

Development of International Insurance Company Solvency Standards



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Introduction

- IAA Risk-Based Capital Solvency Structure Working Party formed in 2002
- Terms of reference:
 - describe principles & methods to quantify total funds needed for solvency
 - foundation for global risk-based solvency capital system for consideration by IAIS
 - identify best ways to measure the exposure to loss from risk & any risk dependencies
 - focus on practical risk measures & internal models

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Key principles

- Multi-pillar approach to supervision
- All types of risks to be included
- Principles based approach preferred to rules based approach
- Integrated balance sheet approach
- Use appropriate risk measures
- Select an appropriate time horizon
- Allow for risk dependencies
- Allow for risk management
- Use of internal risk models is encouraged

Multi-pillar Basel approach to supervision

- A set of target capital requirements is necessary for solvency (Pillar 1)
 - Snap-shot of financial position of insurer
- Supervisory review of insurer (Pillar 2)
 - To better understand the risks faced by the insurer and the way they are managed
- Market disclosure measures (Pillar 3)
 - Disclosure imposes greater market discipline

Pillar 1 Challenges

- Requirements for a capital requirement framework
 - sufficient simple to be applied anywhere
 - sufficiently detailed to reflect company-specific characteristics
 - sufficiently comprehensive to cover all types of risks faced by insurer
- Approach is to start with scientific framework
 - recognize “internal model” as ideal
 - identify risk measures explicitly
 - build risk-based formulas as approximations to internal model

IAIS Risk Classification Scheme

- **Investment Risks:** Various kinds of **asset risks** which are directly or indirectly associated with the insurers' asset management.
- **Technical Risks:** Various kinds of **liability** risks which are directly or indirectly associated with the technical or actuarial bases of calculation for premiums and technical provisions in both life and non-life insurance, as well as risks associated with operating expenses and excessive or uncoordinated growth.
- **Non-Technical Risks:** Various kinds of risk which cannot in any suitable manner be classified as either technical risks or investment risks.

BIS Risk Classification Schemes

- ***Credit Risk*** is the risk of default and change in the credit quality of issuers of securities, counter-parties and intermediaries.
- ***Market Risk*** arises from the level or volatility of market prices of assets. It also includes the exposure of options to movements in the underlying asset price and the exposure to other unanticipated movements in financial variables or to movements in the actual or implied volatility of asset prices and options.
- ***Operational Risk*** is the risk of direct and indirect losses resulting from the failure of processes, systems or people.

IAA Working Party Proposal

- *Underwriting Risk*
- *Credit Risk*
- *Market Risk*
- *Operational Risk*
- *Liquidity Risk*
- *Event Risk*



Specific Risks

1. *Underwriting Risk*

- ✓ Underwriting process risk
- ✓ Pricing risk
- ✓ Product design risk
- ✓ Claims risk (for each peril)
- ✓ Economic environment risk
- ✓ Net retention risk
- ✓ Policyholder behavior risk

■ 2. *Credit Risk*

- ✓ Business credit risk
- ✓ Invested asset credit risk
- ✓ Political risk
- ✓ Reinsurer risk
- ✓ Sovereign risk

Specific Risks

- **3. Market Risk**
 - ✓ Interest rate risk
 - ✓ Equity and property risk
 - ✓ Currency risk
 - ✓ Basis risk
 - ✓ Reinvestment risk
 - ✓ Concentration risk
 - ✓ ALM risk
 - ✓ Off-balance sheet risk
- **4. Operational Risk**
 - ✓ Human capital risk
 - ✓ Management control risk
 - ✓ System risks
 - ✓ Strategic risks

Specific Risks

- **5. *Liquidity Risk***
 - ✓ Liquidation value risk
 - ✓ Affiliated company risk
 - ✓ Capital market risk
- **6. *Event Risk***
 - ✓ Legal risk
 - ✓ Reputation risk
 - ✓ Disaster risk
 - ✓ Regulatory risk
 - ✓ Political risk

Object of this talk

- Discuss three challenges
 1. Risk measures
 2. Dependence
 3. Approximations

1. Risk measures

Total balance sheet requirement is what amount?

- Usually we think in terms of some measure:
 - VaR uses quantile (e.g. 99%)
 - Probability of ruin over some horizon (e.g. 1% over lifetime of existing book of business)
- There are many other possibilities for risk measures for pricing or for solvency purposes.

Risk measures

- ***Coherent*** risk measures satisfy certain criteria:
 - Subadditive. Capital for two risks is not larger than for each risk separately.
 - Risk with no uncertainty requires no capital.
 - Invariant under location and scale transformations, e.g. changing currencies.
 - Additive for comonotonic risks.
- VaR and probability of ruin are **not** coherent risk measures.

Coherent risk measures

- Capital requirement can be expressed as an expectation under a “distorted” measure

$$C = \int x \cdot dg(F_X(x))$$

where $g(x)$ is a concave continuous function on the unit square with $g(0) = 0$ and $g(1) = 1$.

- Every coherent risk measure is characterized by a distortion function.

Recommendation

- TailVaR (CTE, TVaR) as risk measure
 - Find x_q satisfying

$$\Pr\{X > x_q\} = 1 - q$$

where X represents loss to the insurer.

- Total balance sheet requirement (reserves+capital) is

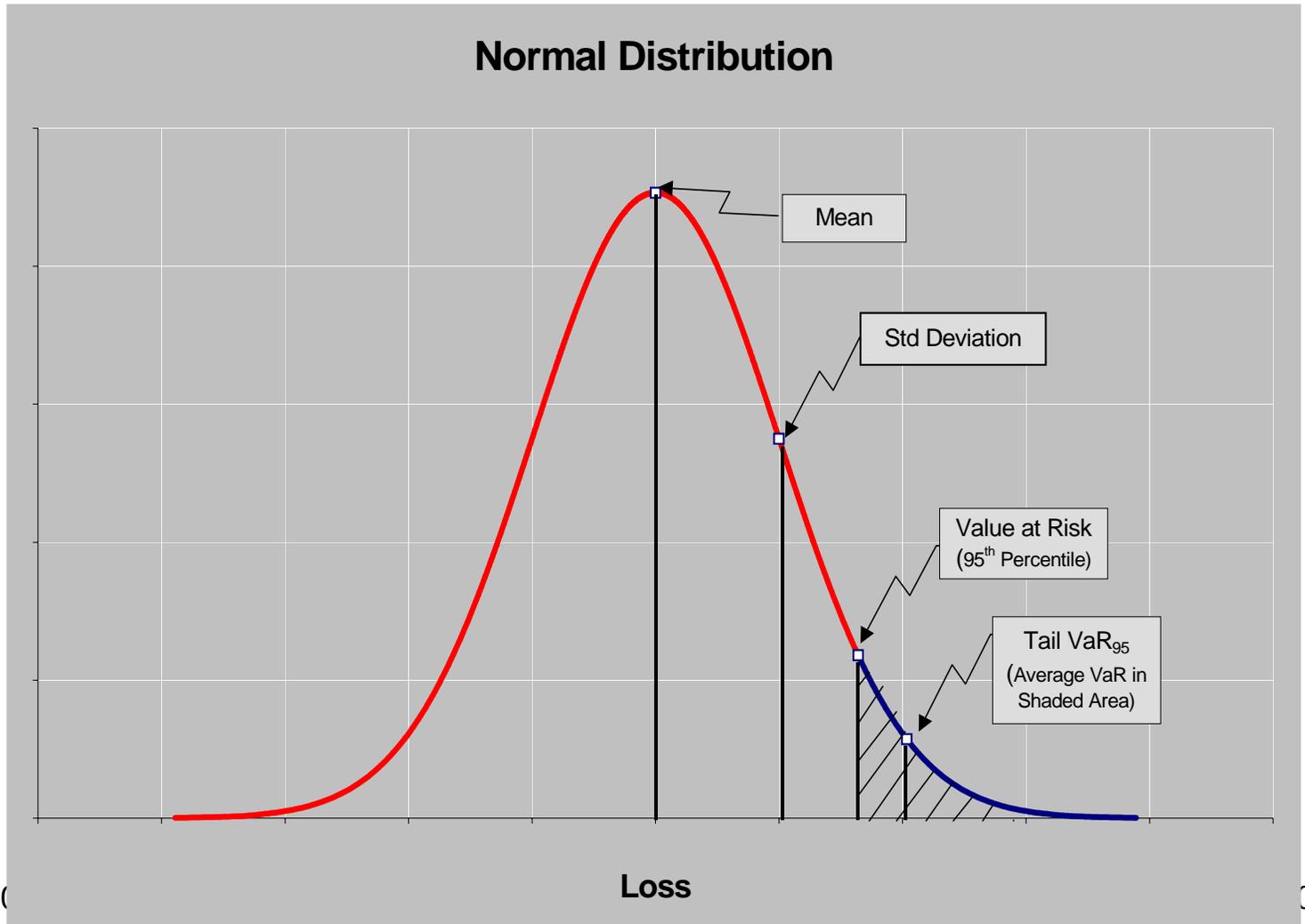
$$E[X | X > x_q]$$

TailVaR

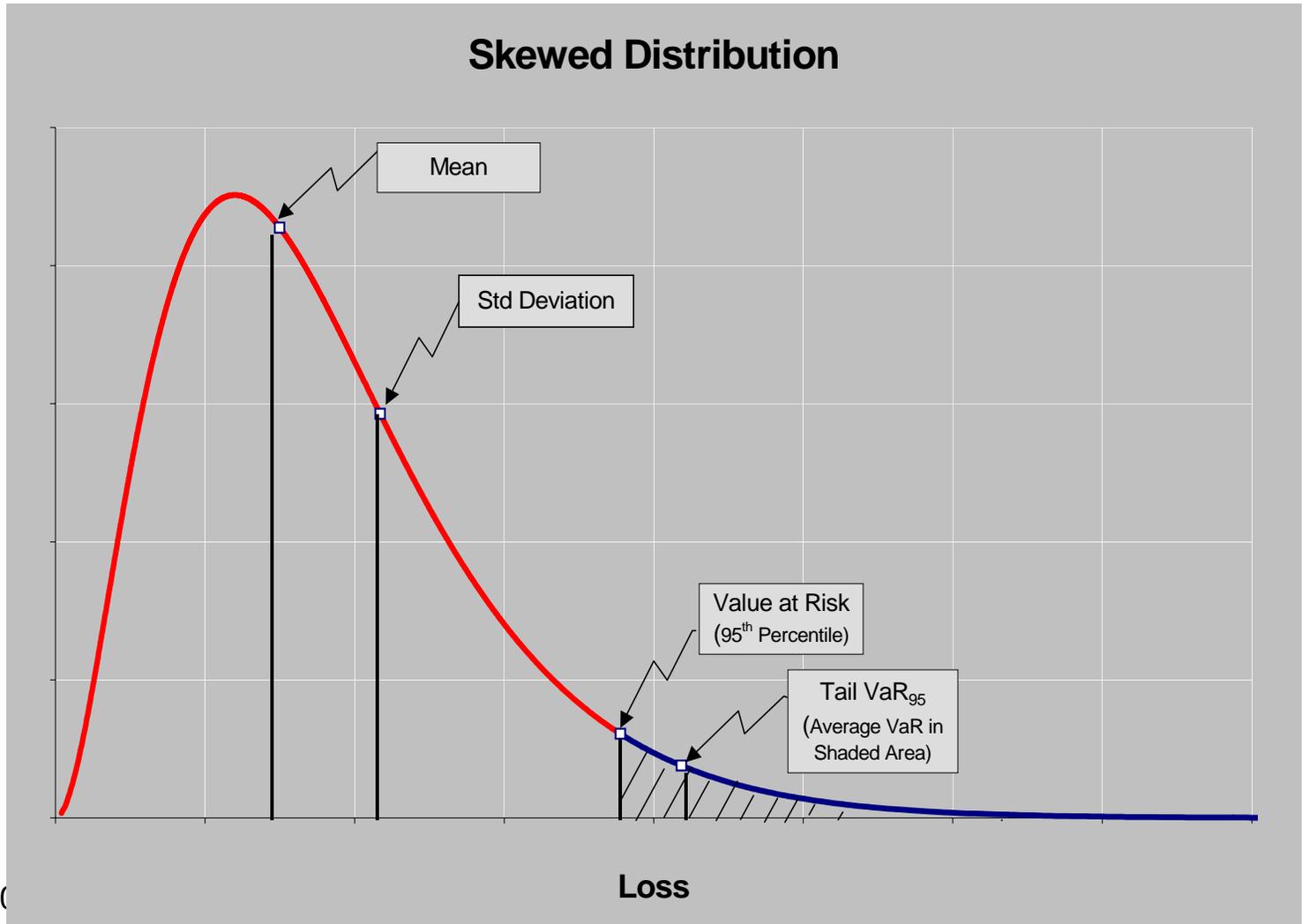
$$\begin{aligned}\text{TailVaR} &= \mathbf{E}[X | X > x_q] \\ &= x_q + \mathbf{E}[X - x_q | X > x_q] \\ &= \text{VaR} + \text{expected "shortfall"}\end{aligned}$$

- Expected **shortfall** is the **net stop loss premium** for excess losses given that a stop-loss claim occurs.
- The **trigger point** x_q can be thought of as the point at which the current assets are just sufficient (on average) to cover current liabilities.

Appropriate Risk Measures



Appropriate Risk Measures



TailVaR

- Easy to estimate from data or from results of a simulation model”
 - Average of top $(1-q)\%$ of observations.
 - Much more stable than quantile, especially when q is small
 - e.g. for a 0.1% level, with 10 000 observations, simple average top 10 observed losses
 - With heavy-tailed distributions, these 10 observations can be quite spread out, so quantile is quite unstable.

Time Horizon

- WP proposes two tests:
 - One is short term, determined for all risks at a very high confidence level (say 99.5% TailVar).
 - The other is long term, determined for all risks at a high confidence level (say 95% TailVaR) for the lifetime of the business.
 - Lower confidence level is appropriate since the insurer can take risk management action throughout the lifetime of the business.

2. Modeling Dependence

- The overall risk of the company can be described as

$$X = X_1 + X_2 + \dots + X_n$$

- i.e. The total risk can be decomposed into risk components.
- In general there are dependencies between risks
 - structural
 - empirical

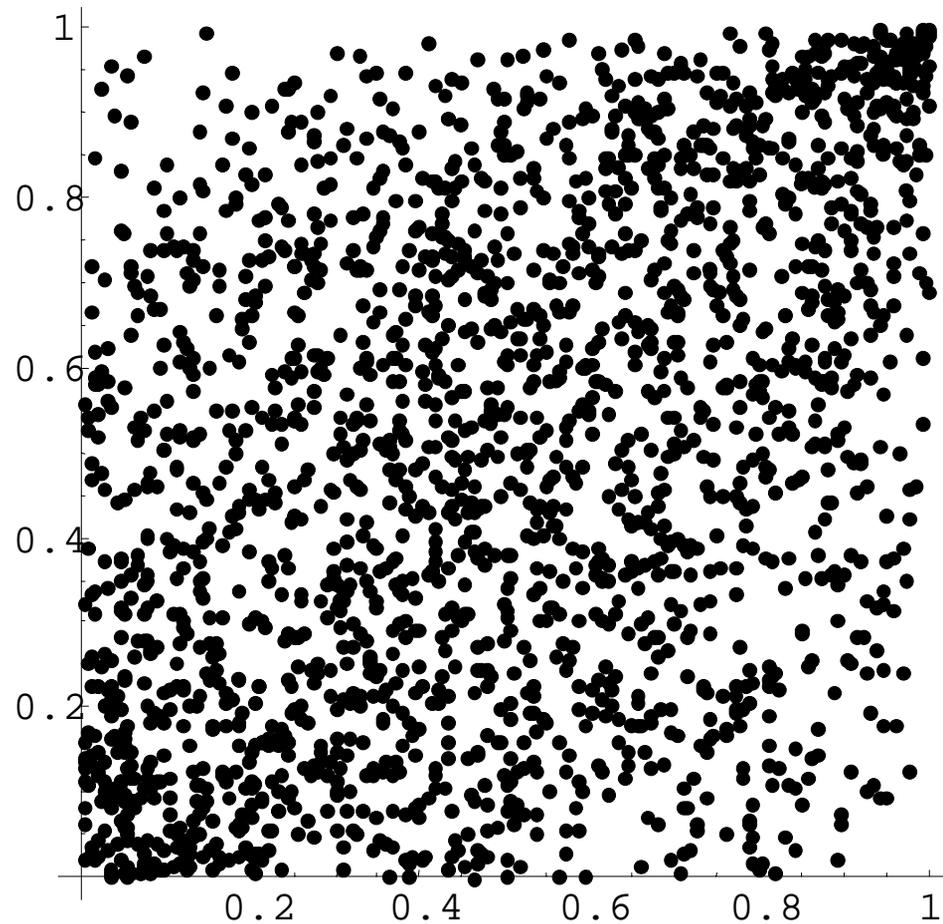
Structural Dependencies

- Loss variables are driven by common variables:
 - Economic factors: inflation drives costs in various lines of insurance
 - Common shocks: an automobile accident can trigger several related claims (BI, damage)
 - Uncertain risk variables: long term mortality changes affect all mortality-related insurance/annuities
 - Catastrophes: 9/11 ripple effect over many lines (life, business interruption, health, property, etc)
- Known relationships can be built into internal models

Empirical Dependencies

- Observed relationships between lines (usually) without necessarily well-defined cause-effect relationships.
 - Relationships may not be simple.
 - Relationships may not be over entire range of losses.
- In practice, observed relationships are at a macro level
 - Detailed data on relationships is often not available.
 - Detailed data on marginal distributions is available.

Dependence?



Copulas

- Begin with marginal distributions
- Construct a multivariate distribution with known marginals
 - e.g. used in connection with time (age) of death of husband and wife (Frees, Valdez, NAAJ)
- Look for specific properties
 - e.g. right-tail dependence, left-tail dependence, or both.
- Calibration of copula is a big issue
 - Applications in stress-testing can lead to deeper understanding of consequences

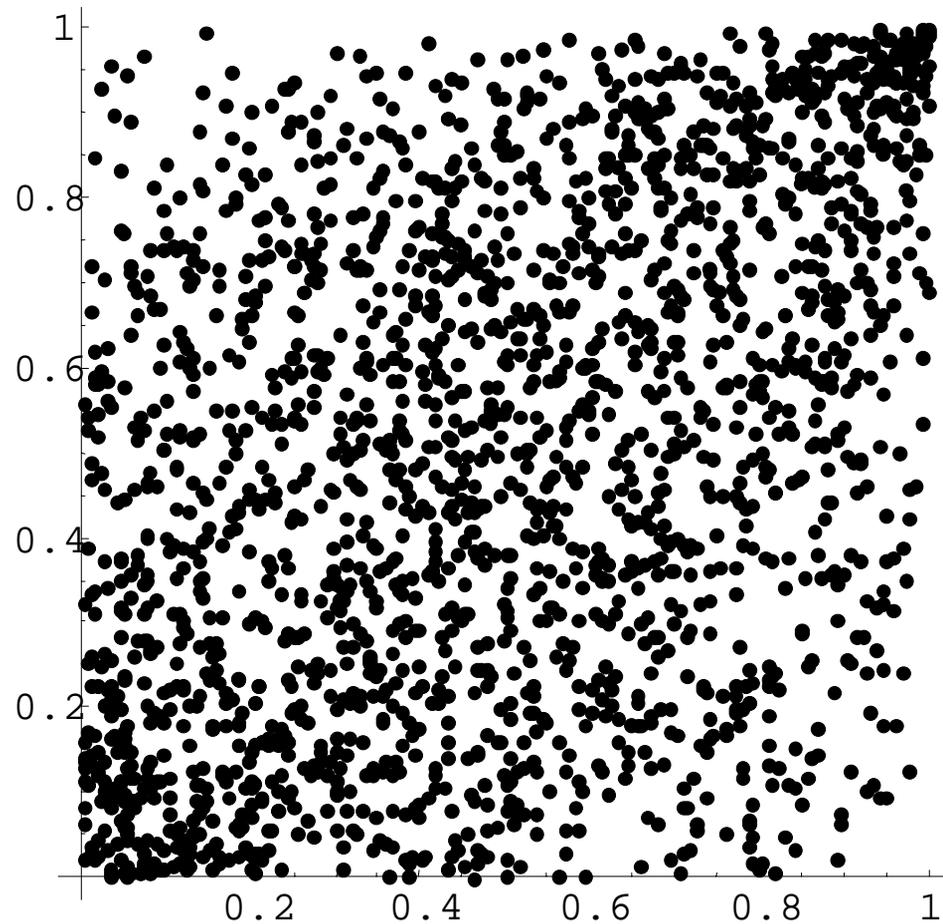
Properties of Copulas

- Tail dependence of X and Y

$$\lambda = \lim_{u \uparrow 1} \Pr \left\{ X > F_X^{-1}(u) \mid Y > F_Y^{-1}(u) \right\}$$

- This is defined in terms of quantiles, not the marginal distributions, so tail dependence can be calculated directly from the copula.
- For a bivariate Normal distribution, the tail dependence is zero if the correlation is less than 1.
 - cannot capture extreme event risk; e.g. 9/11

t Copula results



3. Developing factor-based models

- Ideally, a company should be able to build an “internal model” capturing all aspects of risk and their interactions.
- In practice, a regulator will want to allow for relatively simple methods; consisting of
 - An exposure measure
 - A factor to apply to each exposure measure
 - A formula to combine all the products

Internal Models

- Ideal framework if it captures all key characteristics of a company's risk including
 - All sources of risk under Pillar 1
 - All interactions between different risks
- However, it requires company-specific calibration
 - Data on extreme events is very thin
 - Requires expensive model-development and data collection
 - Results may be very sensitive to calibration, especially in the tail

Example – US RBC for life insurance companies

- C-0: Affiliated Investments
- C-1: Asset Default Risk
 - C-1cs: Unaffiliated Common Stock Risk
 - C-1o: All other C-1 Risk
- C-2: Insurance Risk
- C-3a: Interest Rate Risk
- C-3b: Health Credit Risk
- C-4: Business Risk
- Each category has many risk elements
- Each risk element involves product of exposure measure and a specified factor

US RBC "covariance" adjustment

- $$\text{RBC} = C0 + C4_a + \sqrt{C1_{cs}^2 + (C1_o + C3_a)^2 + C2^2 + C3_b^2 + C4_b^2}$$
 - Recognizes likelihood that not all risks will occur at the same time; i.e. lack of correlation of some risks
 - Uses correlations of either 0 or 1 for simplicity
 - Exact if standard deviation is a risk measure and correlations are correct
- However
 - Insurance company risk is often not Normal
 - Better risk measures should be used to reflect tail risk

Development of formulas

- For an internal model, total balance sheet requirement is

$$TBS = E[X | X > x_q]$$

- This can always be written as $TBS = \mu + k\sigma$.
- The “capital” is obtained as

$$C = TBS - \mu = k\sigma$$

For Normal risks, the value of k can be calculated easily.

- For an entire company the distribution is likely not close to Normal, so more detailed analysis is required; e.g. heavy tailed distributions will have larger values of k .

Adding dependencies

- Models are developed for specific risks within lines of business (LOB) and combined, resulting in

$$C_j = TBS_j - \mu_j = k_j \sigma_j$$

- LOBs are combined recognizing the dependence between them. So some kind of "correlation" is needed, say, $\rho_{i,j}$
- This suggests the simple formula

$$C = \sqrt{\sum_{i,j} C_i C_j \rho_{i,j}}$$

Another representation

$$C_j = k_j \sigma_j = k_j \mu_j \nu_j$$

where ν_j represents the “coefficient of variation”.

- The expected loss can be written as the product of an exposure amount and a standard “risk per unit”.

$$\mu_j = e_j r_j$$

Sources of data

- v_j depend on shape of distribution, and is similar for similar risks for all companies, so this could be based on industry data.
- k_j depends on shape of the distribution and risk appetite of regulator. It is also then similar for all companies.
- r_j is expected loss per unit of risk and depends on industry data.
- e_j is exposure base and depends on company data.
- The “correlations” reflect risk measure, and copula or other measure of correspondence and so can be set by regulator.



Formula Approximations

- Let $X = \sum e_i U_i$
- Fix the joint distribution of the U_i and consider

$$C(e_1, \dots, e_n) = \int x dg[F(x)] - E(X)$$

- Capital function is homogeneous of degree 1 in the exposure variables

$$C(\lambda e_1, \lambda e_2, \dots, \lambda e_n) = \lambda C(e_1, e_2, \dots, e_n)$$

- Choose a target mix of risks $e_1^0, e_2^0, \dots, e_n^0$.

- Put

$$C_0 = C(e_1^0, \dots, e_n^0), \quad C_i = \partial C / \partial e_i^0, \quad C_{ij} = \partial^2 C / \partial e_i^0 \partial e_j^0$$

Formula Approximations

- Theoretical Result: The first two derivatives are given by

$$\frac{\partial C}{\partial e_i} = \int_{-\infty}^{\infty} E[U_i | X = x] dg[F(x)], \quad \frac{\partial^2 C}{\partial e_i \partial e_j} = \int_{-\infty}^{\infty} \text{Cov}[U_i, U_j | X = x] f_X(x) dg'[F(x)].$$

- Some challenges in using these results to estimate derivatives. Second derivatives harder to estimate.
- Some risk measure easier to work with than others.

Formula Approximations

- Let r_i be a vector such that $K = C_0 - \sum_i r_i e_i^0 > 0$ then the homogeneous formula approximation

$$\hat{C}(e_1, e_2, \dots, e_n) = \sum_i r_i e_i + \sqrt{\sum_{i,j} [KC_{ij} + (C_i - r_i)(C_j - r_j)] e_i e_j}$$

agrees with the capital function and its first two derivatives at the target risk mix $e_1^0, e_2^0, \dots, e_n^0$.

- If r_i is a vector such that $K = C_0 - \sum_i r_i e_i^0 < 0$ then a homogeneous formula approximation is

$$\hat{C}(e_1, e_2, \dots, e_n) = \sum_i r_i e_i - \sqrt{\sum_{i,j} [-KC_{ij} + (C_i - r_i)(C_j - r_j)] e_i e_j}$$

Formula Approximation #1

- When $r_i = 0$

$$\begin{aligned}C_g(X) &\approx \sqrt{\sum_{i,j} (C_0 C_{ij} + C_i C_j) e_i e_j} \\&= \sqrt{\sum_{i,j} \frac{(C_0 C_{ij} + C_i C_j)}{c_i c_j} (c_i e_i)(c_j e_j)} \\&= \sqrt{\sum_{i,j} \hat{\rho}_{ij} C_g(X_i) C_g(X_j)}\end{aligned}$$

- Suggests definition of "tail correlation".

$$\hat{\rho}_{ij} = \frac{(C_0 C_{ij} + C_i C_j)}{c_i c_j}$$

Formula Approximation #2

- When $r_i = C_i$ formula is essentially first order

$$C_g(X) \approx \sum_i C_i e_i$$

- "Factors " $C_i < c_i$ already reflect diversification.
- Suggests many existing capital formulas are as good (or bad) as first order Taylor expansions.

Formula Approximation #3

- When $r_i = c_i$ we get

$$\begin{aligned}
 C_g(X) &\approx \sum_i c_i e_i - \sqrt{\sum_{i,j} ((\sum_k c_k e^0_k - C_0) C_{ij} + (C_i - c_i)(C_j - c_j)) e_i e_j} \\
 &= \sum_i c_i e_i - \sqrt{\sum_{i,j} \frac{[(\sum_k c_k e^0_k - C_0) C_{ij} + (C_i - c_i)(C_j - c_j)]}{c_i c_j} (c_i e_i)(c_j e_j)} \\
 &= \sum_i C_g(X_i) - \sqrt{\sum_{i,j} \eta_{ij} C_g(X_i) C_g(X_j)}
 \end{aligned}$$

- Undiversified capital less an adjustment determined by "inverse correlation"

$$\eta_{ij} = \frac{[(\sum_k c_k e^0_k - C_0) C_{ij} + (C_i - c_i)(C_j - c_j)]}{c_i c_j}$$

Formula Approximations: Preliminary Conclusions

- Practical work so far suggests

$$C_g(X) \approx \sqrt{\sum_{i,j} \hat{\rho}_{ij} C_g(X_i) C_g(X_j)}$$

is a more robust approximation. In particular, when the risks are normal

$$\sqrt{\sum_{i,j} \hat{\rho}_{ij} C_g(X_i) C_g(X_j)} \geq \sum_i C_g(X_i) - \sqrt{\sum_{i,j} \eta_{ij} C_g(X_i) C_g(X_j)}$$

- Other homogeneous approximations are possible.

Another Approach

- Models are developed for specific risks within lines of business (LOB) and combined, resulting in

$$C_j = TBS_j - \mu_j = k_j \sigma_j$$

- LOBs are combined recognizing the dependence between them. So some kind of "correlation" is needed, say, $\rho_{i,j}$
- This suggests the simple formula

$$C = \sqrt{\sum_{i,j} C_i C_j \rho_{i,j}}$$

Decomposing k_j

- Now write :

$$k_j = z_g \cdot m_X$$

where

- z_g is as the value of k_j for a Normal(0,1) distribution
- the risk multiplier m_X is a parameter intended to capture the non-normality in a simple way (i.e. $m_X = 1$ in the Normal case.
- The capital formula can be expressed as:

$$C = \sqrt{\sum_{i,j} C_i C_j \cdot \frac{m_X^2}{m_{x_i} m_{x_j}} \cdot \rho_{i,j}}$$

Risk multiplier adjusted correlation

- We define the risk multiplier adjusted correlation

$$\rho_{i,j}^{RM} = \frac{m_X^2}{m_{x_i} m_{x_j}} \cdot \rho_{i,j}$$

- This leads to

$$C = \sqrt{\sum_{i,j} C_i C_j \rho_{i,j}^{RM}}$$

Analytic Examples: TailVaR

- **Pareto**

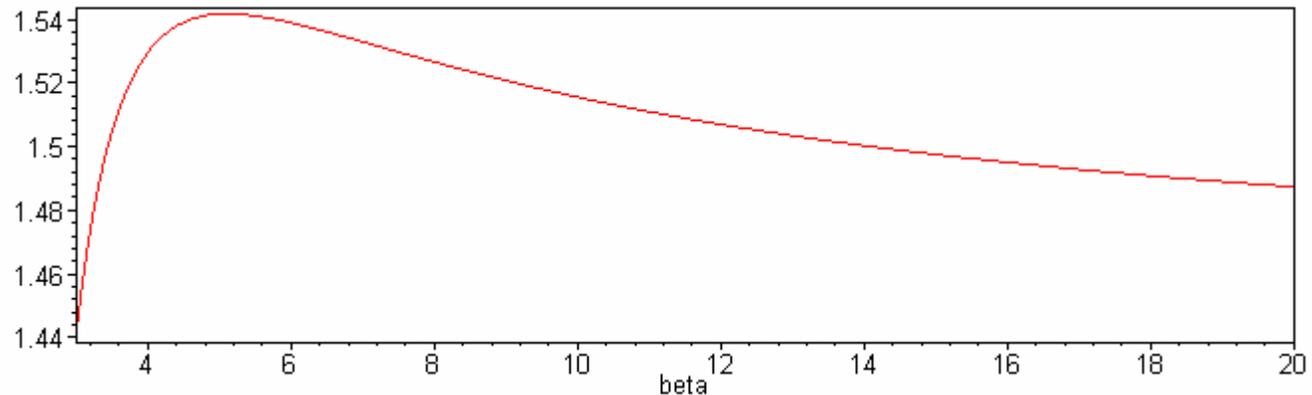
The Risk Multiplier for Pareto distributed random variables with tail parameter β and VaR quantile α is

$$m_x = \frac{\sqrt{\beta(\beta-2)}}{z_g} \left[\frac{1}{(1-\alpha)^{\frac{1}{\beta}}} - 1 \right]$$

For $\alpha = 0.95$, $z_g = 2.061$.

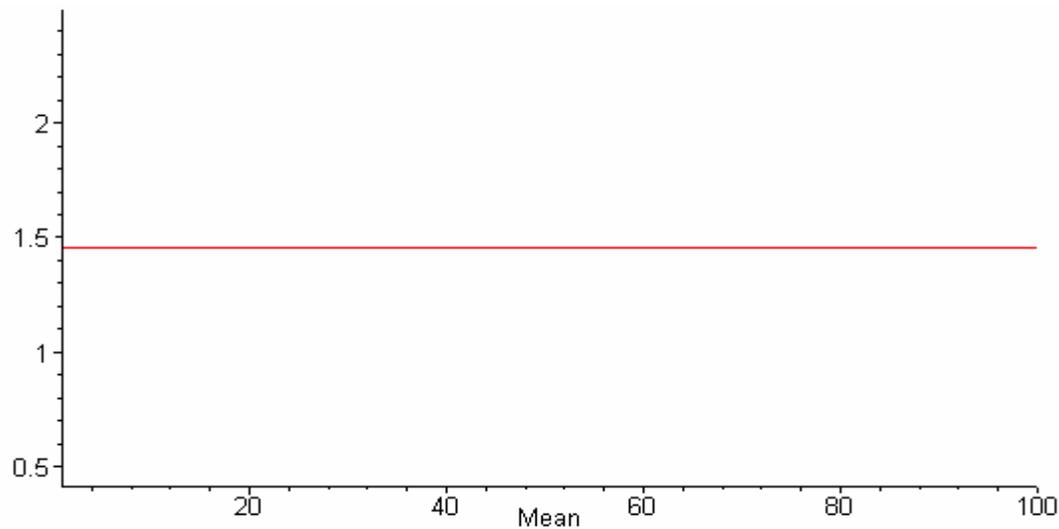
Risk Multiplier: Pareto

- The risk multiplier, as a function of the tail parameter, varies within a narrow range (1.44 to 1.54)



Risk Multiplier: Exponential

- The Risk Multiplier in the Exponential case is constant.
- It only depends on the risk measure and on the significance level.



Further Empirical studies

- Simulation studies show that the risk multiplier is also very stable for an overall portfolio composed of different risks with different distributions for a large range of parameters (light-tailed to heavy-tailed).
- A capital approximation based on the risk multiplier-modified correlation yields similar results for significantly different business mixes and for different risk measures such as VaR, TailVaR, Wang Transform and Block Maximum.

Next Steps

- Report to IAA in 2003 and to IAIS in early 2004
 - not sufficiently detailed for any implementation
 - only provides a “framework”
 - level of detail is major item for IAIS
 - Interaction with accounting standards can be simpler if they are uniform.
 - Further detailed studies of different risk classes will be necessary.