89%	2.00%	5.85%
11	0.0917	0.2758	0.1773	0.5447	0.92%	2.76%	1.77%	5.45%
12	0.0873	0.2603	0.1548	0.5024	0.87%	2.60%	1.55%	5.02%
13	0.1207	0.3661	0.1795	0.6663	1.21%	3.66%	1.80%	6.66%
14	0.1091	0.3147	0.1396	0.5634	1.09%	3.15%	1.40%	5.63%
15	0.0985	0.2705	0.1086	0.4777	0.99%	2.71%	1.09%	4.78%
16	0.9244	1.6563	0.3802	2.9609	9.24%	16.56%	3.80%	29.61%
Total	2.0000	5.0000	3.0000	10.0000	20.00%	50.00%	30.00%	100.00%

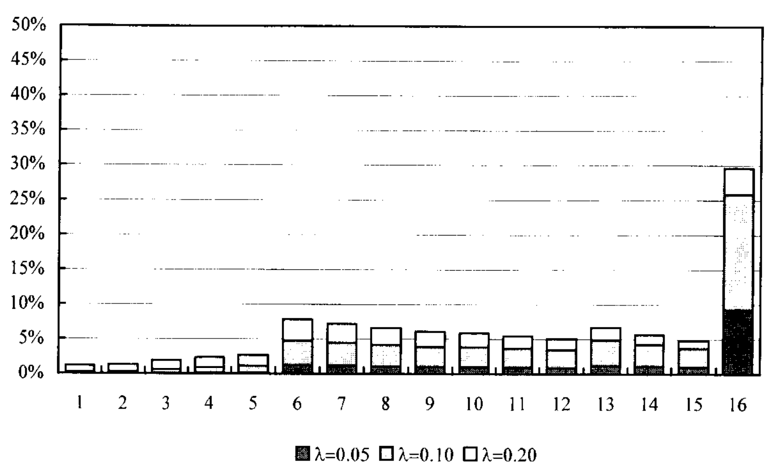

FIGURE 4,2a-2: $\frac{{}^{\beta}y_i^L}{\sum_{i=1}^{16}{}^{\beta}y_i}$, $\frac{{}^{\beta}y_i^N}{\sum_{i=1}^{16}{}^{\beta}y_i}$ and $\frac{{}^{\beta}y_i^H}{\sum_{i=1}^{16}{}^{\beta}y_i}$

c. Calculation of Average Premium and Loss ratio by Risk Level

To compare the average premium and the loss ratio for each rate class and risk level under a bonus-malus system, in contrast to a flat-rate system, we proceed as in 4.1.b..

$$\text{Average premium} = \frac{{}^{\#}\pi \cdot \vec{z} \cdot {}^{\#}\vec{y}^{*}}{\sum_{i=1}^{16} {}^{\#}y_i^{*}}$$

$$\text{Loss ratio} = \frac{C \cdot \lambda^{*} \cdot \sum_{i=1}^{16} {}^{\#}y_i^{*}}{{}^{\#}\pi \cdot \vec{z} \cdot {}^{\#}\vec{y}^{*}}$$

Additionally, average premiums and loss ratios for each risk level are calculated on an aggregate basis for both rate classes, as follows:

$$\text{Average premium} = \frac{{}^{\alpha}\pi \cdot \vec{z} \cdot {}^{\alpha}\vec{y}^{*} + {}^{\beta}\pi \cdot \vec{z} \cdot {}^{\beta}\vec{y}^{*}}{\sum_{i=1}^{16} {}^{\alpha}y_i^{*} + \sum_{i=1}^{16} {}^{\beta}y_i^{*}}$$

$$\text{Loss ratio} = \frac{C \cdot \lambda^{*} \cdot \left(\sum_{i=1}^{16} {}^{\alpha}y_i^{*} + \sum_{i=1}^{16} {}^{\beta}y_i^{*}\right)}{{}^{\alpha}\pi \cdot \vec{z} \cdot {}^{\alpha}\vec{y}^{*} + {}^{\beta}\pi \cdot \vec{z} \cdot {}^{\beta}\vec{y}^{*}}$$

The calculation results are given in TABLE 4,2c.

TABLE 4,2c

Risk Level	Rate Class	Bonus-malus System Average Premium	Bonus-malus System Loss Ratio	Flat-rate System Average Premium	Flat-rate System Loss Ratio
Low	α	62,093	40.26%	66,667	37.50%
	β	84,654	29.53%	100,000	25.00%
	Overall	67,733	36.91%	75,000	33.33%
Neutral	α	68,253	73.26%	66,667	75.00%
	β	93,052	53.73%	100,000	50.00%
	Overall	83,752	59.70%	87,500	57.14%
High	α	89,348	111.92%	66,667	150.00%
	β	121,811	82.09%	100,000	100.00%
	Overall	113,695	87.95%	91,667	109.09%

We see from TABLE 4,2c that the rate-class-attributable premium rate differential between policyholders of the same risk level is less pronounced under the bonus-malus system than under the flat-rate system.

At the same time, rate class subdivisions may result in increased premiums for certain policyholders despite their low-risk status, or vice versa. Clearly, even a bonus-malus system cannot fully absorb differences between policyholder risk levels.

d. Calculation of Loss ratio by Merit Class

To calculate and compare loss ratios, claims paid per policyholder, and the payment coefficient for each merit class, we proceed as in 4.1.c..

$$\text{Loss ratio} = \frac{C \cdot \left(\lambda^{L} \cdot {}^{\#}y_i^{L} + \lambda^{N} \cdot {}^{\#}y_i^{N} + \lambda^{H} \cdot {}^{\#}y_i^{H}\right)}{{}^{\#}\pi \cdot z_i \cdot {}^{\#}y_i}$$

$$\text{Claims paid per policyholder} = \frac{C\cdot\left(\lambda^{L}\cdot{}^{\#}y_i^{L} + \lambda^{N}\cdot{}^{\#}y_i^{N} + \lambda^{H}\cdot{}^{\#}y_i^{H}\right)}{{}^{\#}y_i}$$

$$\text{Payment coefficient} = \frac{C\cdot\left(\lambda^{L}\cdot{}^{\#}y_i^{L} + \lambda^{N}\cdot{}^{\#}y_i^{N} + \lambda^{H}\cdot{}^{\#}y_i^{H}\right)}{{}^{\#}\pi\cdot 0.6\cdot{}^{\#}y_i}$$

The calculation results are given in TABLE 4,2d.

TABLE 4,2d

i	Loss Ratio		Claims Paid Per Policyholder		Payment Coefficient		Premium Rate Coefficient
	α	β	α	β	α	β	
1	46.37%	37.96%	81,164	90,590	1.1591	0.9490	1.50
2	47.50%	39.80%	77,609	88,650	1.1084	0.9287	1.40
3	43.84%	40.08%	66,506	82,892	0.9498	0.8683	1.30
4	44.27%	41.85%	61,998	79,902	0.8854	0.8370	1.20
5	46.09%	44.44%	59,165	77,778	0.8450	0.8148	1.10
6	38.01%	41.34%	44,357	65,778	0.6335	0.6890	1.00
7	41.77%	45.45%	43,874	65,081	0.6266	0.6817	0.90
8	46.47%	50.55%	43,380	64,344	0.6195	0.6740	0.80
9	52.49%	57.08%	42,879	63,575	0.6124	0.6660	0.70
10	60.95%	65.98%	42,678	62,982	0.6095	0.6598	0.60
11	72.23%	78.02%	42,149	62,063	0.6019	0.6501	0.50
12	79.12%	85.28%	41,550	61,059	0.5934	0.6396	0.45
13	82.74%	88.21%	40,556	58,943	0.5792	0.6175	0.42
14	84.83%	90.43%	39,598	57,553	0.5655	0.6029	0.40
15	82.92%	88.32%	38,706	56,210	0.5528	0.5888	0.40
16	72.45%	76.39%	33,821	48,614	0.4830	0.5093	0.40

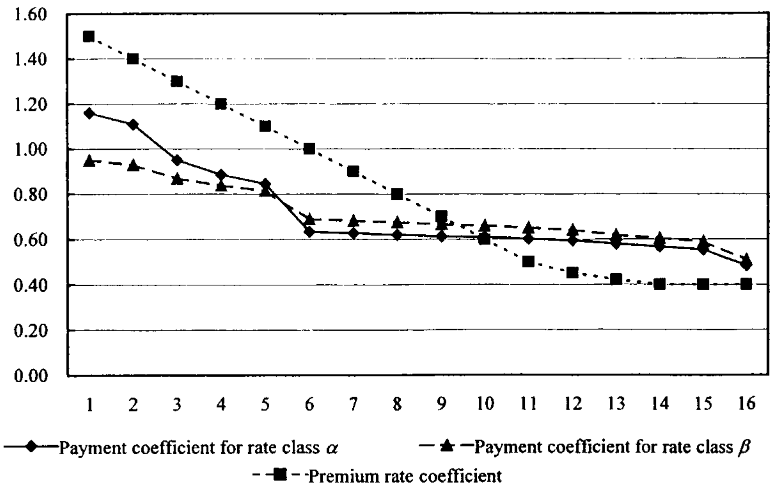

Figure 4,2d: Payment Coefficient and Premium Rate Coefficient

From TABLE 4,2d and FIGURE 4,2d, we can see that for merit classes 6 through 16, the risk-level range attributable to merit class is nearly the same, regardless of rate class. But for merit classes 1 through 5, the risk-level range attributable to merit class varies depending on the rate class.

5. CONCLUSION

We have accomplished the goals stated at the outset, to ascertain the characteristics of bonus-malus systems and to analyze the effects of rate classifications.

We believe this paper may be applied to other similar analyses.

We should note that our analysis is based on certain simplifying assumptions, which inevitably abstract the real-life situation being modeled.

In fact, loss ratios reportedly rise in the merit classes 6 to which new policyholders are assigned. This is believed to be due to the following factors, among others.

- New policyholders include some extremely high-risk cases, but such individuals have low renewal rates and disappear after a short period of time.
- Risk level is initially high when a new policyholder enrolls, but gradually declines over time, eventually stabilizing.

The future direction of the work just discussed may include formulating detailed assumptions using a variety of statistical data, generating results that better reflect reality, and applying those results to help determine company policies, such as appropriate premium rates and changes to existing systems.