Insurance Analytics

Actuarial Tools for Financial Risk Management

Paul Embrechts

ETH Zürich and London School of Economics

Plenary talk at the XXXIV ASTIN Colloquium in Berlin, August 25, 2003.

1. About the title

- **Talk:** **„Actuarial versus Financial Pricing of Insurance"**
  Risk Management in Insurance Firms Workshop,
  Wharton School, May 16, 1996
  As reaction, Till Guldimann coined phrase

- **Guest editorial:** **„Insurance Analytics"**
  British Actuarial Journal, IV,
  639 – 541 (2002)

- **Chapter in P. Embrechts, R. Frey and A. McNeil**
  „Stochastic Methods for Quantitative Risk Management"
  (Book manuscript, 2004, to appear)

2. The economic and regulatory environment around the turn of the millenium

- stockmarket „bubble“
- economic downturn after e-hype
- life insurance crisis: demographic, social, guarantees
- bankassurance: back to the drawing board
- regulation
  - Basel I Amendment and Basel II ($\geq$ 2006)
  - joint supervision of banking and insurance
  - solvency, ALM
  - reinsurance
  - accounting: GAAP, IAS, Statutory
  - embedded value, fair value
- corporate governance: increased importance of technical (actuarial) skills

3. Question: where does that leave the actuary?

The Actuarial Profession:

making financial sense of the future.

- **Actuaries are respected professionals whose innovative approach to making business successful is motivated by a responsibility to the public interest. Actuaries identify solutions to financial problems. They manage assets and liabilities by analysing past events, assessing the present risks involved and modelling what could happen in the future.**

www.actuaries.org.uk

- **Do we live up to this definition?**

4. Insurance analytics: an incomplete list!

- incomplete markets
- premium principles and risk measures
- credibility theory
- tail fitting
  - analytic (models beyond normality)
  - algorithmic (Panjer, FFT)
  - asymptotic (Extreme Value Theory)
- scoring
- dependence beyond correlation
- stress testing techniques
- dynamic solvency testing (ruin, DFA, ...)
- long-term-horizon models

5. Some examples

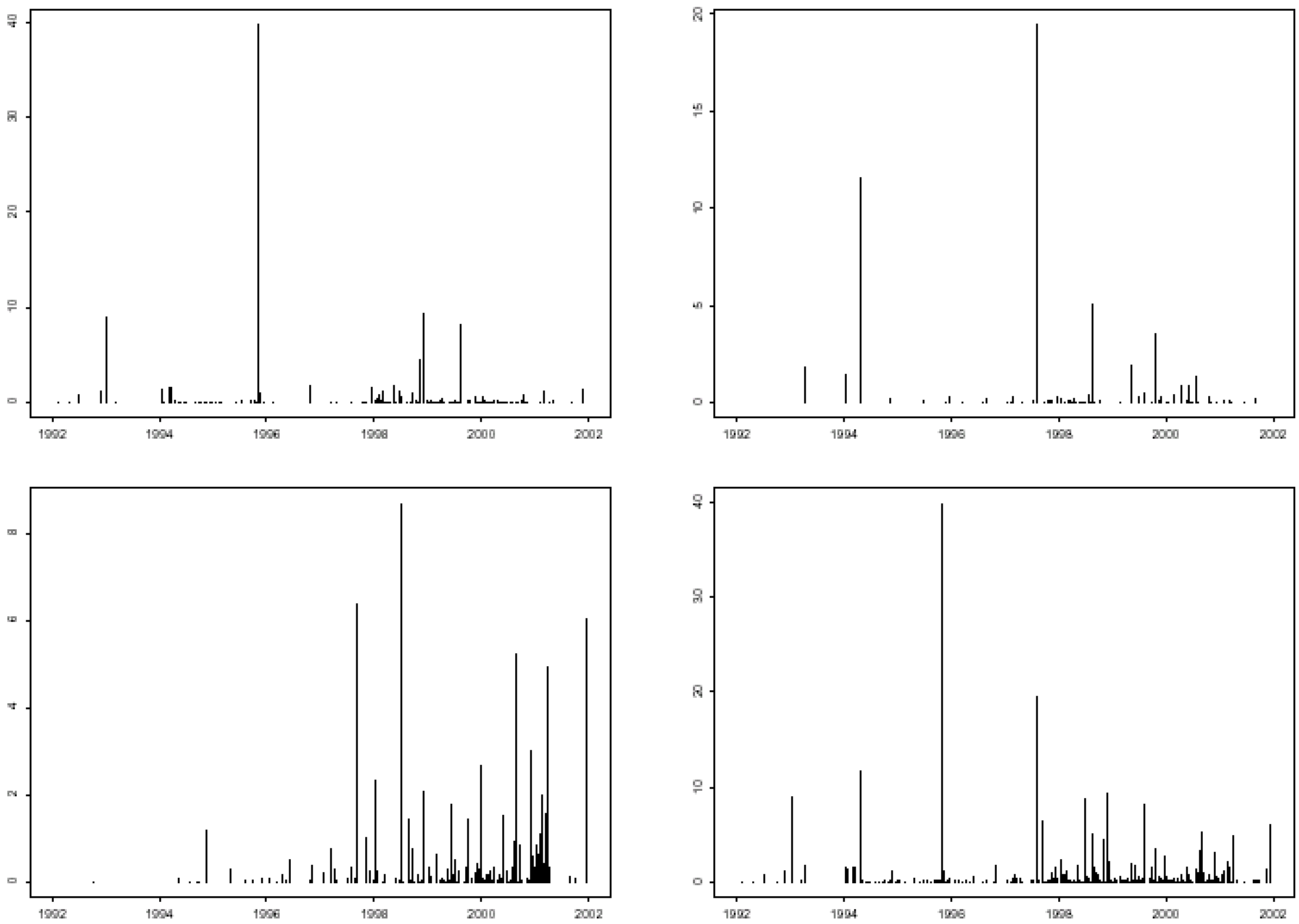
5.1 Operational Risk

- **Basel II Definition: „The risk of losses resulting from inadequate or failed internal processes, people and systems or from external events"**

- **Risk capital (Pillar I) calculation:**
  - **Basic Indicator Approach:** $RC(OR) = \alpha \cdot GI$
  - **Standardized Approach:** $RC(OR) = \sum_{i=1}^{s} \beta_i \cdot GI_i$
  - **Advanced Measurement Approach (AMA)**

- **AMA**
  - **The data:** $\{X_k^{t,i} : t = 1,\ldots,T; i = 1,\ldots,s; k = 1,\ldots,N^{t,i}\}$ **where** t **(years),** s **(loss and/or business types)** $N^{t,i}$ **(total number of losses in year** t **for type** i**)**
  - **Truncation and „**$s=56$**"**

Some data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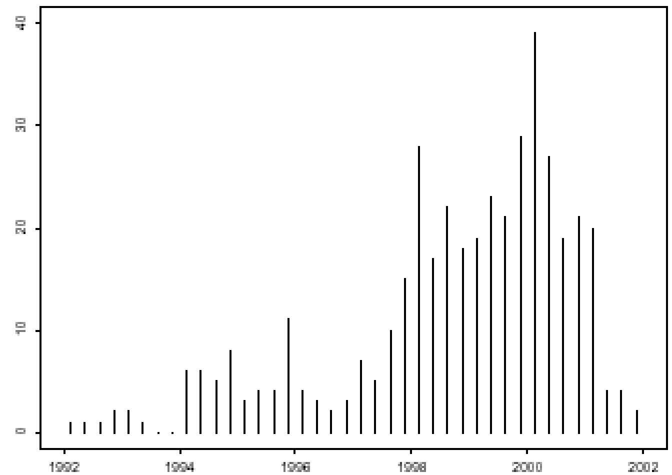
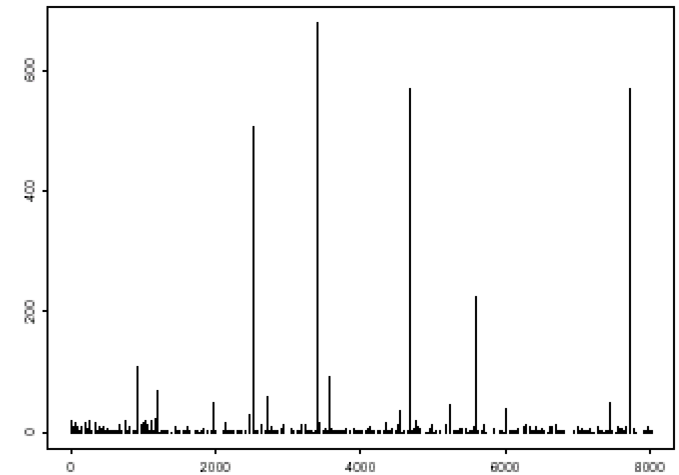
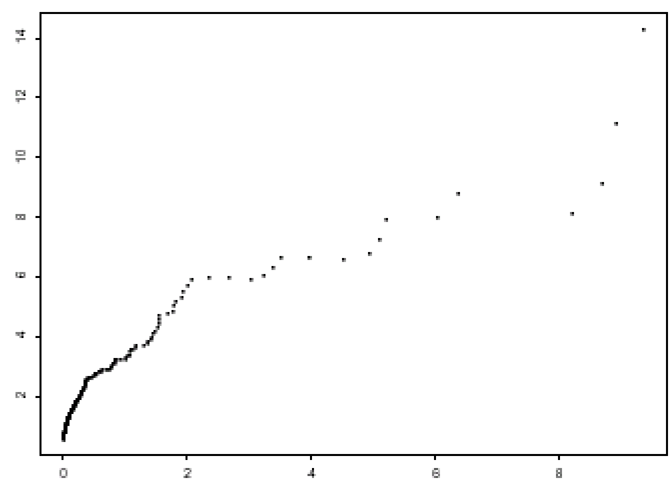
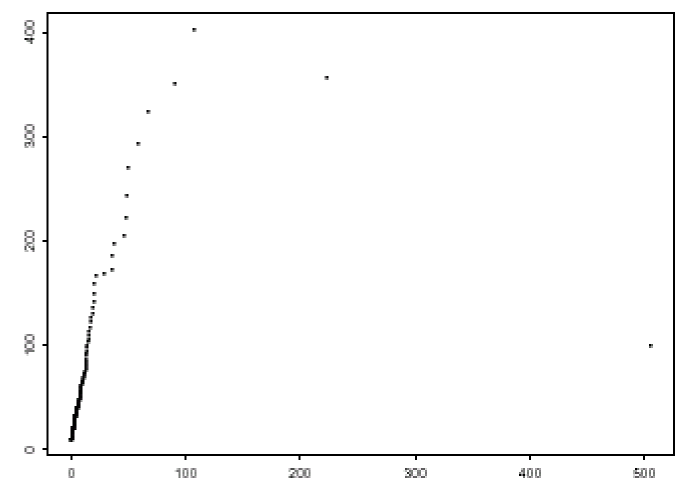


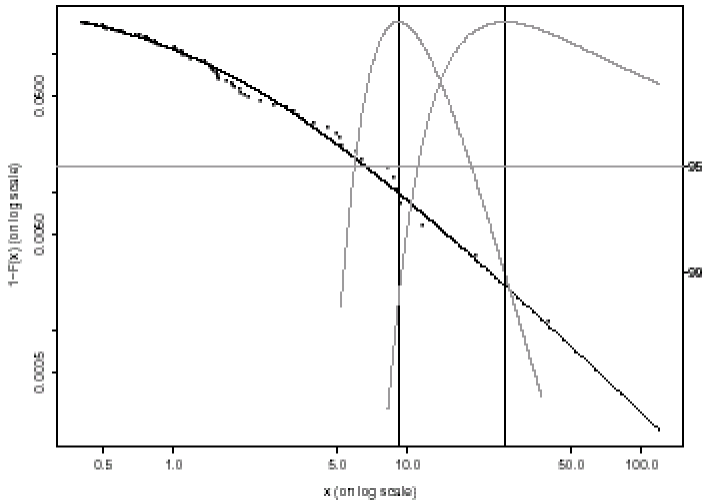
operational risk



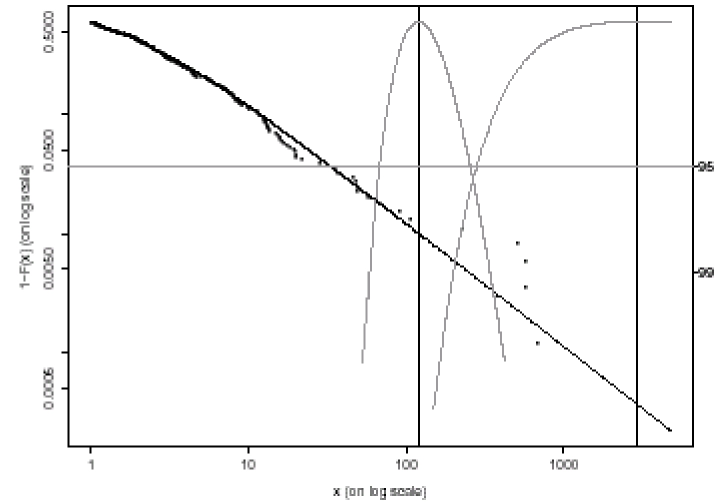
fire loss







operational risk



fire loss

Reference:

P.Embrechts, R. Kaufmann and G. Samorodnitsky (2002), „Ruin theory revisited: stochastic models for operational risk“ (to appear)
Preprint: www.math.ethz.ch/~embrechts

- **The problem: estimate a risk measure for**

(*) $$F_{L_{T+1}}(x) = P\left(\sum_{i=1}^{s} \sum_{k=1}^{N_{i,T+1}} X_k^{T+1,i} \leq x\right)$$

like

$$OR-VaR_{T+1}^{1-\alpha} = F_{L_{T+1}}^{\leftarrow}(1-\alpha),$$ α small (0.03%)

$$OR-CVaR_{T+1}^{1-\alpha} = E\left(L_{T+1} \mid L_{T+1} > OR-VaR_{T+1}^{1-\alpha}\right)$$

- **Discussion: recall the stylised facts**
  i. X‘s are heavy-tailed
  ii. N shows non-stationarity

- **Conclusion:**
  - **(*) is difficult to estimate**
  - **actuarial tools will be useful**
  - **difference btw. repetitive and non-repetitive losses**

5.2 A ruin-theoretic problem motivated by operational risk

Recall for the classical Cramér-Lundberg model

$$Y(t) = u + ct - \sum_{k=1}^{N(t)} X_k = u + ct - S_N^X(t)$$

$$\Psi(u) = P\left(\sup_{t\geq 0}\left(S_N^X(t) - ct\right) > u\right)$$

In the heavy-tailed case:

$$P(X_1 > x) \sim x^{-\beta-1} L(x), x \to \infty, L \text{ slowly varying}$$

implies that

$$\Psi(u) = cte \cdot u^{-\beta} L(u), u \to \infty$$

Important: net-profit condition

$$P\left(\lim_{t\to\infty}\left(S_N^X(t) - ct\right) = -\infty\right) = 1$$

Now assume that for some general loss process *(S(t))*

- $P\left(\lim_{t\to\infty}\left(S(t)-ct\right)=-\infty\right)=1$

(*) • $\Psi_1(u)=P\left(\sup_{t\geq 0}\left(S(t)-ct\right)>u\right)\sim u^{-\beta}L(u),u\to\infty$

Question: how much can we change *S* **keeping (*)?**

Solution: use time change $\left(S_\Delta(t)=S(\Delta(t))\right)$

$$\Psi_\Delta(u)=P\left(\sup_{t\geq 0}\left(S_\Delta(t)-ct\right)>u\right)$$

Under some technical conditions on Δ **and** *S*, **general models are given so that**

$$\lim_{u\to\infty}\frac{\Psi_\Delta(u)}{\Psi_1(u)}=1$$

i.e. ultimate ruin behaves similarly under the time change

Discussion

- time change:
  - Lundberg-Cramér (1930's)
  - W. Doeblin (1940): Itô's lemma
  - Olsen θ-time (1990's)
  - Geman et al. (1990's)
  - Monroe's theorem (1978)
- example:
  - start from the homogeneous Poisson case (classical Cramér-Lundberg, heavy-tailed case)
  - use Δ to transform to changes in intensities motivated by operational risk

Reference:

P. Embrechts and G. Samorodnitsky (2003)
„Ruin problem and how fast stochastic processes mix"
Ann. Appl. Probab. (13), 1-36

5.3 Pricing risk under incomplete information

Suppose $X_1, \ldots, X_d$ **one-period risks**

- $\Psi(X_1, \ldots, X_d)$ **financial or insurance position**

  e.g. $\sum_{k=1}^{d} (X_i - k_i)^+$

  $Max(X_1, \ldots, X_d) \cdot \mathbf{I}_{\{X_1 + \ldots + X_d > q_\alpha\}}$

- ρ **is a „(risk-)measure"**

  e.g. $(C)VaR_\alpha$

  $F_{\Psi(X_1, \ldots, X_d)}$

- **hence** $\rho(\Psi(X_1, \ldots, X_d)) = \rho(\Psi(\mathbf{X}))$

Suppose given:

- **marginal loss distributions** $X_i \sim F_i$, $i = 1, \ldots, d$
- **some idea of dependence** $\mathfrak{D}$ **between** $X_1, \ldots, X_d$

Problem: calculate $\rho(\Psi(\mathbf{X}))$

Remark: not fully specified problem

Solution: find optimal bounds ρ_L, ρ_U **so that**

$$\rho_L\left(\Psi(\mathbf{X})\right) \leq \rho\left(\Psi(\mathbf{X})\right) \leq \rho_U\left(\Psi(\mathbf{X})\right)$$

Examples of $\mathfrak{D}$**:**

- **no information (Fréchet-space problem)**
- **structure on** $\Sigma(\mathbf{X})$
- **positive quadrant dependence:** $C \geq C_I$

Examples:

- **Given** $\Psi(\mathbf{X}) = X_1 + \ldots + X_d$, $\rho = VaR_{1-\alpha}$ **hence find**

$$\min \leq VaR_{1-\alpha}(X_1 + \ldots + X_d) \leq \max$$

e.g. $d = 2$, $F_1 = F_2 = N(0,1)$, $\alpha = 0.05$ (95% - VaR)

$$VaR_{0.95}(X_i) = 1.645$$

$$\max = 3.92 > 2 \times 1.645 = 3.29$$

hence there is a non-coherence gap!

- **For an insurance-related example see**
P. Blum, A. Dias and P. Embrechts (2002)
"The ART of dependence modelling: the latest advances in correlation analysis"
In *Alternative Risk Strategies*, ed. Morton Lane, Risk Waters Group, London, 339 – 356

5.4 Stress testing credit portfolios

Basic tool: copulae

- $F_{\mathbf{X}}(\boldsymbol{x}) = P(X_1 \leq x_1, \dots, X_d \leq x_d)$
- $X_i \sim F_i, i = 1, \dots, d$, **continuous**
- **hence** $U_i = F_i(X_i) \sim UNIF(0,1)$
- **denote** $C(\boldsymbol{u}) = P(U_1 \leq u_1, \dots, U_d \leq u_d)$
- $F_{\mathbf{X}}(\mathbf{x}) = P(F_1(X_1) \leq F_1(x_1), \dots, F_d(X_d) \leq F_d(x_d))$

  $= C(F_1(x_1), \dots, F_d(x_d))$ **(1)**
- **also** $C(u_1, \dots, u_d) = F_{\mathbf{X}}(F_1^{-1}(u_1), \dots, F_d^{-1}(u_d))$ **(2)**
- **Conclusion:** $F_{\mathbf{X}} \Leftrightarrow (F_1, \dots, F_d; C)$

Basic examples:

- $\mathbf{X} \sim N_d(\mathbf{0},\Sigma)$ yields via (2) the normal copula C^N_Σ
- $\mathbf{X} \sim N_d(\mathbf{0},\Sigma)$ independent from $W = \sqrt{\nu / \chi^2_\nu}$ then

$$\mathbf{Y} = W \cdot \mathbf{X} \sim t_{\nu,\Sigma}$$

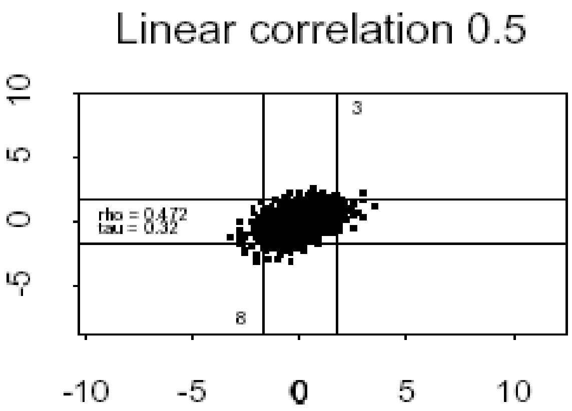
  and yields via (2) the important t-copula $C^t_{\nu,\Sigma}$
- aim of stress testing: joint extremes (~ default correlation)

  use construction (1):
  - for C^N_Σ : no joint extremes
  - for $C^t_{\nu,\Sigma}$: joint extremes
- Conclusion:

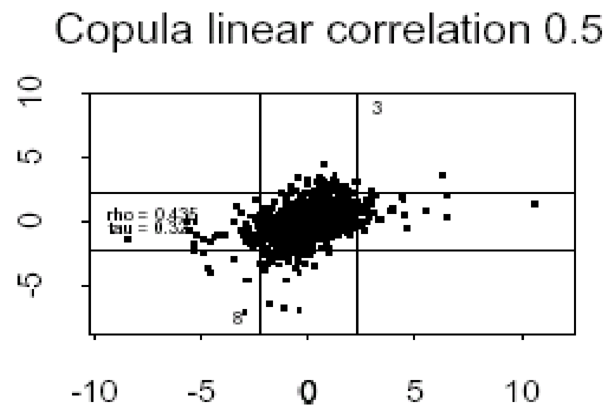
  „In order to produce joint extremes (losses),

  change the copula, not the marginals.“

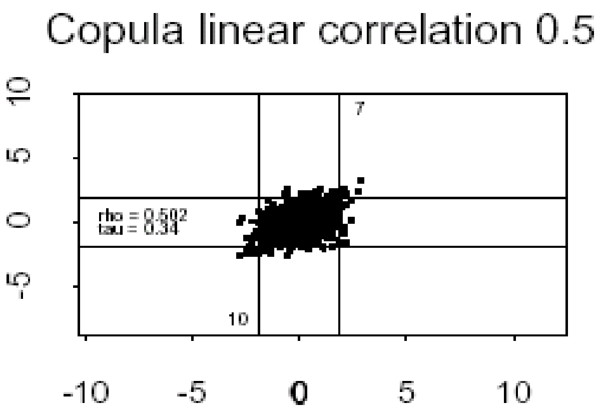
Copula examples



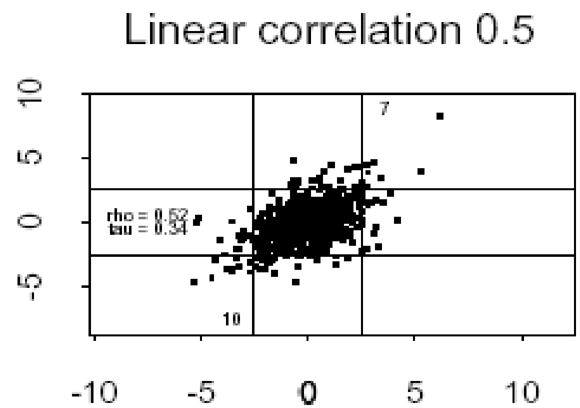
a) Bivariate Normal



b) t4 margins and Gaussian copula



c) Standard Normal margins and t4 copula



d) Bivariate t4

An example: the Merton model for corporate default (firm value model, latent variable model)

- **portfolio** $\{(X_i, k_i) : i = 1, \ldots, d\}$ **firms, obligors**
- **obligor *i* defaults by end of year if $X_i < k_i$ (firm value is less than value of debt)**
- **modelling joint default:** $P(X_1 \leq k_1, \ldots, X_d \leq k_d)$
  - **classical Merton model:** $\mathbf{X} \sim N_d(\boldsymbol{\mu}, \Sigma)$
  - **KMV: calibrate k_i via „distance to default“ data**
  - **CreditMetrics: calibrate k_i using average default probabilities for different rating classes**
  - **Li model: X_i‘s as survival times are assumed exponential and use normal copula**
- **hence standard industry models use normal copula!**
- **improve using t-copula**

The copula is critical

- **standardised equicorrelation ($\rho_i = \rho = 0.038$) matrix Σ calibrated so that for $i = 1, \ldots, d, \;\; P(X_i \leq k_i) = 0.005$ (medium credit quality in KMV/CreditMetrics)**
- **set ν = 10 in t-model and perform 100‘000 simulations on d = 10‘000 companies to find the loss distribution**
- **use VaR concept to compare risks**

Results:

	min	25%	med	mean	75%	90%	95%	max
normal	1	28	43	49.8	64	90	109	331
t	0	1	9	49.9	42	132	235	3238

- **more realistic t-model: block-t-copula (Lindskog, McNeil)**
- **has been used for banking and (re)insurance portfolios**

Conclusion:

- **actuaries have intersting tools to offer:**
  insurance analytics
- **stress testing (insurance and finance) portfolios is crucial**
- **think beyond normal distribution and normal dependence**
- **questions lead to important applications, and**
- **yield interesting academic research**
- **increased importance of integrated risk management**
- **many ... many more important issues exist: future ASTINs**

References

- **check www.math.ethz.ch/finance**