Capital Consumption:
An Alternative Methodology for Pricing Reinsurance

Topic #1: Risk Evaluation
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Abstract

This paper introduces a *capital consumption* methodology for the price evaluation of reinsurance in a stochastic environment. It differs from the common practice of risk-based capital allocation and release by: (i) evaluating the actual contract cash flows at the scenario level; (ii) eliminating the need for contract-level supporting capital allocation and release; (iii) evaluating each scenario's operating deficits as contingent capital calls on the company capital pool; and (iv) reflecting the expected cost of contingent capital calls as an expense load. This method eliminates the need for capital allocation and release; creates scenarios that more closely model actual contract capital usage; allows more flexibility in stochastic modeling; and makes risk-return preferences an explicit part of the pricing decision.

Keywords: capital consumption, reinsurance pricing, utility theory, risk preferences.

1. Introduction

This paper introduces a *capital consumption* methodology for the price evaluation of reinsurance in a stochastic environment. It differs from the common practice of risk-based capital allocation and release by: (i) evaluating the actual contract cash flows at the scenario level; (ii) eliminating the need for contract-level supporting capital allocation and release; (iii) evaluating each scenario's operating deficits as contingent capital calls on the company capital pool; and (iv) reflecting the expected cost of contingent capital calls as an expense load.

This method eliminates the need for capital allocation and release; creates scenarios that more closely model actual contract capital usage; allows more flexibility in stochastic modeling; and makes risk-return preferences an explicit part of the pricing decision.

Section 2 begins with an overview of the capital consumption approach, framing the major differences from capital allocation. Section 3 then presents the details

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of the approach. Section 4 delves further into the concept of contingent capital consumption and its costs. Section 5 shows examples of price evaluation using this approach. Section 6 concludes with linkages to other current research efforts. Appendix A addresses the question: Does insurance capital allocation make sense? Appendix B demonstrates one approach for calibrating to the portfolio level.

2. Capital Consumption Overview

This paper challenges many fundamental conceptual underpinnings of reinsurance pricing. Any attempt at an overview will be difficult. As a start, we will outline the major differences in treatment of capital under an allocation versus consumption framework by considering four questions:

1. What happens to the total capital?
2. How are the segments evaluated?
3. What does being in a portfolio mean?
4. How is relative risk contribution reflected?

Question 1: What happens to the total capital?	
Allocation	**Consumption**
• Divided up among the segments. • Either by explicit allocation, or assignment of the marginal change in the total capital requirement from adding the segment to the remaining portfolio	• Left intact • Each segment has the right to “call” upon the total capital to pay its operating deficits or shortfalls

Allocation splits up the total capital and doles it out to segments. Two critical assumptions underlie this approach: that the capital itself is divisible; and that, similar to manufactured products, insurance products require up-front capital investment to produce. Consumption instead recognizes the right (widely acknowledged among capital allocation proponents) of any contract to consume potentially all the company’s capital.

Question 2: How are the segments evaluated?	
Allocation	**Consumption**
• Give the allocations to each segment • Evaluate each segment's return on their allocated capital • Must clear their hurdle rate	• Give each segment "access rights" to the entire capital • Evaluate each segment's potential calls (both likelihood and magnitude) on the total capital • Must pay for the likelihood and magnitude of their potential calls

Allocation proponents ask, without capital allocation, how can you make either performance evaluations or investment decisions? How can you decide where to grow or shrink your book? How can you divide a bonus pool? They also advocate this as a translation vehicle to results from other industries.

Consumption is a valid alternative for either performance evaluation or portfolio composition decision-making. Both approaches are based upon the premise that riskier segments must pay for their risk. Both approaches are also dependent upon a sound portfolio risk model, the true foundation of stochastic reinsurance pricing.

Question 3: What does being in a portfolio mean?	
Allocation	**Consumption**
• Being standalone with less capital • But still having access to all the capital if necessary, although it is unclear how this is reflected	• Being standalone with potential access to all the capital • But all other segments have similar access rights

This is **the** critical difference. Allocation treats segments as if standalone, with less capital. This means being in a portfolio is like being on your own, but you have to support less capital. Consumption on the other hand treats being in a portfolio like being standalone, with access to potentially all the capital, but with the added wrinkle that all the other segments have similar access rights.

Question 4: How is relative risk contribution reflected?	
Allocation	**Consumption**
• Use a single risk measure to determine required capital • Select a dependence structure for the aggregation of segment distributions into a portfolio aggregate distribution • The marginal impact of adding a segment to the remaining portfolio is that segment's risk contribution	• Use scenario-level detail generated by stochastic modeling • Use explicit risk-return evaluation via utility function • Segment's risk contribution is determined at the scenario level, then aggregated over all scenarios

This is a deep question, one that will be covered in extensive detail in the remainder of the paper. The essential point: once you move to the modeled scenario level, capital allocation becomes increasingly difficult to meaningfully interpret. Allocated capital is determined based on a risk measure of the distribution in total. For any given scenario, though, this overall amount is never the actual required amount — namely, the modeled operating deficit. The allocated capital is excessive for favorable scenarios, and grossly inadequate in severe loss scenarios (unless the capital equals the policy limit)[2].

3. Details of the Capital Consumption Approach

We will demonstrate the capital consumption approach within a stochastic contract analysis framework. We will cover the three major differences from typical risk-based capital allocation approaches: (i) analyzing the contract outcome at the scenario level, (ii) discounting at a default-free rate, and (iii) calculating the contract's capital consumption within each scenario.

Scenario Analysis
The first modification requires maintaining the scenario detail, and analyzing the contract's outcomes at the scenario level. Stochastic modeling is basically

[2] This problem is particularly striking in the evaluation of catastrophe reinsurance contracts, with small probabilities of a full contract limit loss. A "risk-based" capital amount might be some small fraction of the limit—say 5%. What sense does this capital amount make in the limit loss scenario? We held 5% of the limit as capital? And then how exactly did we fund the remaining loss amount? It came from company capital in total. The alternative—holding the full limit as capital—puts an unrealistic return burden on the contract. Current market price levels would likely make the contract look unattractive.

scenario analysis extended to a high level of granularity. Modeling thousands of points of a contract outcome distribution means generating thousands of scenarios. This extension to scenario detail may appear trivial. If the functions are linear, or the distributions symmetric, no benefit will be gained by expanding the detail. Expected values are sufficient for decision-making. Jensen's inequality[3] becomes Jensen's "equality" in these conditions:

$$g(E[x]) = E[g(x)]$$

However, reinsurance contracts have non-linear contract features such as aggregate deductibles, caps, corridors, and co-participations. They also have extremely skewed distributions. In such conditions, we must evaluate ***E*[*g*(*x*)]** to get an accurate result. Evaluating each point of the distribution requires maintenance and use of the scenario detail.

Default-Free Discounting

The second modification involves discounting cash flows at a default-free rate. Scenario analysis (indeed, simulation modeling) is built upon the premise that possible, realizable, plausible outcomes can be generated and analyzed. For the entire process to work, each generated scenario is "**conditionally certain**": given the scenario occurs, its outcome is certain. Where it is not, the entire practice of simulation modeling would be undermined by "meta-uncertainty." The scenarios themselves must withstand the scrutiny of a reality check.

Uncertainty for the contract in total is represented in the distribution across all modeled scenarios, and the probability weights assigned to those scenarios. In other words, *uncertainty is reflected between scenarios, not within them*. Given conditional certainty, scenario cash flows can be discounted at a default-free rate (or a simulated path of default-free forward rates).

Scenario Capital Consumption

The final, and perhaps most controversial, change involves the treatment of capital. Most methodologies focus on up-front allocation of supporting risk-based capital, and its release over time. The capital actually consumed (if any) by each modeled scenario of the contract is the focus here.

Capital is still required at the company level and still needs to be invested in an insurance company. However, it plays a fundamentally different role in an insurer than in a manufacturer. Capital investment in manufacturing is typically up-front, in equipment and raw materials. In contrast, insurance "products" are promises to pay contingent on valid claims. Thus the costs of insurance products are claim-related payments. They are not specific investments, and occur (if at all) in the future.

[3] Jensen's inequality states that for a cumulative probability distribution F(x) and a convex function g(x), E[g(x)] >= g(E[x]). There are countless references on this—e.g., Heyer [7], p. 98.

Insurers receive revenue in the form of premium that includes an estimated provision for their expected costs, plus some volatility loading. Insurance capital acts more like a "claims paying reservoir**,**" an overall buffer for unpredictability and volatility of aggregated product results. This reservoir is subject to unpredictable future inflows and outflows. What has been termed an "allocation" of capital for underwriting new contracts is more like the granting of additional rights to draw upon future capital. The critical issue is, therefore, both the likelihood and magnitude of exposure of capital to possible consumption by contracts.

The cost of maintaining the capital reservoir is an overall cost of business — an overhead expense. This approach essentially assesses contracts for this overhead expense in a "risk-based" manner. The bases for the assessment are likelihood and magnitude of capital reservoir drawdowns.

Maintaining the scenario detail, and recognizing that scenarios are conditionally certain, we can evaluate the capital amounts *actually consumed* by each scenario. A contract can "pay its own way" if its total revenues exceed its total costs. Total revenues include premium and investment income on its own flows. Total costs means expenses and losses. Company capital is needed when the contract runs an **operating deficit** — when its costs exceed its revenues. Philbrick and Painter make this point well:

> "When an insurance company writes a policy, a premium is received. A portion of this policy can be viewed as the loss component. When a particular policy incurs a loss, the company can look to three places to pay the loss. The first place is the loss component (together with the investment income earned) of the policy itself. In many cases, this will not be sufficient to pay the loss. The second source is unused loss components of other policies. In most cases, these two sources will be sufficient to pay the losses. In some years, it will not, and the company will have to look to a third source, the surplus, to pay the losses." [16, p. 124]

To evaluate scenario-level capital consumption and operating deficit, we look at the contract's **experience fund**. An experience fund is a concept from finite risk reinsurance. It is an account containing available revenue (premium net of expenses) plus investment income earned on the fund balance (at an assumed investment rate). All subject losses are paid from the fund. An experience fund allows us to calculate the contract's "terminal value" or cumulative operating result. Consider Example 1, the experience fund of a realistic long-tailed contract.

Example 1
Experience Fund for Long-tailed Contract
120% Loss Ratio Scenario

		Probability	10.0%			
Investment Rate	8.0%	Loss Ratio	120.0%	Ultimate Loss	120,000	

1	2	3	4	5	6	7	8	9
Time	Beginning Fund Balance	Premiums	Expenses	Payment Pattern	Paid Losses	Investment Income	Ending Fund Balance	Capital Call
0	$ -	$ 100,000	$ 15,000	0.0%	$ -	$ -	$ 85,000	$ -
1	$ 85,000	$ -	$ -	50.0%	$ 60,000	$ 2,000	$ 27,000	$ -
2	$ 27,000	$ -	$ -	25.0%	$ 30,000	$ -	$ (3,000)	$ 3,000
3	$ -	$ -	$ -	12.0%	$ 14,400	$ -	$ (14,400)	$ 14,400
4	$ -	$ -	$ -	6.0%	$ 7,200	$ -	$ (7,200)	$ 7,200
5	$ -	$ -	$ -	4.0%	$ 4,800	$ -	$ (4,800)	$ 4,800
6	$ -	$ -	$ -	2.0%	$ 2,400	$ -	$ (2,400)	$ 2,400
7	$ -	$ -	$ -	1.0%	$ 1,200	$ -	$ (1,200)	$ 1,200
8	$ -	$ -	$ -	0.0%	$ -	$ -	$ -	$ -
9	$ -	$ -	$ -	0.0%	$ -	$ -	$ -	$ -
10	$ -	$ -	$ -	0.0%	$ -	$ -	$ -	$ -
TOTAL		**$ 100,000**	**$ 15,000**	**100.0%**	**$ 120,000**		**$ -**	**$ 33,000**
NPV		**$ 100,000**	**$ 15,000**	**86.2%**	**$ 103,479**			**$ 24,775**

Each column is explained in detail:

- Column 1 is time from inception of the contract in years.
- Column 2 is the fund balance at the beginning of each year.
- Column 3 is the premium flow into the fund.
- Column 4 is the expense flow out of the fund.
- Column 5 is the expected payment pattern as a percentage of ultimate loss. Ultimate loss is expressed as a ratio to premium. In this case, it is 120%.
- Column 6 is the product of ultimate losses $120,000 and the pattern in Column 5.
- Column 7 is the investment income earned on the end-of-year fund balance less all payments during the year, assuming those payments are made at the end of the year. This is assumed to go back into the fund for the next year. This could be adjusted to the midpoint of the year if desired.
- Column 8 is the end-of-year fund balance. It equals (2) + (3) – (4) – (6) + (7).
- Column 9 shows the **capital calls**.

Once Column 8 falls below zero, the contract is in an operating deficit position: the fund is empty, yet loss payments must be made. In order to make the payments, a capital call is made for the amount needed to make the required loss payment. Once the fund hits zero, it never rises above it again. Capital is only provided as needed to make the loss payments. Thus the contract makes what amount to a **series of capital calls** stretching into the future.

Time Profile of Capital Consumption

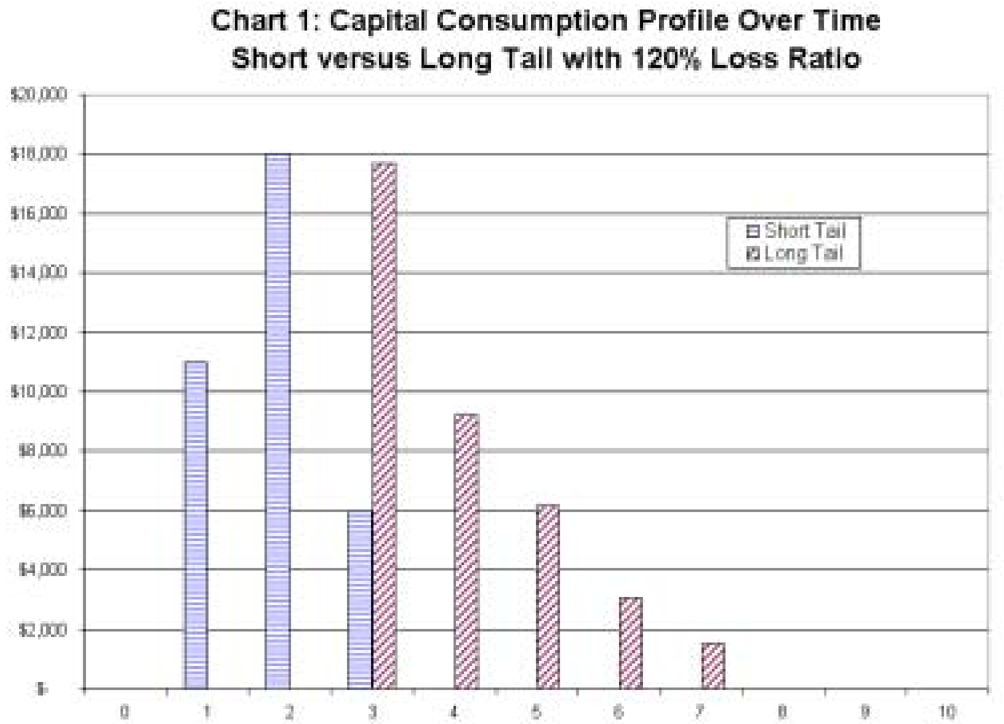
Compare Example 1A, which shows the capital calls for the contract in Example 1 with everything identical except a quicker payment pattern — a shorter tail.

Example 1A
Experience Fund for Short-tailed Contract
120% Loss Ratio Scenario

Investment Rate	**8.0%**	**Loss Ratio**	**120.0%**	**Ultimate Loss**	**120,000**

1	2	3	4	5	6	7	8	9
Time	Beginning Fund Balance	Premiums	Expenses	Payment Pattern	Paid Losses	Investment Income	Ending Fund Balance	Capital Call
0	$ -	$ 100,000	$ 15,000	0.0%	$ -	$ -	$ 85,000	$ -
1	$ 85,000	$ -	$ -	80.0%	$ 96,000	$ -	$ (11,000)	$ 11,000
2	$ -	$ -	$ -	15.0%	$ 18,000	$ -	$ (18,000)	$ 18,000
3	$ -	$ -	$ -	5.0%	$ 6,000	$ -	$ (6,000)	$ 6,000
4	$ -	$ -	$ -	0.0%	$ -	$ -	$ -	$ -
5	$ -	$ -	$ -	0.0%	$ -	$ -	$ -	$ -
6	$ -	$ -	$ -	0.0%	$ -	$ -	$ -	$ -
7	$ -	$ -	$ -	0.0%	$ -	$ -	$ -	$ -
8	$ -	$ -	$ -	0.0%	$ -	$ -	$ -	$ -
9	$ -	$ -	$ -	0.0%	$ -	$ -	$ -	$ -
10	$ -	$ -	$ -	0.0%	$ -	$ -	$ -	$ -
TOTAL		**$ 100,000**	**$ 15,000**	**100.0%**	**$ 120,000**			**$ 35,000**
NPV		**$ 100,000**	**$ 15,000**	**90.9%**	**$ 109,084**			**$ 30,380**

Because of the reduced investment income and shorter tail, the capital calls are larger ($35,000 vs $33,000) and sooner. Chart 1 shows the time profile comparison:



This chart visually depicts a major difference between short and long tail contracts that has yet to be fully understood and integrated into evaluation frameworks. Clearly the capital concept needs a significant extension over *time*. One can envision measures of concentration expanding from scalars to *vectors*, indexed into the future. For example, the impact on the company's future cash position could be reflected in the pricing and underwriting decision for a contract. A contract may have an attractive upside, but may have undesirable structural relationships to other portions of the portfolio (e.g., with respect to a large return premium or reserve adjustment). This concept will be elaborated on in future papers.

Reduced Operating Deficit

Under the 120% Loss Ratio scenario, the Long-tailed contract calls for a total of $33,000 in capital over time. If the loss ratio under another scenario were lower — say 100% — the contract would make smaller capital calls, since its operating deficit would be smaller. Consider Example 2:

Example 2
Experience Fund for Long-tailed Contract
100% Loss Ratio Scenario

		Probability	30.0%
Investment Rate	8.0%	Loss Ratio	100.0%
Ultimate Loss	100,000		

1	2	3	4	5	6	7	8	9
Time	Beginning Fund Balance	Premiums	Expenses	Payment Pattern	Paid Losses	Investment Income	Ending Fund Balance	Capital Call
0	$ -	$ 100,000	$ 15,000	0.0%	$ -	$ -	$ 85,000	$ -
1	$ 85,000	$ -	$ -	50.0%	$ 50,000	$ 2,800	$ 37,800	$ -
2	$ 37,800	$ -	$ -	25.0%	$ 25,000	$ 1,024	$ 13,824	$ -
3	$ 13,824	$ -	$ -	12.0%	$ 12,000	$ 146	$ 1,970	$ -
4	$ 1,970	$ -	$ -	6.0%	$ 6,000	$ -	$ (4,030)	$ 4,030
5	$ -	$ -	$ -	4.0%	$ 4,000	$ -	$ (4,000)	$ 4,000
6	$ -	$ -	$ -	2.0%	$ 2,000	$ -	$ (2,000)	$ 2,000
7	$ -	$ -	$ -	1.0%	$ 1,000	$ -	$ (1,000)	$ 1,000
8	$ -	$ -	$ -	0.0%	$ -	$ -	$ -	$ -
9	$ -	$ -	$ -	0.0%	$ -	$ -	$ -	$ -
10	$ -	$ -	$ -	0.0%	$ -	$ -	$ -	$ -
TOTAL		**$ 100,000**	**$ 15,000**	**100.0%**	**$ 100,000**		**$ -**	**$ 11,030**
NPV		**$ 100,000**	**$ 15,000**	**86.2%**	**$ 86,232**			**$ 7,528**

Now it only asks for $11,030. If the loss ratio were low enough, the contract would make no capital calls, as in Example 3:

Example 3
Experience Fund for Long-tailed Contract
80% Loss Ratio Scenario

		Probability	60.0%
Investment Rate	8.0%	Loss Ratio	80.0%
Ultimate Loss	80,000		

1	2	3	4	5	6	7	8	9
Time	Beginning Fund Balance	Premiums	Expenses	Payment Pattern	Paid Losses	Investment Income	Ending Fund Balance	Capital Call
0	$ -	$ 100,000	$ 15,000	0.0%	$ -	$ -	$ 85,000	$ -
1	$ 85,000	$ -	$ -	50.0%	$ 40,000	$ 3,600	$ 48,600	$ -
2	$ 48,600	$ -	$ -	25.0%	$ 20,000	$ 2,288	$ 30,888	$ -
3	$ 30,888	$ -	$ -	12.0%	$ 9,600	$ 1,703	$ 22,991	$ -
4	$ 22,991	$ -	$ -	6.0%	$ 4,800	$ 1,455	$ 19,646	$ -
5	$ 19,646	$ -	$ -	4.0%	$ 3,200	$ 1,316	$ 17,762	$ -
6	$ 17,762	$ -	$ -	2.0%	$ 1,600	$ 1,293	$ 17,455	$ -
7	$ 17,455	$ -	$ -	1.0%	$ 800	$ 1,332	$ 17,987	$ -
8	$ 17,987	$ -	$ -	0.0%	$ -	$ 1,439	$ 19,426	$ -
9	$ 19,426	$ -	$ -	0.0%	$ -	$ 1,554	$ 20,980	$ -
10	$ 20,980	$ -	$ -	0.0%	$ -	$ 1,678	$ 22,659	$ -
TOTAL		**$ 100,000**	**$ 15,000**	**100.0%**	**$ 80,000**		**$ 22,659**	**$ -**
NPV		**$ 100,000**	**$ 15,000**	**86.2%**	**$ 68,986**			**$ -**

The experience fund and capital calls give us the analytic framework. Now we consider *explicit valuation* of the calls, on our way to price determination.

4. Valuation of Contingent Capital Calls

The *concept* of contingent capital calls was presented in Philbrick and Painter (**emphasis mine**):

> "The entire surplus is available to every policy to pay losses in excess of the aggregate loss component. Some policies are more likely to create this need than others are, even if the expected loss portions are equal. Roughly speaking, for policies with similar expected losses, we would expect the policies with a large variability of possible results to require more contributions from surplus to pay the losses. We can envision an insurance company instituting a **charge for the access to the surplus. This charge should depend, not just on the likelihood that surplus might be needed, but on the amount of such a surplus call**." [17, p. 124]

They continue (my inline comments are [***bold and italicized***]):

> "We can think of a capital allocation method as determining a charge to each line of business that is dependant on the need to access the surplus account [***contingent capital***]. Conceptually, we might want to allocate a specific cost to each line for the right to access the surplus account [***call***]. In practice though, we tend to express it by allocating a portion of surplus to the line, and then requiring that the line earn (on average) an adequate return on surplus. Lines with more of a need for surplus will have a larger portion allocated to them, and hence will have to charge more to the customers to earn an adequate rate of return on the surplus. Effectively, this will create a charge to each line for its fair share of the overall cost of capital." [17, p. 124]

Thus, Philbrick and Painter would like to charge a line of business for the right to access the surplus account — i.e., to make capital calls. However, they are still thinking in terms of a supporting capital allocation framework, within which no such concept exists. In contrast, the capital consumption approach is built around such a charge.

What exactly would such a charge mean? It could have several meanings simultaneously:

1. A risk-based overhead expense loading, though it may not be a payment to an outside entity.
2. A decision variable that influences the attractiveness of certain product types.
3. An explicit expression of the company's risk-return preferences by application of concepts from utility theory.

Each is considered in detail:

1. Risk-based Overhead Expense Loading

As requested by Philbrick and Painter, the charge is based on the magnitude and likelihood of calls upon the common capital pool. Since capital is at the company level only, and available to all contracts, the cost of that capital should be an overhead expense — like rent. However, unlike rent, the cost may not correspond to actual payments being made by the company. However, it is a cost of doing business, in that without capital in total to support the portfolio and guarantee a certain level of perceived claims-paying ability, the contracts could not be sold. All contracts partake of the benefits of the capital pool and, therefore, must be assessed some share of its maintenance cost.

2. Pricing Decision Variable

In order to make informed pricing decisions, product costs must be accurately and objectively assessed for all product types. The "science" of overhead expense allocation is far from exact, yet the stakes are high. Product viability decisions are driven to a large extent by expense figures. What company does not have product managers who feel the overhead cost allocations to their products are unfair or inaccurate? Yet without some kind of objective decision framework, the company may not be reflecting all the costs of a product when determining price adequacy. It is critical that the magnitude and likelihood of capital calls be assessed in a fair and reasonable manner, so that the cost of risk enters the pricing decision.

3. Application of Concepts from Utility Theory

Assessing a cost by scenario is akin to introducing a **utility function**[4]. The Faculty and Institute of Actuaries Subject 109 Financial Economics reading introduces utility as follows:

> "In the application of utility theory to finance it is assumed that a numerical value called the utility can be assigned to each possible value of an investor's wealth by what is known as a preference function or utility function….Decisions are made on the basis of maximising the expected value of utility under the investor's particular beliefs about the probability of different outcomes. Therefore the investor's risk-return preference will be described by the form of his utility function." [4, Unit 1, p. 1]

Introducing a utility function into reinsurance pricing analysis means the company is expressing its risk-return preferences in mathematical form. Borch stated the same thing forty years ago:

[4] Many papers have been written on the application of utility theory to insurance and reinsurance analysis. European actuaries include Karl Borch [1], Hans Gerber and Gerard Pafumi [6], and Hans Buhlmann [2]. In North America, Leigh Halliwell [8], Oakley Van Slyke [19], Alistair Longley-Cook [12], Daniel Heyer [7], and Frank Schnapp [18] have all published articles on utility theory and insurance.

> “To introduce a utility function which the company seeks to maximize, means only that such consistency requirements (in the various subjective judgements made by an insurance company) are put into mathematical form.” [1, p. 23]

This appears to be a profound change in reinsurance pricing practice. However, the change really entails **making the implicit explicit**. Any reinsurance pricing practice includes a utility assumption buried within it. Consider the marginal standard deviation pricing formula from Kreps [10], a de facto reinsurance industry standard pricing method paraphrased here:

> *Our company values the risk of a contract using the marginal impact to the portfolio standard deviation. That is, we take the square root of the expected value of the square of deviations from the mean of the portfolio outcome distribution both with and without the new contract. This difference is used to determine the marginal capital requirement, to which we assess a cost of capital figure.*

Implicit in this method are the following utility assumptions — mathematical expressions of preferences:

> *The marginal impact on the portfolio standard deviation is our chosen functional form for transforming a given distribution of outcomes to a single risk measure.*
>
> *Risk is completely reflected, properly measured and valued by this transform.*
>
> *Upward deviations are treated the same as downward deviations.*

In fact, any risk-based pricing methodology has an implicit underlying utility function[5]. Utility is the mathematical valuation of uncertainty, the essence of reinsurance pricing. One wonders why an industry that exists to purchase risk has not made a point of being explicit about its risk-return preferences.

Cost Functions

How do we assess this capital call cost at the scenario level? We need a **cost function**. The simplest cost function would be a flat percentage of the capital call amount. Table 1 shows the costs for Examples 1 - 3, assuming the scenario probabilities shown in column 3, and a flat capital call cost of **150%** of the capital call amount:

[5] See Section 6.1 of Mango [13] for more on this.

Table 1
Sample Capital Call Costs

1	2	3	4	5
Example	**Loss Ratio**	**Probability**	**Total Capital Call Amount**	**Capital Call Cost = 150% of (4)**
1	120%	10%	$33,000	$49,500
2	100%	30%	$11,030	$16,545
3	80%	60%	$0	$0

Using the assumed probabilities, the expected capital call cost over the three scenarios would be:

> **(10% x $49,500) + (30% x $16,545) + (60% x $0)**
> **= $9,914**

This is ***E[f(x)]***, where ***f(x)*** is our cost function, which is a scenario-dependent, skewed cost function. We need the scenario detail, because the skewness and scenario-dependence imply that ***E[f(x)] >= f(E[x])***.

Kreps [11] proposes a similar approach, one much more deeply grounded in theory. The cost function example here would represent a simplistic special case of what Kreps proposes. His represents one of the few approaches to date in the actuarial literature that recommends modification of the outcomes to reflect risk. Specifically, he suggests:

> "[The] risk load is a probability-weighted average of riskiness over outcomes of the total net loss:
>
> $$R(X) = \int dx\, f(x) r(x) \qquad \text{where } r(x) = (x - \mu)\, g(x) \qquad (1.17)$$
>
> The function g(x) can be thought of as the "riskiness leverage ratio" that multiplies the actual dollar excess that an outcome would entail to get the riskiness. It reflects that not all dollars are equal, especially dollars that trigger analyst or regulatory tests." [11, p. 4]

<u>Risk Neutrality</u>
The simplistic cost assessment is a flat charge: all capital calls cost **150%** of call amount. This is equivalent to a **risk-neutral** utility function. Implementing such a utility function suggests our attitude towards risk is linear with respect to capital call magnitude — e.g., a **$2M** capital call costs a scenario twice as much as a **$1M** call. Such linear scaling implies for example that a **$100M** deficit is "100 times worse" than a **$1M** deficit.

There are some benefits to a simple capital cost charge like this. It is easy to explain, and as Schnapp points out, makes prices additive – a desirable property

[18]. However, the linear charge also implies a constant cost of **marginal** capital utilization. This has troubling implications; for example,

> Two scenarios consuming an additional **$1M** in capital would be charged the same for that additional capital call magnitude, despite the fact that one is increasing its call from **$1M** to **$2M**, while the other is increasing from **$99M** to **$100M**.

Risk Aversion

We may in practice believe capital consumption costs are not linear with respect to magnitude. There are two arguments for considering a non-linear cost function.

First, at the company level, there are definitely non-linear effects of loss magnitude. Certain catastrophe loss scenarios are intolerable, because they will impair our ability to continue as a going concern. Losses of a certain size may also trigger rating agency downgrades, rendering us uncompetitive. We might even segment decreases to a company's capital into qualitative "bands" or "tiers" whose properties change non-linearly[6]:

1. *Acceptable* (0-10%) – acceptable variation, cost of doing business.
2. *Troubling* (10%-20%) – enough deviation to cause material concern, disclosure to shareholders and rating agencies.
3. *Impairment/rating downgrade* (20%-30%) – hinders functioning of firm as a going concern.
4. *Regulatory control* (30%-50%) – substantial intervention and rehabilitation.
5. *Insolvency* (>50%)

Second, reinsurance pricing actuaries know any pricing methodology implies preferences and cost allocations, creates incentives, and ultimately steers the composition of the portfolio. It is in essence a ranking and scoring scheme. Linear marginal consumption costs may steer us toward product lines with higher risk or greater downside potential than we are comfortable with. Whether or not there are non-linear effects observable at the contract level, from a consistency and portfolio management viewpoint, we may want to have non-linear cost assessment.

A non-linear, increasing marginal cost of capital is equivalent to a **risk-averse** utility function. Rather than having a constant implicit marginal cost of capital, a risk-averse utility function will increase the capital call cost rate (non-linearly) as a function of capital call magnitude.

A risk-averse utility function need not be expressed in a closed-form. A perfectly valid risk-averse capital call cost function can be a lookup table like Table 2:

[6] See Mango [13] for additional detail on this concept.

Table 2
Sample Risk-Averse Capital Call Function

Capital Call Magnitude	Capital Call Charge
$0 - $5,000	125%
$5,001 - $10,000	150%
$10,001 - $20,000	200%
Over $20,000	400%

Table 3 shows the capital call costs using the risk-averse capital call function.

Table 3
Sample Capital Call Costs
Using Risk-Averse Capital Call Function

1	2	3	4	5	5
Example	**Loss Ratio**	**Probability**	**Total Capital Call Amount**	**Capital Call Charge**	**Capital Call Cost**
1	120%	10%	$33,000	400%	$132,000
2	100%	30%	$11,030	200%	$22,060
3	80%	60%	$0	125%	$0

Now the expected cost is

(10% x $132,000) + (30% x $22,060) + (60% x $0)
= $19,818

Compared to the risk-neutral cost of **$9,914**, the risk-averse function resulted in a higher capital call cost. This is the response we would expect, since we are mathematically stating that we have an increasing aversion to risk.

Portfolio Calibration

Ultimately, the cost function is a critical portfolio management decision, since the implicit risk-return preferences embedded in it will heavily influence the eventual portfolio composition. It represents the mathematical expression of a firm's risk appetite. This represents perhaps the most dramatic recommendation in this paper. Critics may understandably argue it is too theoretical, that mathematical expression of preferences is practically impossible. Capital allocation techniques have the apparent advantage of "observability." A "cost of capital" or "risk-adjusted discount rate" can be derived using CAPM (see Section IV of Feldblum [5]), which *appears* to ground the result in the capital markets, giving many a sense of comfort.

Unfortunately, the comfort is illusory at best. Hanging pricing decisions on a CAPM-derived cost of capital merely pushes the parameterization problem onto the capital markets. Reality checks are of course important, as a firm that wishes to have returns far in excess of any of its competitors will be in for a rude awakening. Cost function calibration **will** be difficult; but, it is groundbreaking work, so it **should** be difficult. This is true research, and involves the elucidation of intuitive risk return preferences that guide a firm's decision process. This will require framing of the decision process in a progressively more refined, analytical manner. It will mean constant feedback loops, testing of assumptions, portrayal of tradeoffs via graphical depictions, and reframing of preferences to provide different perspectives. It will be an ongoing process involving a cross-functional team of senior personnel throughout the company.

Difficult calibration is also not unique to the capital consumption approach. In fact, the comparable calibration of allocated capital to total capital is at least as difficult in its own right. Here is a sampling of issues related to capital allocation which have yet to be adequately resolved.

- *Static or Dynamic?*
  Is capital allocated annually at plan time, or "real-time" as actual premium volumes by line come in? If it is annual, what happens when an underwriting unit "hits its goal"? Are the remaining contracts free?
- *Top-Down or Bottom-Up?*
  Perform a true allocation, or build up from contract or segment level values? This quickly becomes a calibration nightmare[7].
- *Capital Good or Bad?*
  Does allocated capital represent underwriting capacity or an expense burden? In other words, do underwriting units want more or less capital?
- *Ongoing or Runoff?*
  Should capital be allocated to reserves, assets, latent, or runoff lines?
- *Zero sum game?*
  That is, is the total capital fixed? If so, if one segment's capital requirement decreases, does that mean all the other segments' capital increases?
- *Additivity?*
  Do you allocate on a marginal basis? Do you re-balance so it adds up to the total? What about order dependency?

Calibration of a utility function should be no harder than calibration of a capital allocation exercise. The end result could arguably be of more value, being a tested, explicit, mathematical representation of a company's risk-return preferences. Appendix B explains one method that can be used to calibrate by-segment pricing targets to a portfolio measure.

[7] Mango and Sandor [12] explains in detail an experimental study of "bottom-up" capital allocation and calibration to a portfolio measure.

5. Examples: Reinsurance Price Evaluation

We have proposed the following principles of stochastic reinsurance contract evaluation:

- Contingent capital calls have a cost associated with them, which is a function of the magnitude of the call.
- This cost is assessed at the scenario level.
- The expected value of the cost over all scenarios is treated as an overhead expense loading in the contract pricing evaluation.
- We determine the risk-adjusted net present value of the contract as the expected net present value of contract cash flows minus the expected value of the capital call cost.

We will now demonstrate these principles on two example reinsurance contracts.

Long-Tail
We will look at the pricing of the Long-tailed contract from Examples 1 - 3, using Table 2, the Risk-Averse capital call cost function. For a $100,000 premium, we can pull the results from Table 3 with an additional column for NPV:

Table 3
Sample Capital Call Costs
Using Risk-Averse Capital Call Function

1	***2***	***3***	***4***	***5***
Example	**Loss Ratio**	**Probability**	**Capital Call Cost**	**NPV of Capital Call Cost**
1	120%	10%	$132,000	$99,099
2	100%	30%	$22,060	$15,057
3	80%	60%	$0	$0

The total costs on a discounted basis would be:

1	***2***	***3***	***4***	***5***	***6***	***7***
Example	**NPV Premium**	**NPV Expenses**	**NPV Losses**	**Underwriting NPV = (2) – (3) – (4)**	**NPV of Capital Call Cost**	**Overall NPV = (5) – (6)**
1	$100,000	$15,000	$103,479	($18,479)	$99,099	($117,578)
2	$100,000	$15,000	$86,232	($1,232)	$15,057	($16,289)
3	$100,000	$15,000	$68,986	$16,014	$0	16,014

The expected value of the Underwriting NPV is

10% * ($18,479) + 30% * ($1,232) + 60% * 16,014
= $7,391

The expected value of the Overall NPV *including capital costs* is

10% * ($117,578) + 30% * ($16,289) + 60% * 16,014
= ($7,036)

Thus, reflecting all costs, this deal is below break-even. Assuming constant expenses and the same ultimate loss dollars, we find the risk-adjusted "break-even" premium to be **$103,305.** The term "break-even" should not imply that overall the company is earning no return. The cost function can be calibrated to any desired level of portfolio return measure. A more appropriate term would probably be "target premium." Here are the figures at target:

1	***2***	***3***	***4***	***5***	***6***	***7***
Example	**NPV Premium**	**NPV Expenses**	**NPV Losses**	**Underwriting NPV = (2) – (3) – (4)**	**NPV of Capital Call Cost**	**Overall NPV = (5) – (6)**
1	$103,305	$15,000	$103,479	($15,173)	$86,858	($102,031)
2	$103,305	$15,000	$86,232	$2,073	$6,702	($4,629)
3	$103,305	$15,000	$68,986	$19,320	$0	19,320

The expected value of the Overall NPV *including capital costs* would be

10% * ($102,031) + 30% * ($4,629) + 60% * 19,320
= $0

Property Catastrophe
Consider a high-layer contract, with a 2% chance of being hit (1 in 50 years). However, when it is hit, it suffers a full limit loss. Example 4 shows the details:

Example 4
Property Catastrophe Contract

Premium	**$**	**1,000,000**
Limit	**$**	**10,000,000**

		No Loss Scenario		***Loss Scenario***
Probability		**98.0%**		**2.0%**
Premiums	**$**	**1,000,000**	**$**	**1,000,000**
Expenses	**$**	**-**	**$**	**-**
Losses	**$**	**-**	**$**	**10,000,000**
Capital Call Amount	**$**	**-**	**$**	**9,000,000**
Capital Call Factor		**0.0%**		**400.0%**
Capital Call Charge	**$**	**-**	**$**	**36,000,000**
Expected NPV	**$**	**800,000**		
Expected Capital Call Cost	**$**	**720,000**		
Expected Risk-adjusted NPV	**$**	**80,000**		

In the full limit loss scenario, the capital call is for $9M. Hitting this with a 400% capital call charge factor, the expected risk-adjusted NPV is $80,000 – close to target.

6. Conclusions

This paper has ties to much current work in both actuarial and financial literature. In particular, it is linked to:

- Pricing via probability measure change – from voluminous capital markets literature;
- Utility theory in pricing – from Halliwell, Heyer and Schnapp;
- The Wang Transform – from Wang;
- The market cost of risk – from Van Slyke; and
- Additive Co-Measures – from Kreps.

Of particular note are comments made by Kreps [11]:

> "[It] seems plausible that for managing the company the risk load for an outcome:
>
> (1) should be a down side measure (the accountant's point of view);
> (2) should be proportional to that excess over the mean for excess small compared to surplus (risk of not making plan, but also not a disaster);
> (3) should become much larger for excess significantly impacting surplus; and
> (4) should flatten out for excess significantly exceeding surplus – once you are buried it doesn't matter how much dirt is on top. " [11, p. 9]

The proposed approach focuses on downside, and can support discontinuous, non-linear risk measures as functions of surplus.

This approach also represents an attempt at conscious, intentional and explicit introduction of "information content" into reinsurance market prices. Perhaps one of the more dubious assumptions of competitive market theory is that market prices reflect all the information available. Even setting aside the enormous informational asymmetries in the reinsurance arrangement, one cannot ignore the large role of interpretation. In order for reinsurance market prices to contain all this information — i.e., to really "mean something" — the submission information must be converted into prices. The process is one of interpretation by reinsurance underwriters and actuaries, including subjective and objective considerations, market intelligence, internal strategy, tips and hints from the broker or client, relationship, bank…. With all this confluence of strategies and signals, the discipline of an explicit, objective utility approach seems desperately needed and sorely overdue.

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Appendix A
Does Capital Allocation Make Sense for Insurance?

This appendix addresses whether the practice of capital allocation even makes sense for insurance. Capital allocation was originally applied to manufacturing firms. However, the nature of their usage of capital is fundamentally different from insurers.

Manufacturing
Consider a representative example from Halliwell:

> "A company is considering entering the widget business, which entails the purchase of a machine to produce widgets. The company estimates that the machine will last five years, and the profits from the sale of its widgets over those five years will be $100,000, $125,000, $125,000, $100,000, and $75,000." [8, p. 73]

This example typifies the manufacturing capital analysis framework. Capital is invested up front, and profits (hopefully) come in the future. This approach was designed for analysis of investment opportunities in industries where production comes before revenue collection. We might term such industries "*spend-then-receive*." These industries must invest capital into production and distribution costs before they can hope to collect revenue. There are no products to sell without capital. Manufacturers have the following time dynamic with respect to capital usage:

- Capital investment costs are mostly up-front and well known, while
- Revenues are in the future and unknown.

Manufacturing capital also must cover operating deficits. In order for the firm to continue operations when revenues are less than costs, additional capital must be invested[8].

It is important to note that capital investment represents a cost, an expenditure of a known amount. Large manufacturing organizations that "allocate capital" between business units actually spend the capital they are allocating. Capital is "consumed by production." There is nothing theoretical about either the total amount available, or the amounts allocated to various product lines. Since real spending of real money is involved, capital allocation decisions receive a tremendous amount of attention, scrutiny and peer review. They lie at the heart of strategic planning for manufacturing firms. Capital allocation is the lifeblood of a manufacturing business unit, the means to continue activities.

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[8] Venture capitalists often refer to the "burn rate" of a start-up company: the rate at which the operation consumes capital during its start up period, when there are typically no revenues, only costs.

The manufacturer hopes this capital investment will be followed by profits in the future, but is uncertain how much revenue will come, or when. Typically, this uncertainty influences the decision in the form of a risk-adjusted discount rate applied to future revenue projections[9].

<u>*Insurance*</u>
In contrast to manufacturing, which is "*spend-then-receive*," insurance is a "*receive-then-spend*" industry. What we term production is really revenue collection. Our revenues are fairly predictable, even by product line. Demand is somewhat inelastic, given the legal and regulatory requirements. Insurers can plan their premium volume with a good degree of accuracy. They struggle to assess the loss cost of their products that come in the future. Comparing the time dynamic of insurance and manufacturing is illuminating:

Item	***Manufacturing***	***Insurance***
Revenue	In the future, unknown	Up front, well known
Costs	Up front, well known	In the future, unknown

Insurers are actually something of a "temporal mirror image" of manufacturers: they collect revenue up front, and hope the future costs aren't too high or too soon. There is no question that insurers need capital in total to secure claims paying ability. However, it is critical to recognize this distinction: insurers can collect revenue on products **without having had to invest any capital in product production**. Insurance "production costs" are actually loss payments.

It is surprising that such a striking difference in capital usage has not resulted in any materially different capital treatment in the insurance IRR framework. In fact, actuaries have kept the manufacturing capital usage profile, treating insurance products as if they require supporting capital to be invested up-front, then released. This insurance IRR framework is actually *pseudo-manufacturing*: the capital amount is "risk-based," derived from stochastic analysis; yet it is invested in an essentially deterministic framework that ignores the reality that insurance products do not require capital investment to produce.

Several major problems with this hybrid approach are immediately apparent.

First, when evaluated in a stochastic environment, **the allocated supporting capital makes no sense at the modeled scenario level**. The risk-based supporting capital is determined based on a risk measure of the distribution in

[9] The end goal of "dis-counting" – literally, reducing the value of – uncertain future revenues is appropriate. The method of risk-adjusted discounting – effecting that reduction in value by using compounded discounting at a higher rate – is unnecessary and (as Halliwell [8] has shown) fraught with inconsistencies. It represents an example of "overloading an operator," piling additional functional burden onto what should be a single purpose operator.

total. For any given scenario, though, this overall amount is never the actual required amount — namely, the modeled operating deficit. The allocated capital is excessive for favorable scenarios, and grossly inadequate in severe loss scenarios (unless capital equals the policy limit). This problem is particularly striking in the evaluation of catastrophe reinsurance contracts, with small probabilities of a full contract limit loss. A "risk-based" capital amount might be some small fraction of the limit — say 5%. What sense does this capital amount make in the limit loss scenario? We held 5% of the limit as capital? And then how exactly did we fund the remaining loss amount? It came from company capital in total. The alternative — holding the full limit as capital — puts an unrealistic return burden on the contract. Current market price levels would likely make the contract look unattractive.

Second, **insurance contracts use capital in the future**. Insurance "production" costs are in fact distribution and revenue collection costs. Manufacturing capital funds true production costs, as well as operating deficits. Since the vast majority of insurance costs come in the future in the form of loss payments, insurance capital usage belongs in the future as well. However, capital would only be needed if the contract began running an operating deficit or loss — a negative cash position reflecting all sources of revenue, including investment income.

Third, **allocated supporting capital is completely theoretical**. In contrast to manufacturing, where allocation means actual spending of actual known amounts, allocated supporting capital simply does not exist at the contract level. Nothing is actually spent or invested. The strong ties to reality inherent in the manufacturing framework have been lost, and with them go much of the discipline and meaning of capital allocation.

Finally, on a more philosophical level, **supporting capital is a portfolio concept, and may not be meaningfully divisible**. There is no question that supporting capital in total is *essential to the insurance operation*. The product we sell is current and future claims paying ability; however, this ability applies to the insurer in total as a going concern. Future claims paying ability is heavily dependent on total supporting capital. There is also no question that allocation of supporting capital is *possible*. The issue is with the meaning of that allocated capital. There are many holistic phenomena that have no divisible component pieces. An historical example is found in the ancient scientists' search in vain for the "seat of the soul" in the brain. They sought a physically grounded, identifiable location for what is now believed to be a "field" phenomenon. One might well consider trying to allocate life to the component organs of the body, or allocating the success of the Lakers to individual players: 38% to Shaq, 33% to Kobe…. Since actuaries like to communicate mathematically, in equation form:

$$\sum \left(Promises\ To\ Pay \right) \neq Promise\ To\ Pay\left(\sum \right)$$

Appendix B
One Approach to Portfolio Calibration

This appendix will outline one approach for calibration of a contract capital call cost function with a company total cost of capital. This approach can be used to develop risk-adjusted target combined ratios by LOB without allocating capital. No matter which cost function is used, a testing period is recommended where several possible functions and/or parameter sets can be evaluated across a significant sample of the portfolio. Once the results are aggregated, the total assessed cost of risk can be estimated and expressed as a percentage of a base such as expected losses or premium.

Here are the steps of the suggested process:

1. Generate modeled scenarios of company operating income and individual line of business underwriting result over a projected three calendar year period. Include reserves as well as prospective business in the definition of a line of business. Also include asset risk and linkages with generated economic scenarios.

2. Apply a risk-averse utility function to the company's operating income distribution to assess a "capital depletion" cost at the scenario level to those scenarios with negative operation income. Calibrate the expected value of this cost over all scenarios to a desired target cost of capital measure.

3. Allocate the scenario capital depletion costs back to line of business at the scenario level in proportion among all lines having an underwriting loss in that scenario.

4. Calculate the expected value of allocated depletion cost by line of business over all scenarios. Express this charge as a percentage of expected loss.

5. Generate the other components of a break-even risk-adjusted combined ratio, namely discounted loss ratio and expense ratio.

Example 5 shows a simplified flowchart of steps 1-4.

Example 5
Company Risk Adjustment Methodology Flowchart

Step 1: Generate Operating and U/W Results

Underwriting Results

Iteration	LOB 1	LOB 2	LOB 3	TOTAL	Inv Inc	Company Operating Result	Total Risk Charge
1	(100)	10	(200)	(290)	350	60	-
2	100	(200)	(400)	(500)	350	(150)	(122.67)
3	(300)	50	(200)	(450)	350	(100)	(48.94)
4	(200)	(500)	100	(600)	350	(250)	(426.87)

Step 2: Apply Risk Adjustment Formula to Operating Result

Step 3: Allocate Total Risk Charge to LOB

Neg U/W Results - Shares

Iteration	LOB 1	LOB 2	LOB 3
1	33.3%	0.0%	66.7%
2	0.0%	33.3%	66.7%
3	60.0%	0.0%	40.0%
4	28.6%	71.4%	0.0%

Allocated Risk Charges

	LOB 1	LOB 2	LOB 3	Total
	-	-	-	-
	-	(41)	(82)	(123)
	(29)	-	(20)	(49)
	(122)	(305)	-	(427)
Expected Risk Charges	(38)	(86)	(25)	(150)

Calibrate with Overall Cost of Capital

PHS	*1,000*
Expected Risk Charge as % of PHS	*15.0%*

Step 4: Calculate the Expected Risk Charges

Expected Losses

LOB 1	LOB 2	LOB 3
1,000	800	1,300

Expected Risk Charges

LOB 1	LOB 2	LOB 3
3.8%	10.8%	1.9%

We will cover each step in more detail.

Step 1: Model Company Operating Income and LOB Underwriting Income
We use company calendar year operating income as the risk measure at the scenario level. Negative operating income depletes capital, so the cost of capital depletion is assessed here, based on the magnitude of the depletion. We also model the line of business (LOB) calendar year underwriting income at the scenario level. The calendar year variation includes the random effects of reserve runoff for carried reserves as of the start of the simulation period.

The main strength of this approach lies in its inclusion of so many modeled dependencies and interactions: between reserve runoff LOB's, between reserves and prospective business; between liabilities and assets via the economic scenarios, etc.

Step 2: Apply a Utility Function to Assess the Cost of Capital Depletion
As can be seen in the demo flowchart, a risk-averse exponential utility function assesses a capital depletion charge to scenarios with negative operating income. The calibration of the expected assessed charge with a portfolio capital cost is straightforward.

Step 3: Allocate Capital Depletion Cost back to LOB
At the scenario level, the calculated depletion cost is allocated back to those LOB with underwriting losses, in proportion to their underwriting loss as a percent of the total of underwriting losses for LOB with an underwriting loss. This is one

allocation rule, and obviously not the only or even best. The point is that an allocation rule can be applied at the scenario level.

Step 4: Calculate Expected Capital Depletion Cost by LOB
Each LOB has an expected value of the allocated depletion costs over all scenarios. This figure is expressed as a percent of expected loss to facilitate inclusion in the break-even risk-adjusted combined ratio calculation.

Step 5: Calculate the Break-even Risk-adjusted Combined Ratio
The additional required elements are a discounted loss payment pattern and expense load. The goal is to calculate economically break-even combined ratios — i.e., 100% discounted combined ratio — including a load for the cost of capital.

Advantages
This approach shows one way to implement insurance portfolio management using a dynamic portfolio model. The approach links corporate cost of capital needs with LOB pricing targets in a simple, coherent framework, without allocating capital. A dynamic portfolio model also has other advantages, including the development of a more complete picture of current capital adequacy; the ability to introduce a time dimension into risk modeling; and a framework for introducing systematic risk from the insurance and capital markets.