

**ASSESSING THE UNDERWRITING RISK
OF A COMPOSITE INSURANCE COMPANY**

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Abstract:

The paper shows a practical way to assess the underwriting risk of an insurance company. This is done by deriving the distribution of the annual underwriting result and describing the implications as to the management's decision on the risk to be taken.

Keywords:

Risk, underwriting risk, log-normal distribution, total loss distribution, monte-carlo-simulation, probability of ruin.

The underwriting risk of an insurance company is evident in the random occurrences of losses leading to fluctuations in the underwriting result.

A company is not only interested in surviving adverse loss scenarios but even more, at the end of a fiscal year, in showing an acceptable profit and loss statement and, hopefully, even fulfill the shareholders' expectations regarding the payment of dividends.

Unfortunately, the distribution of the annual underwriting result is not readily available since, for instance, historical data is only of little use. Usually, the portfolio of business changes during time and also, extreme values might not have occurred in a representative number due to the restricted time horizon for which data is available.

The approach taken in this paper consists of 3 steps:

- a) split of portfolio in uncorrelated segments
- b) find distributions for each segment
- c) run simulations to derive the desired distribution of annual results

1. Segmentation

Let us take a fictitious composite insurance company named SECURITAS and split its business into 5 uncorrelated segments: motor, fire, liability, marine and other classes.

Let us suppose that we are to derive the respective distributions for the following fiscal year.

For each segment, SECURITAS has a good knowledge about the expected earned premium and the expected result. (If this is not the case, the company should immediately hire new underwriters and/or actuaries.)

The expected result is expressed in terms of the combined ratio which is meant to consist of all loss elements and all cost elements including proportionate overhead expenses.

Besides assessing premium and result, the qualified staff also has a good knowledge about possible loss scenarios and their expected frequency. For our purposes, it is necessary to determine an annual combined ratio which has the probability of 1:100 to be exceeded. Naturally, this estimation depends on the specific kind of business planned to be written in the following year and, also on the planned reinsurance protection since we are interested in the net risk (of course, the same analysis can be carried out on gross figures as well).

The assessments are made as follows:

Segment	Expected premium (in some currency unit)	Expected combined ratio (in %)	Combined ratio (1%-percentile)
Motor	200	105	130
Fire	100	95	200
Liability	100	120	200
Marine	50	100	150
other classes	50	80	150
	Sum : 500	Average: 103	

A lot can be said about possible distributions to be used for an individual segment. As already mentioned, historical data are of only limited use so that a decision is always – at least to a certain extent – arbitrary. Clearly, a distribution of the combined ratios should fulfill 3 criteria:

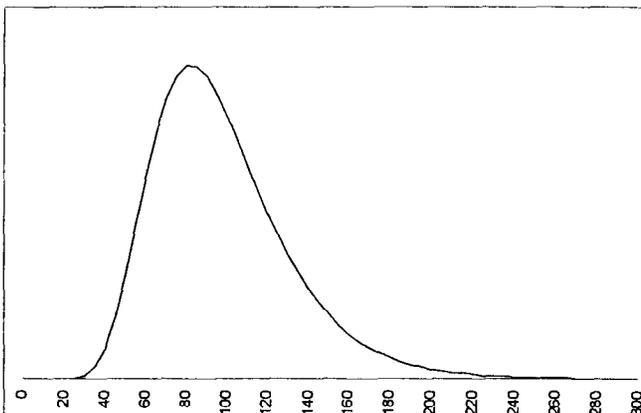
- it should reflect the above assessment
- it should be non-negative
- it should be skewed

For this type of analysis, I suggest the use of the log-normal distribution which not only fulfills the above criteria but also shows an overall plausible shape.

The segment „fire“ is used as an example. According to the assessments in the above table, the log-normal parameters can be derived by an iterative procedure:

$$\begin{aligned} \text{expected value} &= 4.4941 \\ \text{standard deviation} &= 0.3457 \end{aligned}$$

The resulting distribution shows the following shape:



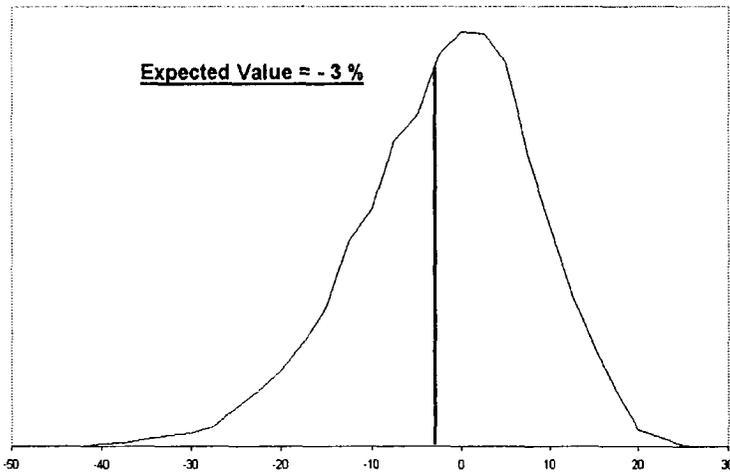
The complete list of the respective log-normal parameters is as follows:

Segment	Expectation	standard deviation
Motor	4.6496	0.0937
Fire	4.4941	0.3457
Liability	4.7608	0.2311
Marine	4.5887	0.1814
other classes	4.3405	0.2880

2.Simulation

Now, the simulation procedure is started.

For each segment, a random combined ratio is generated which – after applying the weighted premiums – leads to the underwriting result of a single year. By simulating a multitude of years (e.g. 100,000), the distribution of the annual underwriting result (in % of premium) is found to look like this:

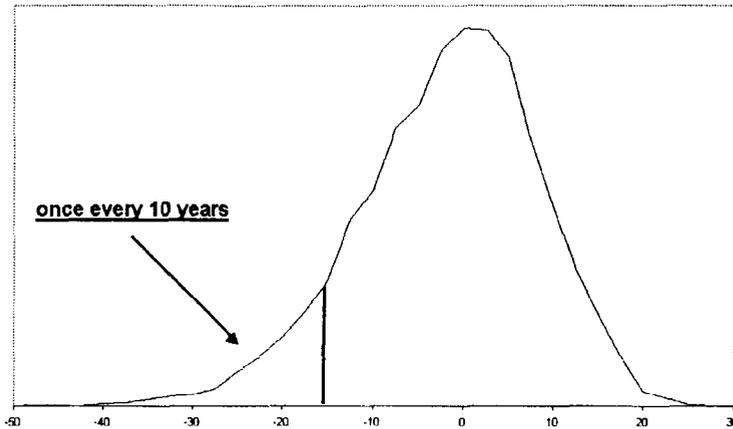


In the following, I would like to show how practical information on the inherent risk can be derived in order to guideline management decisions.

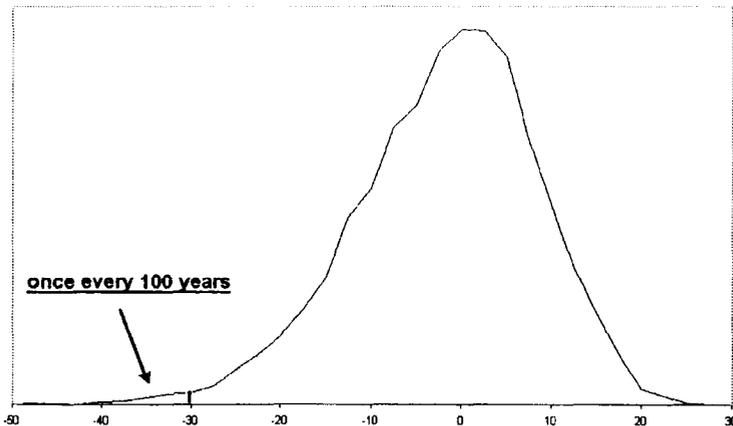
As mentioned in the beginning, the underwriting risk is reflected in the variation of the annual underwriting result, i.e. the above distribution is the ultimate tool for evaluating the underwriting risk on a company level.

First of all, the distribution indicates the probabilities with which certain loss levels are exceeded.

In our example, a negative result of 16,2 % is exceeded with probability 0.1, i.e. we would expect such an event to occur every 10 years if conditions were kept unchanged.



With probability 0.01 or once every 100 years, the underwriting loss will exceed - 30.1 %:



Whether the above results are considered acceptable or not, will mainly depend on the amount of surplus and the investment gains of SECURITAS.

Let's suppose the surplus to be 300 and the investment gains 50 currency units. (I am aware that the amount of the investments gains also depends on random occurrences. Since I am restricting myself to modelling the underwriting section, I would suggest as investment gains either a conservative estimate or a benchmark figure.)

The above distribution allows us to derive additional information on the risk.

We can, for instance, assess the probability of ruin, i.e. the event that the annual loss exceeds the investment gains and the company's surplus.

This probability is found to be 0.00001 for the next year and, therefore, negligible.

In this context, I would like to add that it does not make sense to try to assess the probability of ruin for a longer time horizon. The reason being that after each fiscal year, a company usually reconsiders underwriting policy and reinsurance protection. Conditions, therefore, will not at all be stable over a longer period of time and assumptions would be needed about the company's decisions for any kind of situation.

In any case, the probability of ruin usually – as well as in this particular case, fortunately for SECURITAS – is so small that it hardly can be used to justify any action to be taken.

Another important issue for management is the potential effect on the surplus.

A surplus reduction occurs when the underwriting loss exceeds the investment gains. In our example, the respective probability amounts to 23 %.

Management might prefer another measure of risk. It might have determined to put a maximum of 10 % of the surplus „at risk“ and is interested to know the probability that this promise can be kept. For SECURITAS, this is true with 90 % probability.

3. Alterations

Although the above considerations and results give a rational basis for the decision making by management it must be taken into account that the underlying assessments are made with some degree of uncertainty, especially regarding the 1 %-percentiles.

It is, therefore, advisable to perform a sensitivity analysis with a different set of assumptions. For demonstration, I have doubled the figure for the 1 %-percentiles for all segments and kept everything else unchanged.

The resulting distribution shows the following risk features:

- a) every 10 years: annual loss exceeding – 42.6 %
- b) every 100 years: annual loss exceeding – 97 %
- c) probability of ruin: 3.1 %
- d) probability of surplus reduction: 35 %
- e) probability of surplus reduction by more than 10 %: 29 %

Obviously, such a situation would necessitate a reduction of risk, for instance by buying additional reinsurance.

Hopefully, such extremely different assessments will not occur in practice. Otherwise, the quality of the respective underwriters and actuaries is quite doubtful. The qualified actuaries of SECURITAS come to the conclusion that their original assessments are more credible, and the respective results are presented to management.

In the meantime familiar with probabilistic figures, management is quite satisfied with the outcome except for the probability that more than 10 % of the surplus is lost. In order to achieve a higher degree of security for SECURITAS, it requires such an event to occur with a return period of at least 15 years which the actuary translates into a maximum probability of 6.67 %.

To this aim, several strategies are feasible:

- an increase in surplus
- a different mix of business
- more reinsurance protection

The model can now be used to test any of the above strategies.

Since the last option is the easiest one to accomplish in practice, we will consider the effect of reinsurance.

Besides decreasing the net earned premium, reinsurance has under normal market conditions two impacts on the protected segment: an increase of the expected combined ratio (due to the reinsurer's loading) and a reduction of the 1 %-percentile.

In our hypothetical case, a friendly reinsurance company is found providing a fire protection that changes the respective assessments for SECURITAS as follows:

Net premium	: 80 currency units
Expected combined ratio	: 100 %
1 %-percentile	: 110 %

After another simulation run, the critical probability is found to be 6.4 %, thus satisfying the limit set by the management.

SECURITAS has now not only achieved a very good knowledge about the structure of the risk but has also steered this risk to be within acceptable boundaries.

4. Optimization

It may be argued that the above approach does not necessarily lead to an optimal solution.

From a practitioner's point of view, I would like to add some comments on this aspect.

Every insurance company is striving for at least 2 essential goals: profitability and security. Unfortunately, these goals are in conflict with each other and cannot be maximized simultaneously. For instance, additional reinsurance increases security but decreases expected profitability.

An optimal solution, therefore, can only be found by assessing the trade-off between these goals in terms of the utility defined by management. Although management may be already familiar with probabilities, I doubt that it can define its utility function and is probably unwilling to find the trade-offs by carrying out a number of theoretical experiments.

Alternatively, I propose that management sets minimum criteria for security, and profitability is then maximized under these constraints.

In practice, the starting point of analysis is given by the current situation of the company regarding profitability and security, especially the existing reinsurance protection.

If the model leads to the conclusion that the security is too low (resp. too high), then usually there is only a limited number of options available, for instance buying certain additional (resp. dropping certain) reinsurance covers. All practical options can then be tested in the model and evaluated according to their results on security and profitability.

The option finally decided upon may, therefore, not be optimal in a general sense, but is best regarding the alternatives available in the particular circumstances.

The presented model shows a practical approach for the analysis and evaluation of the underwriting risk. „Practical“ essentially means that all necessary assumptions can in practice be assessed with a high degree of accuracy and that results can be communicated to non-actuarial decision makers.

If decision makers, in a futuristic scenario, are actuaries then modelling and reality will have already merged with some unknown positive probability into virtual reality, i. e. matter is still reality but reality does not matter, anymore.