MAKING USE OF INTERNAL CAPITAL MODELS

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The need for an internal capital model is often associated with its use for calculating capital requirements, or, more exclusively, for Solvency II, when it comes to European companies. But internal models are not just about capital requirements and regulatory compliance. Using an internal capital model goes well beyond the scope of regulatory compliance by providing greater insight into company's risk profile and assisting management in strategic decision making. In particular, it is essential for monitoring company's health and navigating towards more efficient use of company's capital and enhanced shareholder value, as well as greater appreciation of this by external stakeholders like regulators, investment analysts, rating agencies, etc. This paper provides a discussion of what is required to build an efficient capital model, and how it can be pragmatically used to fully extract the benefits of model use for both regulatory and business requirements.

Keywords: internal capital models, ERM, Solvency II.

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1 INTRODUCTION

Many insurance companies have invested a significant amount of resources into designing, developing and calibrating their internal capital models for the purpose of determining capital requirements and being compliant with Solvency II standards. Now, with the recent announcement of the delay in implementation of Solvency II Directive one would naturally expect insurers to react to this differently - perhaps those well prepared would be disappointed, while those falling behind the schedule would only welcome the extra time. However, real figures from various recent market surveys suggest that about two thirds of insurers have reduced their Solvency II implementation as they now expect the directive to come into force a couple years later.

This reduction in the activity could have generally negative implications for many insurers, especially for large listed European insurers. In particular, it would allow insurers to assume more risk than regulators would allow under the new directive and also favour less advanced companies that would be struggling to comply with the new standards. This in turn would make those companies look more riskier in the eyes of stock analysts and rating agencies, and also leave the whole industry with more information asymmetry, and thus less transparent, in the eyes of investors and policyholders.

Obviously, the role of regulators cannot be understated here. It is generally agreed that the efficient regulatory policy should be designed so that it creates the environment adhering as closely as possible to the conditions of a competitive market in which all players have access to relevant information. Three pillars of the new Solvency II Directive, for example, provide such environment through business licensing, capital requirements and supervising (Pillar 1), and using the process of structured early intervention and resolution (Pillar 2 and Pillar 3). But is it only regulatory measures that could create such environment?

This paper argues that in order to maintain their going concern position insurers should use some of those regulatory measures, if not all, as business as usual regardless of the type of regulatory policy. In particular, to a certain extent companies are incetivised to run their own risk and health assessment process and impose some form of discipline around risk-reward monitoring in order to ensure sustainable performance in long run. This incentive may be partially justified using agency theory, where insurance managers demonstrate good stewardship to their risk averse principals and maximise shareholders value by minimising frictional cost and reducing specific risk.

Integrating an internal model into company's enterprise risk processes could actually bring many advantages in addition to model's value in just meeting regulatory requirements. Typical examples of model use include assessing company's risk profile, and deriving optimal capital structure by optimising underwriting portfolio mix, reinsurance and investment strategies, and testing them against company's risk appetite.

The structure of this paper is as follows. In section 2, we provide an overview of risk and capital in insurance, and highlight how they are linked to each other. We explain the role of internal capital models in shareholders value creation and how using them could turn ordinary regulatory compliance into a competitive advantage. In section 3, we explain what it takes to build a good efficient model, and also provide principles of internal capital modelling, drawing attention on potential pitfalls and challenges one should expect when designing and implementing the model. Finally, brief conclusions are given in section 4.

2.1 *Risk and capital in insurance - an overview*

By and large insurers, and in particular stock insurers, leverage themselves by issuing 'risky debt' in the form of insurance policies (Swiss Re [17]). In turn, assumed risk is financed by

- *shareholders* providing equity (free) capital required to support insurer's solvency; and
- *policyholders* supplying risky debt capital as a part of operating capital required to generate underwriting profit.

On one hand, an insurer uses free capital as a buffer against variability in its earnings that avoids 'ruin' and enables it to continue operating in a manner consistent with achieving stated financial objectives. For an insurer, capital takes on a greater level of importance given the extent of insurance liabilities that are held on the promise of future payment.
On the other hand, both shareholders and policyholders require a reasonable degree of confidence in the viability of the organisation to realise value from their investment or premiums.
Examples of typical risks faced by the insurer include

- Large catastrophe event (e.g. high category windstorm or high intensity earthquake) striking densely populated and high wealth households areas;
- Excessive social (seperimposed) inflation in bodily injury classes;
- Prolonged and unexpected economic inflation;
- Collapse in asset values, particularly equities, as the result of global financial crisis;
- Pandemics leading to widespread business interruption;
- Severe reputational damage leading to loss of confidence or protest actions;
- Massive business interruption due to loss of a building;
- Loss of reinsurance recovery due to reinsurer's default;
- IT systems failure preventing all processing for a prolonged period;
- Longevity risk;
- Lack of innovation in insurance products;
- Failure of underwriting controls resulting in high risk policies being written;
- Large scale fraud, embezzlement;
- New products with no data to establish an appropriate price.

Link between risk and capital

In general, risk and capital in an insurance organisation are linked to each other via so called 'risk appetite'. Although there is no universal definition of risk appetite, it may be conceptually viewed as a risk management function representing an optimal solution to the main optimisation problem of insurance organisation - optimising shareholders value using its risk averseness driven utility function under the constraints of external factors, such as regulators, agencies, competitive advantages, etc.

Any unexpected departure from that optimal course of business could lead to change in shareholding, and thus reduced share price and lower shareholders value. For example, excessive exposure to investment or cat risk might not be palatable to existing shareholders as they may see company's main competitive advantage to be in selling insurance policies rather than specialising in managing investment risk and/or prefer the company to transfer extreme cats to reinsurance market rather than resemble a reinsurance company.

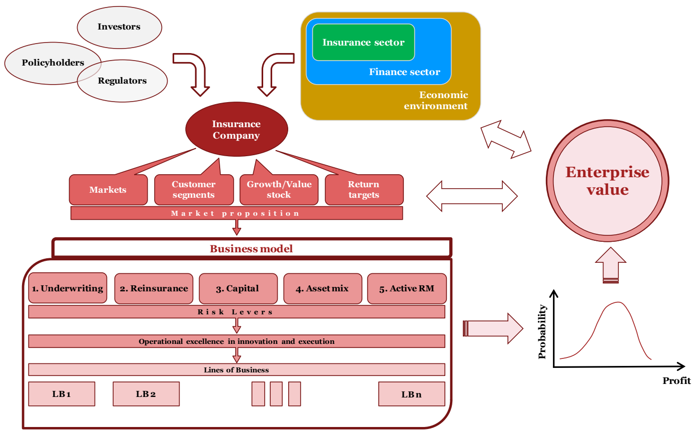
The following subsection focuses on the specific framework of risk appetite for insurance companies developed by Insurance Australia Group (IAG).

An example of a good Risk Appetite: IAG Framework of Risk Appetite for Insurance Companies

A typical insurance company usually develops a market proposition and company's strategy after consideration of the exogenous environment (e.g., insurance sector, finance sector, economic and competitive environments, etc.) and its stakeholders (e.g., policyholders, shareholders, regulators and agencies). This broadly defines the nature of market risk the organisation will absorb and manage in order to deliver enterprise/shareholders value. There are five fundamental business risk levers that define the risk appetite in delivering this enterprise value:

- *Capital and capital allocation*;
- *Underwriting* - i.e. type and composite of underwriting business;
- *Reinsurance*;
- *Asset allocation*; and
- *Active risk management strategies & control environment* - e.g., portfolio mix and business planning, dividend policy, contingent capital, limits of catastrophe and credit risk exposure, etc.

These levers are arranged into a business model that has a focus on alignment, accountability and operational excellence in innovation and execution. Risk appetite is the way in which the fundamental risk levers are applied to produce an earnings profile (i.e. company's risk profile), that supports the overall market proposition and corporate strategy in achieving enterprise value objectives. This is schematically illustrated in the diagram below.



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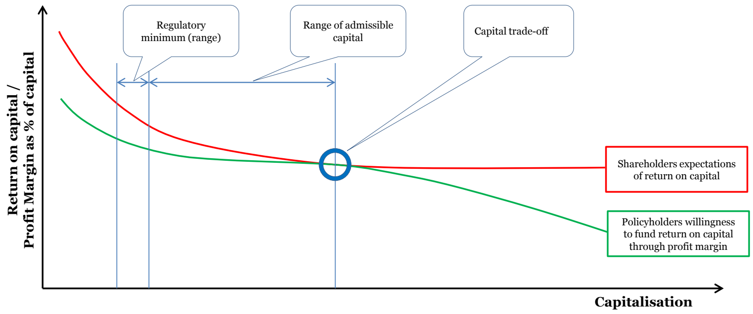
It should be further noted that company's risk appetite plays a key role in optimising level of capitalisation as well as capital structure within the company. Heuristically speaking, running business at a very low level of capitalisation makes the company instantaneously insolvent, whereas holding too much of capital is not economically viable, as it would be impossible to deliver the return on capital desired by the shareholders. The optimal level of capitalisation - a capital trade-off, is somewhere between those two extremes.

Indeed, on one hand shareholders would progressively require a lower return on an increasing capital base due to decreasing level of risk undertaken by the insurance company. This is represented by the curve depicted in red in the exhibit below.



On the other hand, policyholders would be willing to pay an increasing profit margin for additional security directly associated with insurers capitalisation/solvency level. But this will taper off to a maximum profit margin at some point. In particular, at lower levels of capitalisation policyholders would pay a disproportionately lower profit margin to get compensated for the higher risk of insurer's financial distress and insolvency (i.e. insolvency cost or the value of insolvency exchange put option, e.g. see Krvavych and Sherris [11]). When expressed as a percentage of capital, the profit margin has a shape of the curve depicted in green.

The capital trade-off is the equilibrium point where the two curves meet - market clears and desired amount of insurance business is supplied by insurers backed by the appropriate level of capital from adequately awarded investors.

2.2 *The nature and purpose of internal capital models*

The Internal Capital Model (ICM) of an insurer is a stochastic system of financial forecasting designed to support risk management decision making. The ICM is based around the insurer's budget and current balance sheet. It works by placing a statistical distribution based on past experience of key insurance portfolios around the key elements of the budget (premium volumes, loss ratios, investment income, etc) so as to produce many equally likely scenarios for future emerging balance sheets and income statements, starting from the current balance sheet position. Its stochastic nature enables the ICM to be used to make predictions, within specified probabilities, of future key performance indicators of the modelled insurer such as profit and loss statement, return on equity and capital adequacy relative to minimum capital requirements specified by a regulator.

The ICM is an alternative to the regulator's prescriptive model, like the standard model for Solvency II, and is designed to address enterprise risk management questions such as

- How likely is it that the insurer will be unable to meet policyholder claims as they fall due, resulting in absolute or economic ruin?

- How likely is it that the Economic Capital will fall below the minimum regulatory capital requirements, resulting in regulatory ruin?
- How likely is it that the Economic Capital will fall below a certain threshold, leading to a financial distress and triggering potential intervention by the regulator?
- What is the optimal level of capitalisation (solvency ratio)?
- Which lines of business are the most volatile and/or consume more capital, and hence require the most capital? - typical question asked when allocating capital to business classes;
- Which lines of business should be expanded, and which ones need to be contracted?
- What type of reinsurance should the insurer purchase?
- What is the optimal portfolio of reinsurance counterparties?
- How should the insurer optimally invest its assets?

The ICM incorporates identification, calibration, modelling of a company's key risks and their interraction/dependency, and also aggregation of risks via their dependency structure. Although both the ICM and the regulator's prescriptive model are likely to quantify the same set of key risks, there are the following conceptual differences between them:

- The regulator's prescriptive model is a 'one size fits all' type of risk factor model, whereas the ICM is a stochastic model with the structures and specifications that best depict the company's unique business. For example, the company-specific data are used to parameterise the ICM and the use of simulation is in place of prescriptive factors to quantify risk.
- Unlike the prescriptive model, the ICM is capable to
  - explicitly model all material risks as well as their interactions and dependency structures; and
  - recognise and allow for active management, and model risk mitigation structures.
- While being able to replace the prescriptive model in the calculations of regulatory requirements - often defined over the one year period of time, the ICM is flexible enough to accommodate multi-period model runs that could be used in various ERM analyses to provide insightful management information and assist in strategic risk management decision making. In particular, the ability to run the ICM under the setting of multi-year time horizon is essential in order to get a longer term view of company's risk profile:
  - Financial year balance sheet basis more realistic than run-off measures.
  - One year is insufficient to capture evolving risks such as inflation that impacts both premium and provisioning adequacy. This is particularly relevant for insurers writing long tail classes with material balance sheet provisions.
  - One year is insufficient to allow the impacts of an insurance cycle to emerge.

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– Three-year time horizon, for example, could be considered as optimal - the ability to predict the exposure growth, business mix and profitability in a dynamic market diminishes beyond three years. Capital modelling does not typically include actions taken to address an ailing solvency position. Three years represents a compromise between allowing risks to emerge and being able to effectively mitigate an adverse position.

Benefits of using ICM

Insurers with internal capital models in place have the possibility to analyse numerous risk management strategies that would not be possible to assess otherwise using much simpler risk factor based prescriptive model. They are more likely to run their businesses more effectively - understand and measure company's risks better through insightful risk attribution analyses and thus manage the risk efficiently and cost-effectively, and also focus on the profitable areas to maximise shareholders value.

When fully approved by the regulator, the ICM can be used to replace the regulator's prescriptive model in the calculation of regulatory capital requirements, and extract extra benefits in addition to those offered by the prescriptive model. For example, in the case of Solvency II using the ICM could provide the following additional benefits:

- *Pillar 1* - potential capital savings, as switching from the Standard Formula approach to ICM could lead to reduced capital requirements;
- *Pillar 2* - increased ability to demonstrate integrated enterprise risk management and model governance capabilities to the regulator. Examples include stating risk appetite, testing various risk strategies (risk levers) against achieving the targets specified in the risk appetite statement, or applying Own Risk and Solvency Assessment (ORSA);
- *Pillar 3* - more advanced reporting suite and increased transparency of company's risk profile.

Perhaps the most important role the ICM could play in insurance business is its ability to provide the management with insightful information about company's risk levers and how they could be arranged into business model in the most optimal way. Examples of ICM use in an insurance organisation include:

- underwriting portfolio mix analysis and M&A;
- capital adequacy (optimal capitalisation) and capital allocation;
- reinsurance analysis; and
- strategic asset allocation.

PORTFOLIO MIX ANALYSIS. This type of analysis could assist management in budgeting providing insight into how and where the business should grow, what lines need to be expanded/contracted. A particular insurer may contemplate between writing short and long tail lines, and/or personal and commercial lines. This analysis is not as simple as it might look at first glance. It often requires taking a holistic view of risks and understanding how they interact. For example, putting more weight on to long tail businesses would not just increase riskiness of insurer's earning profile due to more riskier nature of long tail classes, but also impose additional concentration risk in the tail of the distribution due to more pronounced effect of superimposed inflation (a commonality factor). The ICM would help to deal with these kind of challenges.

CAPITAL ALLOCATION. This exercise is very important in company's business cycle. The results of capital assignments to lines of business are further used in performance monitoring which ultimately helps management to focus on most profitable segments and maximise the return on capital. There are so many different ways of allocating risk, and each one would perhaps be appropriate under certain conditions (e.g. see J. Dhaene and Vanduffel [8]). But it is always important to be able to have that holistic view of nature of risk, its origin and carrier, and its relation to company's risk appetite, in order to be able to come up with the most suitable methodology of capital allocation. It seems the ICM is the best tool that offers all of this.

- *Understanding risk, defining it from first principles and linking it to risk appetite*

  The 'risk' is generally defined as a possibility of having adverse performance result (insurance, investment, or company's overall result) that results in 'low capital performance' (i.e. a return on capital below the shareholders opportunity cost of capital) and/or erosion of current shareholders value (i.e. capital consumptions).

  It is perceived differently by different stakeholders (policyholders, shareholders, management, regulators, etc.). For example, company's management would consider the risk to start materialising when the rate of return on capital falls below the opportunity cost of capital and continues to the point where the capital is eroded to the level of minimum capital requirements (MCR). In the event that capital falls below MCR the regulator would take management control of the company - theoretically the management no longer have a direct interest in the company. On the other hand, from the shareholders point of view the risk range is wider, spanning even further down to the point of absolute ruin (full capital erosion). The diagram below illustrates the capital layers supporting the risk range.

- *Understanding where the risk is coming from, what carries it and how it 'consumes' capital.* This helps to define and eventually model the risk carrier.

- *Understanding the risk carrier interaction across various risk classes.* This helps to determine the most suitable approach to modelling dependencies and allocating capital.

- *Deriving capital assignments and communicating them via convenient business proxies (premium, claim liabilities, etc.)*

REINSURANCE ANALYSIS. The Internal Capital Model is often capable of assessing the value achieved from the existing reinsurance programme and investigating the benefits to be gained from changes. Reinsurance is the key risk lever used within Risk Appetite Framework to protect against insolvency, maintain stability of earnings volatility and provide capital relief. It is also used as a form of rented capital to replace more expensive paid-in capital. Managing retained risk volatility should not be limited to a simple reduction in volatility, but also go beyond this and focus on higher orders of retained risk. Some investors, depending on their risk appetite and risk averseness, may prefer to simply 'leapfrog' a standard CoV metric and put more emphasis on the third

order of retained risk, i.e. skewness (e.g. see Powers [15]). For instance, they might want to know how a certain reinsurance option could efficiently reduce adverse earnings outcomes at say 1-in-4, 10, 20 years level.
In essence, reinsurance plays a key role in optimising company's capital structure. Although reinsurance optimisation becomes irrelevant from 'naive theory' point of view when insurers are risk neutral and economic environment is frictionless, it is relevant in real situations when frictional cost is present and insurers are risk averse (e.g. see Krvavych and Sherris [11]). Insurers would usually try and modify their reinsurance program by expanding (contracting) the cover - e.g. vertically and horizontally in the case of CAT program - until they achieve the maximum value of return on equity (ROE). In this case, a particular layer/reinstatement would be added (dropped) if marginal ROE is above (below) the target ROE.

STRATEGIC ASSET ALLOCATION (SAA). This exercise seeks to maximise shareholders value by maximising return on risk based capital, subject to various risk considerations and constraints. The theoretically optimal SAA of the insurer is constrained by such external factors as: *regulators*, *rating agencies* and *investors & stock analysts*. Ultimately, the insurer's risk appetite will stipulate the key risk constraints that would narrow down the set of feasible investment strategies, e.g. strategies that are not exceeding a tolerable level of assumed investment risk (say no more than 30% of 1-in-200 capital requirements) and not departing beyond the boundaries of tolerable asset mix (say no more than 65% of growth asset). The optimal asset mix strategy would then be chosen out of this constrained feasible set.
The ICM is an ideal tool for determining an optimal SAA.

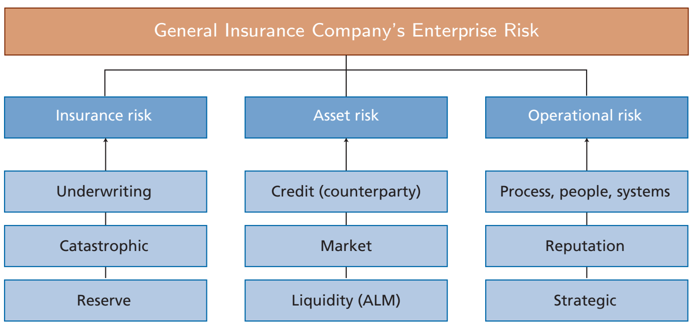
3 PRINCIPLES OF INTERNAL CAPITAL MODELLING

This section focuses on how internal capital models are structured, how they work and what they could and should model, and what is required to build a good model.

3.1 *Risk taxonomy of internal model and modelling approach*

A good internal capital model should first of all have capabilities of quantitative modelling of key material risks and their interactions/dependencies. This also includes capabilities of allowing for company's active management and modelling various risk mitigation structures.

Broadly speaking, there are three distinctive risk categories that are essential to each insurance organisation: *insurance risk*, *asset risk* and *operational risk*. Each of those categories would have its own specialty risks material enough to be addressed and modelled by internal model. The exhibit below represents the risk taxonomy of a typical internal capital model in general insurance.



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Whilst it is not a primary goal of this paper to delve into the details of risk modelling, we do highlight the key features of the modelling approach any good model would use.

UNDERWRITING RISK. Modelling of underwriting risk is concerned with modelling premiums and attritional losses of non-catastrophe nature. Modelling premiums is usually done stochastically by perturbing the budget level of premiums written by a stochastic factor of premium rating strength. Combining modelled premiums with premium earning patterns will form primary risk exposure. This becomes important, as unit of primary risk exposure generates a certain amount of claims incurred - an incurred loss ratio, that is specific to each class of business. This feature is used in scaling underwriting risk, which is central when it comes to modelling and testing underwriting portfolio mix strategies.

With respect to modelling claims, internal models usually start with modelling gross attritional claims and then applying risk mitigating structures to them, like proportional and non-proportional reinsurance. Here, attritional claims could also be split into working (small) attritional claims and individual large claims. Both types of losses could be modelled using either the following two methods - *frequency and severity method* or *loss ratio method*. The first method requires separate modelling of frequency and severity, whereas the second one models aggregate loss by applying modelled loss ratio to primary risk exposure (i.e. earned premium). Losses are drawn from statistical distributions that are parameterised using company's own loss experience and exposure rating.

It is often the case that the first method is more suitable for modelling individual large claims, especially when they are subject to Per Risk XoL cover requiring claims details at the event level. On the other hand, the second method might be more appropriate for modelling small attritional losses, as detailed knowledge of small claims arrivals may bring no additional insight into overall underwriting risk profile.

The approach to modelling underwriting risk may also include elements of underwriting cycle, including even separate trends for premiums and losses. The need for extra complexity like this one will always be determined by business use.

A good coverage of the main principles of underwriting risk modelling can be found in Daykin et al. [2] and Kaas et al. [9].

CATASTROPHE RISK. This type of risk is a key attributor and driver of capital requirements. Modelling of catastrophe risk is concerned with modelling catastrophe loss events. This is often done with the help of vendors' models providing vital statistical information required for the simulation of cat loss events. Industrial vendors' models are physical models that are using physical data and applying geo-spatial statistical analyses to calibrate a so called Event Loss Table (ELT) - a set of all possible events, each defined and represented by a statistical distribution of event frequency and severity. The ELT is not a list of simulated events that are ready to be used in the model, but rather a database of all possible cat events. Using ELT alone with other information, like Occurrence Exceedance Probability (OEP) curve, is often sufficient in order to accurately model cat losses up to the level comparable with capital requirements (say 1-in-200 to 500 years).

Obviously, here we are dealing with low-frequency and high-severity events, and thus some care needs to be taken in order to avoid excessive sampling error and ensure stability of model outputs. For example, straight Monte Carlo simulation are likely to produce large sampling error, but applying variance reduction techniques could prevent this.

A lot of useful information on modelling catastrophe events can be found in Grossi et al. [7] and Woo [19]. Technical details of modelling extreme events can be found in Embrechts et al. [4] and Falk et al. [6].

RESERVING RISK. This is the risk that provisions for past exposures will be inadequate to meet the ultimate costs when the business is run off to extinction. The risk of reserves developing other than expected is significant for general insurers, especially for long tail lines of business. In addition to being random in their frequency and severity, insurance losses are characterised by gaps between: 1) the date the insurance event occurs and the date it is reported; and 2) the date it is reported and the date it is eventually settled.

Since these lags, along with claim frequencies and severities, are stochastic, booked claims liabilities have substantial risk that their actual value realised will adversely deviate from the expectation.

When it comes to modelling reserving risk one immediately faces a problem of choosing a modelling approach from a plethora of those stochastic reserving methods developed over the last three decades (e.g., see Taylor [18], Wüthrich and Merz [20], England et al. [5]). The choice of the most suitable approach should obviously be driven by business use and pragmatism. One could, for instance, employ a general approach that is based on principles of modelling uncertainty, and incorporate:

- the uncertainty in the estimation of the reserves at model start date, by applying a stochastic factor of reserve fluctuation; and also
- recognise the uncertainty arising in respect of events that will occur after the model start date (a calendar year effect).

MARKET RISK. Insurers are naturally exposed to market risk as insurance operations interact with financial markets that are constantly varying causing changes in asset values and other market rates affecting both asset and liability. The part of the ICM dealing with the modelling of market rates scenarios - investment asset returns, interest rate yield curves, inflation rates, currency exchange rates, etc., is often called Economic Scenario Generator (ESG). The ESG may be developed in-house as part of the ICM, or could be supplied by external source and integrated to the main model. All fundamental relationships between various asset classes are usually modelled explicitly. When simulating ESG scenarios the following considerations should be taken into account:

- the model would usually aim at producing real-world scenarios, but if risk-neutral probabilities are used to simulate market affects, they must be adjusted to real world distributions to be used in the model;
- credit risk on investments would normally be modelled as part of market risk;
- dependencies among the market effects - usually modelled within the ESG;
- allowance for active management - e.g., dynamic portfolio rebalancing, etc.; and
- allowance for liquidity risk - liquidity risk for an insurer is very different to that of a bank or similar financial institution where customers can make claims upon the institution at very short notice based upon their perception of its financial strength. In the case of a non-life insurer, the largest unanticipated call on cash would likely arise in the event of a large insured catastrophe.

Comprehensive coverage of market risk modelling can be found in Sandström [16].

Credit risk. In general, insurers are exposed to three types of credit risk: investment, reinsurance and premium debtors. Credit risk on investments is traditionally modelled by ESG as part of market risk. The modelling of credit risk of premium debtors is straightforward - e.g., it can be modelled stochastically using asset impairment rate. The nature of events leading to impairment of premium debtors is mainly associated with brokers' inability/unwillingness to pay.

In order to accurately model the reinsurance credit risk a great deal of care needs to be taken. This is mainly because the reinsurance credit risk significantly differs from credit risk of a conventional bond. This means that traditional methods often used to assess bond issuer default cannot be easily transferred to assessment of reinsurance default. Differences arise in relation to probability of default, exposure, and recoveries. In regards to default event, the reinsurer default is usually defined as an asset impairment event, whereas the credit markets consider an issuer default as having occurred when there is a shortfall in interest and/or principal payments on its obligation. Under financial distress reinsurers often go into runoff and usually enter into a commutation agreement with cedants, and therefore it is often unclear if the default defined in financial sense has taken place or not.

The exposure profile of an insurer is also different. The insolvency of a reinsurer will lead to costs on the part of the insurer:

- any amounts owing as a result of claims settled with the reinsurer but not yet paid will be impaired;
- amounts potentially owing in respect of claims incurred but not yet advised to the ceding insurer are potentially not recoverable;
- any potential recoveries in respect of policies in force are unlikely to be met in full;
- the ceding company may be required to purchase replacement cover to remain protected against events from the time of reinsurance default to the time the contract would have expired.

Within the constructs of the ICM there is a further difficulty, in that exposure may arise in respect of contracts not in force at the calculation date, but expected to be written in respect of future years. All of those modelling issues should be addressed in the multi-period settings of the ICM. A good example of a practical DFA approach to modelling reinsurance credit risk can be found in Britt and Krvavych [1].

Operational risk. In practice, many insurers quantify operational risk as a certain percentage of their capital requirements ranging from 5% to 20%. Whilst this could be seen as a quick pragmatic approach, it is subjective one and such that could lead to serious overstatement. For example, quantifying operational risk to be at the level 20% may be reasonable for banks, but quite inadequate for insurance sector.

As an alternative to this, insurers may employ Loss Distribution Approach (LDA, e.g., see Panjer [14] and references therein) to modelling operational risk scenarios. This approach is a frequency-severity approach utilising relevant historical operational losses as well as experts judgment to parameterise distribution parameters. When modelling operational risk using LDA the following should be taken into account:

- allowing for risk mitigation structures like D&O insurance and ERM processes; and
- avoiding double counting of operational risk - e.g., allowing for traces of operational risk in the modelling of other type of risk like reserving, underwriting catastrophe, etc.

RISK DEPENDENCIES. Accurate modelling of interaction between risks is of paramount importance, as it will ultimately determine the shape of the distribution of aggregate risk used in the assessment of capital requirements. Assessing risk interactions using copulae is recommended instead of using commonly used linear correlations. Copulae emulate the interaction between risks across their full distribution, providing greater insight into the tail of the distribution of aggregate risk - the part of the distribution driving capital requirements. When modelling dependency structures it is important to keep the right balance between the complexity and reality dictated by business pragmatism. For example, in practice insurers often opt to constructing gigantic correlation matrices without realising that this may compromise the quality of capital modelling leading to longer model run time and instability modelled results. The modelling approach should be simple and pragmatic, and such that could be easily calibrated and interpreted. One of the examples of good robust approach to modelling dependencies is pair-copula structure. For comprehensive treatment of dependency modelling using copulae please refer to McNeil et al. [13] and Mai and Scherer [12].

RISK MEASURES. For the company's own flexibility risk should be calculable using various risk measures. The choice of risk measure is mainly dependent on how the retained risk impacts the benefits anticipated from the risk management processes. For example, when considering volatility of insurance results one would traditionally opt for standard deviation or even one-sided standard deviation of insurance results falling below their expected value. On the other hand, when looking at solvency protection, tail-sensitive risk measures are common choice in risk management.

In general, the use of variance-based risk measures in risk minimisation can be justified mathematically by either/both of assumptions: 1) shareholders' (risk managers') utility function is quadratic; 2) the distribution of risk carrier is normal, implying that the standard deviation is fully characterises the distribution. Unfortunately, neither of those two assumptions are realistic in insurance, as insurance losses are extremely skewed and shareholders' absolute risk averseness decreases with wealth[1] in contrast to investors with quadratic utility. While the use of variance-based risk measures in insurance, in particular when pricing underwriting risk (attritional losses), could still be justified through diversification of large risk portfolio (asymptotically converging to 'normality'), it is not appropriate for measuring risks with highly skewed and/or heavy tailed distribution, e.g. catastrophe losses or losses from reinsurance default. Therefore, risk measures capturing most, if not all, of the characteristics of distribution are more desirable. When measuring and minimising risk one could, for example, 'leapfrog' the variance and consider higher moments, measuring skewness and tail heaviness of risk distribution (as it was done in Powers [15]), or even a quantile-based risk measure, like a TVaR measure averaging risks in the distribution tail above their central estimate.

When measuring risk insolvency (financial distress) both insurers and regulators often employ tail-sensitive risk measures like VaR or TVaR. In particular, when deriving risk capital of an insurer under solvency restriction VaR is preferable candidate. This can been explained from the following two points of view. From the regulator's point of view, when considering simultaneous minimisation of policyholders deficit and cost of capital, VaR comes up as an optimal solution to this problem (e.g. see pp. 73-75 in Denuit et al. [3]). From insurer's point of view, risk managers are not interested in 'how bad is bad' in the event of insolvency, as at that instance they will be replaced and/or control of the company will be taken over by regulator. Hence, VaR is optimal.

1 As was shown in Krvavych [10] the maximization of shareholder value under solvency restrictions is approximately equivalent to the maximization of shareholder value using utility approach with a specific isoelastic utility function.

Criteria for a good efficient model

A good internal capital model should satisfy the following criteria:

- *Fit for purpose* - model outputs must reflect what was meant to be modelled.
- *Adaptable* - the model is flexible enough to handle different aspects of risk modelling. It follows principles of global settings/coding. For example, using variation of existing reinsurance contract/treaty should not trigger a model change, but rather be accommodated via a combination of existing model functions. Model adaptability is also determined by the level of model structure intelligence.
- *Robust* - the model is stable enough to demonstrate its invariance to re-sampling and resilience to biasness. Typical examples of the issue leading to model instability would include sampling error of modelled catastrophes.
- *Simple and pragmatic* - the methodology/approach used in the model should be simple enough so that the model is auditable and such that can be interrogated at the scenario level when required. Although, it should not be overly simple and the reality should not be sacrificed for elegance or simplicity. The model structure should be lean and optimal (intelligent) so that the model run time is within reasonable boundaries.

Modelling challenges and pitfalls

Most modern commercial modelling platforms (e.g. Igloo, ReMetrica, etc.) are specifically designed to cope with large scale linear operations. Risks and their components are usually assumed to be modelled using straight forward sampling from distribution curves. Once all the types of risk are modelled the model could utilise the power of modelling platform to swiftly process financial accounts at the class level and consolidate them across classes and business divisions.

The big hurdle for modelling platforms here though is to get through the labyrinths of 'non-linear' algorithms of modelling certain types of risk - for instance, modelling of cats and credit risk involving simulation of low-frequency and high-severity events and/or regime switching, or implementing reinsurance structures of non-linear nature like excess of loss, or non-traditional contracts like Loss Portfolio Transfer or Adverse Development Cover.

Inability of a particular modelling platform to accommodate efficient implementation of a certain non-linear modelling approach could compromise model robustness and/or lead to increased model run time.

RARE EVENT AND EXTREME VALUE MODELLING. This involves dealing with low frequent and high severe events/losses. In the case of cat risk modelling the external vendors' models, like RMS, Catrader, EQECAT are used. The external models are not simulation engines - they rather provide a useful information, such as ELT and OEP curve, that can be further unfold to simulate actual cat events.

Special care needs to be taken when simulating cat event losses. The following things could contribute to inaccurate modelling of risk in the tail and thus have a tremendous impact on capital requirements:

- *Straightforward Monte Carlo* - it works against us and thus some form of variance reduction techniques is required;

- *Poor understanding of external model limitations* - for example, possible event clustering and dependency could lead to overdispersion of event frequency (i.e. the case of non-Poissonian frequency).

Scarcity of the historical data of individual large non-cat losses would prevent us from accurate distribution fit. Inappropriate approach to modelling infrequent extreme events could lead to model instability.

REINSURANCE MODELLING. Modelling reinsurance covers with interactions between layers, perils, contracts could be challenging. Execution of numerous non-linear transformations in one model run, like excess of loss or stop-loss, may increase the model run time. This could happen when, for example, implementing reinsurance program at the contract level rather than at the treaty level.

CREDIT RISK MODELLING. Modelling of reinsurance credit risk often involves generating very infrequent default events as most reinsurers would be of good credit quality due to limits of credit risk exposure imposed by active risk management. Inappropriate approach to modelling rare default events could lead to model instability. Also additional complexity of the modelling approach does not necessarily make model more realistic and could even slow down performance of the model. For example, modelling credit migration of reinsurers is not necessary, as in reality active management would take care of this, and the additional complexity increases model run time.

DEPENDENCY MODELLING. Modelling a particular dependency structure by means of using one gigantic correlation matrix is often inefficient and such that is difficult to calibrate, interpret and operate with. This could easily lead to model instability and inefficiency. In this case, using, for example, pair-copula structures (vine copulae) instead would be one of the ways around this issue.

Model design considerations

When building an efficient capital model it is always desirable to use an intelligent model design. Today many modern modelling platforms offer this via functionality of so called modular ('Lego') approach. The benefits of using modular approach to building internal capital model structures are that it:

- avoids rerunning global modules (when unnecessary) and saves on run time;
- allows cut-down versions of the model that might be useful for a certain narrowly defined tasks; and also
- makes the model structure more transparent, auditable and such that is easy to interrogate for specific scenarios leading to the outcome under investigation.

In addition to this the modeller may contemplate between different risk-class structures, say, single risk class vs. multi-array class structure. When modelling class specific risks like underwriting, cats, reserving one may consider the following two options:

- Modelling risks separately for each class; or
- Modelling risks using multi-array class data structure.

The first option is difficult from the model governance point of view, but is relatively more efficient from the model performance point of view. In contrast, the second option is more convenient when updating the model with the new data inputs, but it could slow down the model performance.

4 CONCLUSIONS

To summarize this paper, we believe that for many insurers having a good, efficient and robust internal capital model, and using it internally in the normal course of company's business is the right approach to modern risk management. In addition to its value in meeting regulatory requirements, the internal capital model can be used to provide insightful management information and navigate towards more efficient use of company's capital and enhanced shareholder value. This paper provides a discussion of what the internal capital models are made of, what are the benefits of using them, and what it takes to build a good efficient model.

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