Techniques for valuation a general insurance company within the framework of IAS standards: some proposals.

(ASTIN TOPIC 3)

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Abstract

The European Commission in Brussels has set the IAS as the unique accounting standards in order to build an integrated market of financial services within the European Union would become mandatory as from 2005 in particular for insurance companies listed on the stock exchange.

For these standards, valuation of assets and liabilities in “Fair Value” is the basic principle. However as it is difficult to assess the market value of P & C insurance contracts, the valuation of liabilities would be carried on the “Entity Specific Value” principle.

This principle relies on a prospective valuation of future cash-flows from the current book of contracts with a margin for risk and uncertainty and a discount of technical reserves.

This paper is providing some technical proposals, based on Generalized Linear Models.

Keywords

Keywords: General insurance, IAS standards, discounted future cash-flows, Market Value Margin, stochastic reserving, GLM, bootstrap.

1. INTRODUCTION.

There are different paths to estimate the valuation of a general insurance company, which require different accounting methods for assets and liabilities.

Financial assets can be accounted for in different ways, historical cost, market value, present value of future cash flows, etc.

Different reserving methods, especially in loss reserving, are available to measure insurance liabilities.

Among its strategic priorities, one aim of the Brussels Commission is to create a European financial market. The use of consistent accounting standards is one necessary condition for producing understandable, relevant, reliable, and comparable financial reporting.

These standards should allow different users of financial statements, especially financial analysts, investors and rating agencies to take decision.

The EEC has published on July 19th, 2002 a compulsory rule[1] to use from 2005 onwards the IAS standards for the consolidated accounts of listed. However, before the application of this requirement, current standards must be approved by the ARC[2]

Every state member will be able to extend these requirements to other type of companies.

It is important to note that there is a large "consensus" in France against the use of the current IAS standards especially those regarding the financial products[3]. One of the main argumentation is based on the increased volatility of financial accounts.

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[1] Regulation nº 1606/2002

[2] ARC stands for Accounting Regulatory Committee of the European Union.

[3] IAS standards 32 and 39

2. IAS STANDARDS

The IAS project developed since 1991 by the IASB (International Accounting Standards Board), part of a private independent organism, requires that an asset and liability approach be used, under “Fair Value” valuations of assets and liabilities that are approximatively market values or estimated market values.

An asset and liability measurement approach is one that measures the assets and liabilities at the end of each exercise. The profit is recognised through the relative change in these two quantities from one year to the following. As the volatility of financial market has recently increased, it can be fear that the volatility of financial company results will be increased too.

Insurance products must be viewed as a quite distinct product. The implementation of fair value for insurance products was delegated to a Steering Committee in 1997. This Steering Committee published in June 2001 a reference report, which took the form of Draft Statement of principles (DSOP) containing 13 Chapters. After review and eventually amendments of the IASB, this report will constitute the IFRS (International Financial Reporting Standards) for insurance contracts.

2.1 DSOP Principles

Most of the general insurance contracts fulfil the insurance contract definition given by the DSOP. It can be noted that there is no difference between the treatment of insurance contracts and the treatment of reinsurance contracts. However credit risk and some of life insurance contracts are excluded from the definition of the insurance contracts because they are considered as financial products and so must be treated according to the IAS 32 and 39 Standards.

As there is no market sufficiently liquid for general insurance liabilities, the fair value of such liabilities can be obtained via the Entity Specific Value (ESV). The ESV represents the value of assets and liabilities using company assumptions and not market one. It can be stressed that benchmarking and comparability may be affected by this pragmatic approach.

In a prospective approach the ESV is the sum of the discounted future cash flows. Assumptions used by the company to carry out cash flows projections can be chosen between its own history or using market benchmarks These assumptions[4] should be reasonable and explicit, reflecting all future events, including changes in legislation, future technological progress, inflation etc…

In determining the ESV, the following items are taken into account:

- Premium
- Claims
- Added Value Taxes

[4] If necessary assumptions have to be periodically revised

- Expenses
- Recoveries

The following should not be included in determining the future cash flows for liabilities:

- Income tax payments
- Payments to and from reinsurers
- Investment returns
- Cash flows between different components of the reporting entity

Within the current proposals, ESV chosen is a closed book approach meaning that all future contracts from renewals or new business are not taken into account to determine the ESV.

The DSOP recommends doing an evaluation by book of contracts presenting the same risks, taking into account the diversification and the correlation within each book of contracts. On the contrary, the correlation between books of contracts must be ignored.

According to the DSOP, the investment strategy of the company must not have impact on the ESV of the liabilities. Moreover, the credit risk of the company must not be taken into account in the ESV.

2.2 Risk and Uncertainty Adjustment

(Known as Market Value Margin in the DSOP)

Insurance contracts future cash flows are always subject to uncertainty and risk due to various reasons:

For the recurrent claims:

- The claims count can be different than expected
- The severity of each claim may be different than expected
- The payment pattern can be different than expected

It is although possible that arise some catastrophic events in the book.

These uncertainties may affect the claims payment, the payment pattern or both. The DSOP requires the Entity Specific Value to reflect always risk and uncertainties of insurance assets and liabilities.

This risk adjustment is made preferably in the cash flows but the DSOP accepts this adjustment to be included in the discount rate even if an adjustment included in the discount rate is less accessible, implying a lack of transparency.

The IASB splits the risk in three parts: risk model, parameter risk and process risk.

- The risk model refers to the choice of an inaccurate model. For example, the company can assume that the ultimate losses follow a model that does not correspond to the reality.

- The parameter risk (or estimation risk) concerns an inaccurate estimation of the parameters of the model.

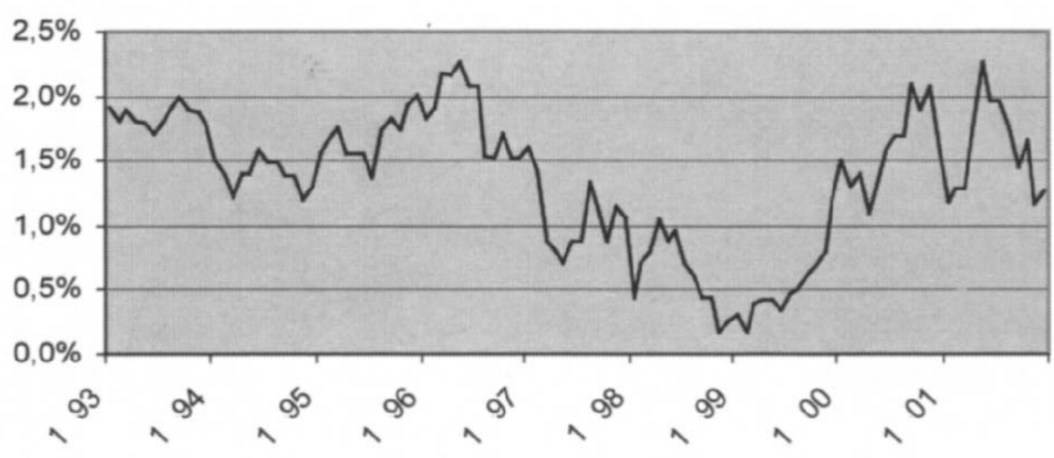
- The process risk refers to the unexpectedly occurrence of exceptional risk even if the model is accurate and the parameters good estimated.

The DSOP gives no advice to estimate the risk and uncertainty adjustment but requires that it is "additive".

2.3 Inflation

The graph 1, representing the evolution of the yearly general inflation rate (France: 1993-2001), shows that the inflation rate didn't stay constant during this period.

GRAPH 1



When the inflation is variable year to year, the DSOP requires projecting deflated cash flows. It is although well known that the Consumer Price Index does not allow treating the inflation claims in different lines of business of general insurance.

It can be noted that the inflation rate evolution is strongly linked with the interest rate evolution, mentioned in the following paragraph.

2.4 Discounted Cash Flows

The DSOP recommends discounting cash flows including the adjustment for risk and uncertainty with the risk free discount rate. It implies the use of the interest rate curve. The construction of such a curve, i.e. the analysis of the term structure of interest rates, has been lot studied and has given lots of publications.

Roughly, we can mention the model based on the short-term interest rate from Cox Ingersoll Ross (Kaufmann, 2001) or the models based on autoregressive inflation process from Wilkie. (Daykin et al., 1994).

If the risk market margin is integrated in the discount rate, it is possible to use the deflators. Deflators are a technique that allows discounting stochastically cash flows, integrating risks (Jarvis et al.).One other alternative would be based on the CAPM (Cummins, 1999).

2.5 Asset and Liability approach

Only the contracts present in the portfolio at the date of valuation are taken into account.

A market value approach is used to valuate the assets, in accordance with the IAS 32 or 39 Standards.

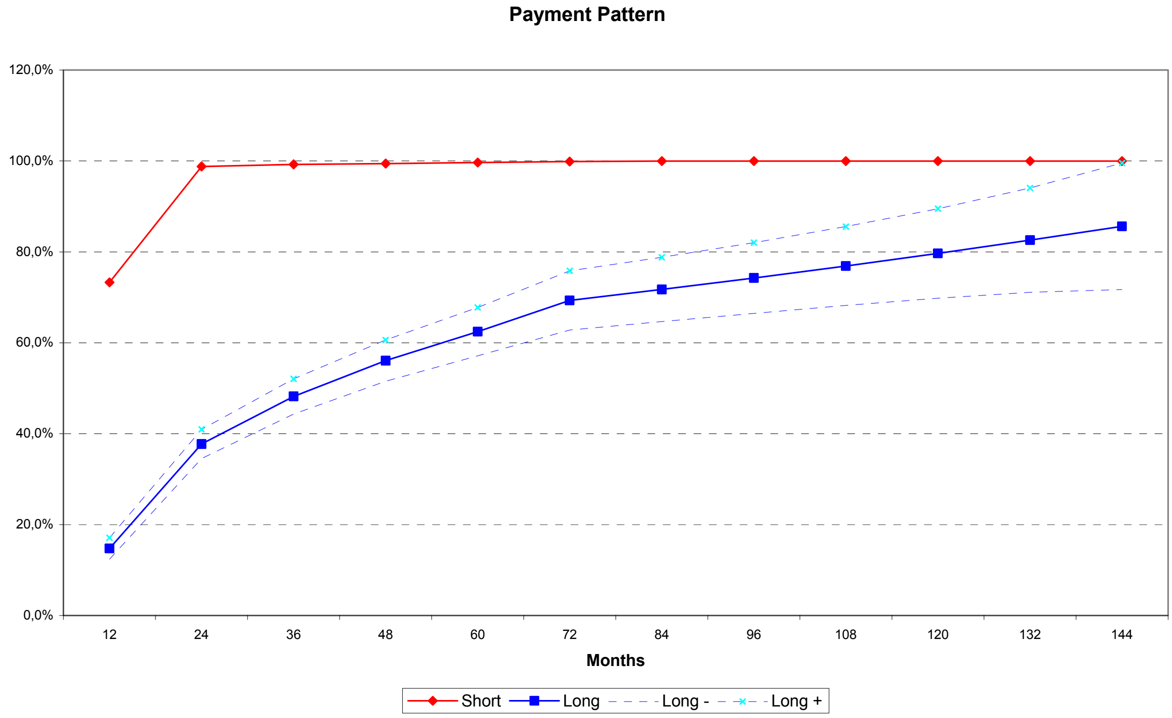
Concerning the liabilities, the premium and expenses are treated with deterministic classical methods, which are used to project Profit & losses in the present accounting system.
The valuation of the claims payment (and recoveries) is as usual essential in determining the ESV.
All the items are gross of reinsurance in liabilities. In asset claims paid by the reinsurers generate future cash flows affected by risk defaults.

3. IMPACT OF PAYMENT PATTERN

In determining the future claims cash flows, the DSOP recommends to use stochastic model but accept that deterministic model like Chain Ladder method for example are used. We can note that Chain ladder method implied the same payment pattern for each exercise. Dynamic models of payment pattern evolution may be although used.

Timing of payment has more and more impact when we are starting to discuss discounting on reserves valuation.

In the graph 2 we are comparing the payment pattern of two different products:



According to the different pattern the discount will have a strong effect which can be summarised in the table below.

IMPACT OF PAYMENT PATTERN ON DIFFERENT DISCOUNTING EFFECT				
Discount Rate	Short	Long	Long -	Long +
0%	1,00	1,00	1,00	1,00
2%	0,97	0,90	0,88	0,92
4%	0,95	0,81	0,79	0,84
6%	0,93	0,74	0,71	0,78
8%	0,91	0,69	0,65	0,73
10%	0,89	0,64	0,60	0,68

From the table provided above, we can conclude that payment speed will have an impact which is as least as important as the choice of discount rate. Therefore, the theory can be developed on the different framework to get the discount rate, but it is as important to discuss payment speed.

The following aspects must be taken into account

- Impact of financial market on the speed of payment. It can be analysed that when financial conditions are poor client will try to increase their settlements of claims to make sure that they get the money. When financial environment is on a better shape, it can be observed that payment speed is reducing as claimants will try to increase their settlements; therefore payments speed will be reduced.

- Impact of inflation on payment speed. If inflation is increasing, insurers will try to increase their speed of settlements in order to limit their ultimate costs.

- Impact of lines correlation (or dependencies) on payment speed. For package business usually all guaranties are settled on the same time, therefore there will be some impact of each other.

From the previous example, we can notice that payment speed will not be regular and must be taken into account.

The next steps will be to try to model payment pattern. The following approach could be taken:

- Some of the methods to estimate ultimate cost are based upon payment triangles. Identical statistical distribution could be used to simulate payment pattern. For example, the Thomas Mack methods of ultimate cost can be based on payment triangles. The same approach could be used to evaluate each claim payment

- Payment pattern could be analysed as a cumulative function. Testing different distributions such as some beta distributions provide a good fit Beta distributions are classical distributions for fitting payment patterns. As the two parameters define respectively the left tail and right tail of the distribution it is possible with some indication on the future trend of the parameters to derive projected payment pattern per year of occurrence of losses and suppress the traditional assumption of fixed payment pattern for all occurence years

- Correlation of payment pattern could be built in from external factor.

Nevertheless, when simulating payment pattern two different routes could be used:

- One where each and every payment are simulated, then the impact from one on the other period could be simulated. It could be assimilate as a stochastic process

- One where the cumulative function is simulated but we will miss the information ate every time on what happen the previous period.

4. CLAIMS CASH-FLOWS

To analyse cash flows, we use naturally the incremental claims payments triangle. We remember that they are deflated payments. As a closed book approach is adopted, the origin years are the underwriting ones.

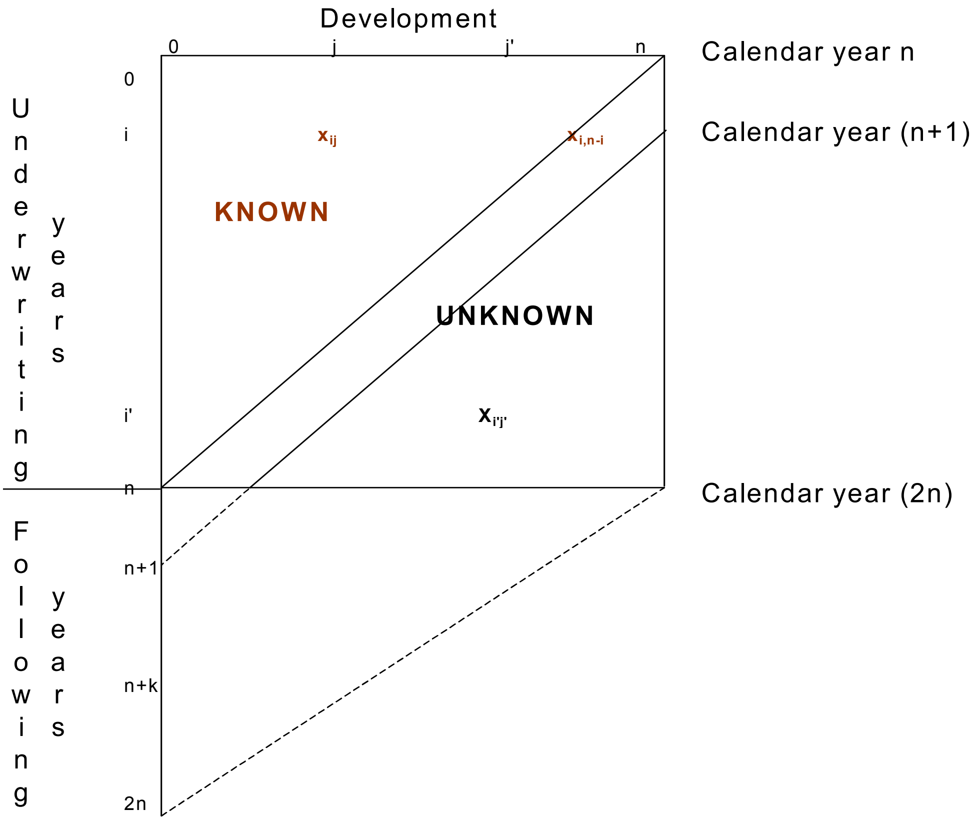
In determining the future cash flows, the DSOP recommends to use stochastic models but accept that deterministic models like chain ladder method are used.

With a stochastic model, we will project the recurrent claims of a contracts book. The catastrophic claims must be treated separately and specifically.

4.1 Notations

The table 2 contains the claims run-off triangle on which the analysis is based. The valuation date is 31/12/n for a book of contracts with a claims development of (n+1) years

Table 2: Run-off triangle



In this figure X_{ij} $(i,j=1,\ldots,n)$ is the random variable (r.v.) amount of claims paid for the underwriting year i with the delay j. The amounts paid until the 31/12/n (known) appear in the upper triangle: x_{ij} $(i+j \leq n)$ is the realization of X_{ij}.

Assumption: X_{ij} are independent r.v. ($i,j = 1,\ldots,n$)

Under these notations, the future claims cash-flows for the calendar year $(n+k)$,

$1 \leq k \leq n$, are given by the r.v.

$$CF_{n+k} = \sum_{\substack{i,j \\ i+j=n+k}} X_{ij}$$

and in the aggregate

$$CF = \sum_{k=1}^{n} CF_{n+k} \qquad \text{without any discount}$$

If $(\tau_{n+k})_{1\leq k\leq n}$ is the sequence of discount rates attached to years $(n+1),\ldots, 2n$, we get

$$CF^{(a)} = \sum_{k=1}^{n} \frac{CF_{n+k}}{(1+\tau_{n+k})^k}$$

as total discounted future cash-flows r.v.

4.2 Actuarial parameters

Let F the cumulative distribution function (c.d.f.) of CF (same approach for $CF^{(a)}$). The main problem consists to estimate a chosen parameter $\Pi(F)$, certain but unknown, using the upper triangle data. This parameter could be

1. Some central value indicator: mean $E(CF)$, choice of the DSOP, median $M(F)$,…
2. Some volatility indicator: variance $V(CF)$, standard deviation $\sigma(CF)$,…
3. An insufficiency probability $P(CF > A)$,
   where A is an asset representing CF.
4. A risk measure : VaR defined by $P(CF > VaR_\varepsilon) = \varepsilon$
   Tail VaR defined by $E(CF / CF > VaR_\varepsilon)$
5. A risk and uncertainty margin (Market Value Margin)

The first component of this margin, corresponding to the process risk, would be similar to a security loading in ratemaking and proportional to

$$E(CF), V(CF), \sigma(CF), VaR(CF) - E(CF), \ldots.$$

6. The distribution de CF given by its d.f.. or moments generating function $M(s) = E(s^{CF})$ with, X_{ij} being independent ,

$$M(s)=\prod_{k=1}^{n} M_{CF_{n+k}}(s)=\prod_{k=1}^{n}\prod_{i+j=k} M_{X_{ij}}(s).$$

Numerical inversion of M (Fast Fourier Transform) gives F.

4.3 Estimation

If $\hat{\Pi}=\hat{\Pi}\left[\left(X_{ij}\right)_{i+j\le n}\right]$ is an estimator[5] of $\Pi(F)$, the corresponding uncertainty is classically measured by the Mean Square Error function:

$$MSE\left(\hat{\Pi}\right)=E\left\{\left[\hat{\Pi}-\Pi(F)\right]^2\right\}$$

$$=V(\hat{\Pi})+\left[E(\hat{\Pi})-\Pi(F)\right]^2$$

$$=V\left(\hat{\Pi}\right) \quad \text{if } \hat{\Pi} \text{ is an unbiaised estimator.}$$

or by the standard error

$$s.e.(\hat{\Pi})=\sqrt{V(\hat{\Pi})}$$

It is worthwhile to note that, for an asymptotically normal estimator, it is common to use asymptotic variance or standard error, denoted respectively $V_{as}(\hat{\Pi})$ and $s.e._{as}(\hat{\Pi})$. These functions are itself estimated by $\widehat{MSE}\left(\hat{\Pi}\right)$ and $\widehat{s.e.}\left(\hat{\Pi}\right)$, using for instance the substitution principle.

In addition or alternatively, we can obtain a confidence interval for $\Pi(F)$ with level 0.95, for instance.

The bounds $A\left[\left(X_{ij}\right)_{i+j\le n}\right]$ and $B\left[\left(X_{ij}\right)_{i+j\le n}\right]$ are defined by

$$P\left\{A\left[(X_{ij})_{i+j\le n}\right]\le \Pi(F)\le B\left[(X_{ij})_{i+j\le n}\right]\right\}=0.95.$$

Remark: The DSOP recommends to <u>estimate</u> $E(CF)+$ Margin. It may also seem natural to look for a prediction of a r.v. $f\left[(X_{ij})_{i+j>n}\right]$ depending on the lower triangle. Such functions could be for instance X_{ij}, CF_{n+k}, CF.

[5] In claims reserving the"best estimate" of the reserve is a very classical concept. However its definition is far from being clear and unique. Mostly authors agree to consider that this best estimate is an estimate of a central value.

A predictor of the r.v. f is a function $h\left[\left(X_{ij}\right)_{i+j\leq n}\right]$ of the r.v. in the upper triangle, with corresponding prediction: $h\left[\left(x_{ij}\right)_{i+j\leq n}\right]$.

Uncertainty associated to this prediction is measured by the (prediction) mean square error.

$$MSEP(h)=E\left\{\left[h\left(X_{ij}\right)-f\left(X_{hk}\right)\right]^2\right\}$$

where expectation is taken on variables from upper and lower triangles.

Independence of r.v.. $f\left[\left(X_{hk}\right)_{h+k\geq n}\right]$ and $h\left[\left(X_{ij}\right)_{i+j\leq n}\right]$ implies

$$MSEP(h)=V\left\{\left[f\left(X_{hk}\right)_{h+k>n}\right]\right\}+E\left\{\left\{h\left(X_{ij}\right)_{i+j\leq n}-E\left[f\left(X_{hk}\right)_{h+k>n}\right]\right\}^2\right\}$$

where $V\left\{\left[f\left(X_{hk}\right)_{h+k>n}\right]\right\}$ is an uncertainty specific to the model (independent of the prediction process).The second term of this equality is the (quadratic) estimation error of $E\left\{\left[f\left(X_{hk}\right)_{h+k>n}\right]\right\}$ by $h\left[\left(X_{ij}\right)_{i+j\leq n}\right]$.

The first term, additional relatively to the estimation process, is present because in a prediction process, we need to integrate the r.v.'s volatility.

In the same manner as for confidence interval for a parameter, we can derive a prediction interval for $f\left(X_{hk}\right)_{h+k>n}$:

Lower $A_1\left[\left(X_{ij}\right)_{i+j\leq n}\right]$ and upper $B_1\left[\left(X_{ij}\right)_{i+j\leq n}\right]$ bounds are defined by

$$P\left\{A_1\left[\left(X_{ij}\right)_{i+j\leq n}\right]\leq f\left[\left(X_{hk}\right)_{h+k>n}\right]\leq B_1\left[\left(X_{ij}\right)_{i+j\leq n}\right]\right\}=0,95\ .$$

5. GLM MODELLING

Yearly cash-flows being compound of increments, it seems reasonable to develop the stochastic valuation process inside the GLM modelling, with computing support for instance of Genmod procedure in SAS.
Probably the conditional model of Mack (Mack, 1993) would offer some alternative solutions.

5.1 Models

We recall that a Generalized Linear Model is given by the following characteristics :

A. Random component

We have independent "response" r.v. $X_{ij}\left(i,j=0,...,n\right)$ with exponential type probability distribution. meaning that the "density" of $X_{ij}(i,j=0,...,n)$ is given by

$$f\left(x_{ij};\theta_{ij},\phi\right)=\exp\left\{\left[\theta_{ij}x_{ij}-b\left(\theta_{ij}\right)\right]\big/\phi+C\left(x_{ij},\phi\right)\right\}$$

where θ_{ij} is a real parameter, called "natural" parameter.

$\phi>0$ ($\phi=1$ eventually) is a scale parameter being independent of i and j.

b and c are regular functions, specific of the distribution.

We can show :

$$\mu_{ij}=E\left(X_{ij}\right)=b'\left(\theta_{ij}\right) \text{ and } V(X_{ij})=\phi b''\left[b'^{-1}(\mu_{ij})\right]=\phi V(\mu_{ij})$$

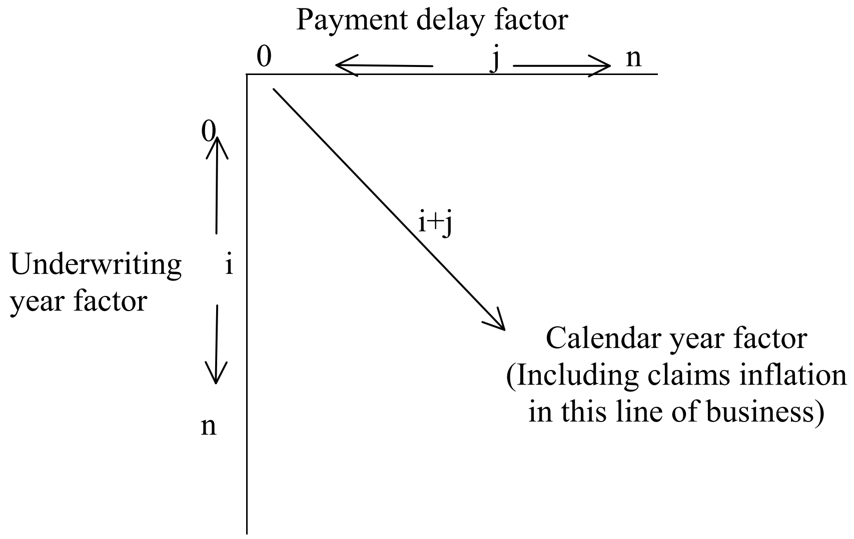
The function V is called variance function of the distribution.

Remark: (μ_{ij},ϕ) would be an alternative parametrisation.

This family contains distributions like Bernoulli, Poisson with $V(\mu)=\mu$ and $\phi=1$, Normal, Gamma, …

B. Factors, systematic component, link function

Factors occurring for modelling a run-off triangle are given by its three « natural » directions:



Under a constant inflation rate, we associate to these factors the real parameters

$$\alpha_i, \beta_j, \mu_{i+j} = \mu \ \left(i, j = 0, ..., n\right)$$

with some identifiability constraints (for instance $\alpha_0 = \beta_0 = 0$ if we choose levels 0 as references).

Let ${}^t\gamma = (\mu, \alpha_1, ..., \alpha_n, \beta_1, ..., \beta_n)$ be the parameters vector.

The systematic component is $\eta_{ij} = \mu + \alpha_i + \beta_j \ \left(i, j = 0, ..., n\right)$

The link function sets up a bridge between random and systematic components as a strictly monotonic differentiable real function g such that :

$$\eta_{ij} = g\left(\mu_{ij}\right) \text{ or } \mu_{ij} = g^{-1}\left(\eta_{ij}\right).$$

Standards links are:

Identity link: $\eta_{ij} = \mu_{ij}$

log link: $\eta_{ij} = \ln \mu_{ij}$ or $\mu_{ij} = e^{\eta_{ij}}$

C. In fact the natural parameters θ_{ij} appearing in the "density" being not used, it is worthwhile to note that a GLM model is summarized by:

- A probability distribution for response variable X_{ij}
- Expectation and variance function, with eventually a dispersion ϕ

$$E\left(X_{ij}\right)=\mu_{ij}\,,\ V\left(X_{ij}\right)=\phi V\left(\mu_{ij}\right).$$

- A link function $\eta_{ij}=g\left(\mu_{ij}\right)$.

5.2 Estimation

The independence assumption gives the likelihood function associated to the upper triangle data:

$$L\left[\left(x_{ij}\right)_{i+j\le n};\mu,\left(\alpha_i\right),\left(\beta_j\right),\phi\right]$$

Setting to 0 all partial derivatives of Log L relatively to parameters $\mu,\left(\alpha_i\right),\left(\beta_j\right)$ leads to the following system of equations (Wedderburn system)

$$(S)\left\|\sum_{\substack{i,j=1\\ i+j\le n}}^{n}\frac{\left(x_{ij}-\mu_{ij}\right)}{V\left(\mu_{ij}\right)}\frac{\delta\mu_{ij}}{\delta\eta_{ij}}b_{ij}^{h}=0\ ,\ k=1,\ldots,p\right.$$

where ${}^{t}b_{ij}=\left(b_{ij}^{(k)}\right)$ is the row vector corresponding to the chosen parameters $\mu,\left(\alpha_i\right),\left(\beta_j\right)$ for derivative.

Remark: The system (S) does not contain the parameter ϕ.Adding to (S) the equation $\frac{\partial LogL}{\partial\phi}=0$ gives, if we need it, an estimation of ϕ.

System (S), which can be only numerically solved using standard algorithms such that Newton-Raphson or score, gives the maximum likelihood estimation (mle) $\hat{\mu},\left(\hat{\alpha}_i\right),\left(\hat{\beta}_j\right)$ of $\mu,\left(\alpha_i\right),\left(\beta_j\right)$.[6]

Using the invariance property of maximum likelihood, we conclude that $\hat{\mu}_{ij}=g^{-1}(\hat{\eta}_{ij})$ with $\hat{\eta}_{ij}=\hat{\mu}+\hat{\alpha}_i+\hat{\beta}_j$ is the mle of $\mu_{ij}=E(X_{ij})$.

$\hat{\mu}_{ij}$ is the"predicted" value by the model.

Remark: we recall that under Poissonian model and Log link, predicted values are strictly equal to the results of the chain ladder method.

[6] These estimates are outputs of each statistical software including a GLM procedure.

In the same way $\hat{E}(CF_{n+k}) = \sum_{i+j=n+k} \hat{\mu}_{ij}$ is mle of $E(CF_{n+k})$

and $\hat{E}(CF) = \sum_k \sum_{i+j=n+k} \hat{\mu}_{ij}$ of $E(CF)$.

Remark: We can consider $\hat{X}_{ij} = \hat{\mu}_{ij}$ as a predictor of $X_{ij}\,(i+j>n)$,
$\hat{C}F_{n+k} = \sum_{i+j=n+k} \hat{X}_{ij}$ and $\hat{C}F = \sum_k \sum_{i+j=n+k} \hat{X}_{ij}$ for CF_{n+k} and CF.

In order to assess the risk estimation and computing $V\left[\hat{E}(CF_{n+k})\right]$ and $V\left[\hat{E}(CF)\right]$ we need variances and covariances of $\hat{\mu}_{ij}$ because

$$V\left[\hat{E}(CF_{n+k})\right] = V\left[\sum_{i+j=n+k} \hat{\mu}_{ij}\right] = \sum_{i+j=n+k} V(\hat{\mu}_{ij}) + 2 \sum_{i_1+j_1=n+k} \sum_{i_2+j_2=n+k} \mathrm{cov}(\hat{\mu}_{i_1 j_1}, \hat{\mu}_{i_2 j_2}).$$

In most cases deriving these quantities is a difficult task. However if we can use asymptotic[7] results [$m = \frac{(n+1)(n+2)}{2}$ observations are present in the upper triangle), we obtain asymptotic variances and covariances of ${}^t\hat{\gamma} = (\hat{\mu}, \hat{\alpha}_1, \ldots, \hat{\alpha}_n, \hat{\beta}_1, \ldots, \hat{\beta}_n)$ as components of the inverse of Fisher information matrix $I_m(\gamma)$:

$$I_m^{-1}(\gamma) = \begin{bmatrix} \sigma^2_{as}(\hat{\mu}) & \mathrm{cov}_{as}(\hat{\mu}, \hat{\alpha}_i) & - - - - - - - - - - - & \mathrm{cov}_{as}(\hat{\mu}, \hat{\beta}_i) \\ & \sigma^2_{as}(\hat{\alpha}_1) & & \\ & & \ddots & \\ & & \sigma^2_{as}(\hat{\alpha}_n) & \\ & & \sigma^2_{as}(\hat{\beta}_1) & \\ & & & \ddots \\ \mathrm{cov}_{as}(\hat{\mu}, \hat{\beta}_n) & - - - - - - - - - - - & - - - - & \sigma^2_{as}(\hat{\beta}_n) \end{bmatrix}$$

with an asymptotic normality property for $\hat{\gamma}$:

$$\hat{\gamma} \;\; \text{Asympt.}\, N(\gamma, I_m^{-1}(\gamma))$$

We recall that $I(\gamma)$ matrix is derived by computing expectation of second partial derivatives of $LogL$.

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[7] Asymptotics need to be used cautiously because the number of parameters $2n+1$ is linked to the data size.

The asymptotic variance $\sigma_{as}{}^2\left(\hat{\eta}_{ij}\right)$ is easily deduced from previous variances and covariances :

$$\sigma_{as}{}^2\left(\hat{\eta}_{ij}\right)=\sigma_{as}{}^2\left(\hat{\mu}\right)+\sigma_{as}{}^2\left(\hat{\alpha}_i\right)+\sigma_{as}{}^2\left(\hat{\beta}_j\right)+2\operatorname{cov}_{as}\left(\hat{\mu},\hat{\alpha}_i\right)+2\operatorname{cov}_{as}\left(\hat{\mu},\hat{\beta}_j\right)+2\operatorname{cov}_{as}\left(\hat{\alpha}_i,\hat{\beta}_j\right)$$

For a regular real function g (with a derivative not equal to 0), Delta method gives the asymptotic variance of $\hat{\mu}_{ij}$:

$$\sigma_{as}^2\left(\hat{\mu}_{ij}\right)=\left[\left(g^{-1}\right)'\left(\eta_{ij}\right)\right]^2\sigma_{as}^2\left(\hat{\eta}_{ij}\right).$$

Then

$$V_{as}\left[\hat{E}(CF_{n+k})\right]=\sum_{i+j=n+k}\sigma_{as}^2(\hat{\mu}_{ij})+2\sum_{i_1+j_1=n+k}\sum_{i_2+j_2=n+k}\operatorname{cov}_{as}(\hat{\mu}_{i_1j_1},\hat{\mu}_{i_2j_2})$$

and similarly for $V_{as}\left[\hat{E}(CF)\right]$.

As alternative to this asymptotic approach we can use a now standard approach of variance estimation, based on resampling techniques, namely Jackknife or bootstrap (cf Efron and Tibshirani, 1993; Shao and Tu, 1995) applied to residuals of regression model.

5.3 Residuals. Checking models

5.3.1 Cell residuals

After fitting a model it is very useful to compare observed and predicted value in each cell
(i,,j) of the upper triangle.($i+j\le n$).
For the (i,j) cell we consider the following residuals:

- Residual: $r_{ij}=x_{ij}-\hat{\mu}_{ij}$
- (unstandardised) Pearson Residual: $r_{ij}{}^{(p)}=\dfrac{x_{ij}-\hat{\mu}_{ij}}{\sqrt{V\left(\hat{\mu}_{ij}\right)}}$

Remark : (standardised) Pearson residual is

$$\frac{x_{ij}-\hat{\mu}_{ij}}{\sqrt{\hat{V}\left(x_{ij}\right)}}=\frac{x_{ij}-\hat{\mu}_{ij}}{\sqrt{\hat{\phi}V\left(\hat{\mu}_{ij}\right)}}$$

where $\hat{\phi}$ is an estimation of the dispersion parameter.

- Deviance residual : defined by,

$$r_{ij}^{(D)} = \mathrm{sgn}\left(x_{ij} - \hat{\mu}_{ij}\right)\sqrt{\left(d_{ij}\right)} \quad \text{for } d_{ij} = 2\left\{x_{ij}\left(\tilde{\theta}_{ij} - \hat{\theta}_{ij}\right) - \left[b\left(\tilde{\theta}_{ij}\right) - b\left(\hat{\theta}_{ij}\right)\right]\right\}$$

$$\text{with} \quad \tilde{\theta}_{ij} = b'^{-1}\left(x_{ij}\right) \text{ and } \hat{\theta}_{ij} = b'^{-1}\left(\hat{\mu}_{ij}\right)$$

Cell by cell analysis of residuals allows to detect possible untypical values in the upper triangle with a need for a deeper analysis for these cells.
Plotting residuals against observed values, predicted values and factors must only show random structures. Otherwise diagnostic of no agreement with model assumptions could be done.

5.3.2 Global measures

Similarly to the Normal linear model, indicators of goodness of fit of a model to the upper data triangle are given by the following

- Generalised χ^2 statistics (Pearson)

$$X^2 = \sum_{i+j \le n} \frac{\left(x_{ij} - \hat{\mu}_{ij}\right)^2}{V\left(\hat{\mu}_{ij}\right)}$$

- Deviance

With the above notations

$$D = \sum_{i+j \le n} (r_{ij}^{(D)})^2 = 2\sum_{i+j=n} \int_{\hat{\mu}_{ij}}^{x_{ij}} \frac{x_{ij} - u}{V(u)} du$$

Examples :

1. Normal $\mathrm{N}\left(\mu, \sigma^2\right)$

$$D = X^2 = \sum_{i+j \le n} \left(x_{ij} - \hat{\mu}_{ij}\right)^2$$

Standard sum of square residuals

2. Poisson $\mathsf{P}(\lambda)$

$$D = 2\sum_{i+j \le n} \left[x_{ij} \ln \frac{x_{ij}}{\hat{\mu}_{ij}} - \left(x_{ij} - \hat{\mu}_{ij}\right)\right]$$

$$X^2 = \sum_{i+j\le n} \frac{\left(x_{ij} - \hat{\mu}_{ij}\right)^2}{\hat{\mu}_{ij}}$$

We know that a direct comparison of the values taken by these indicators is useful for selecting the most significant factors in the framework of a GLM model (Normal, Poisson,…). It is the important purpose of ratemaking procedures.
However in reserving factors are predetermined and the important problem is to compare different GLM models using the same factors. But simple comparisons of the above indicators for two different models are not available.

6. RISK MODEL

To reduce this risk we intend to get the best model (using a best fitting measure) within a large class of GLM models.
The estimation process of $E(CF)$ and the measure of risk estimation are based on ϕ and solutions of the Wedderburn system .These solutions depend only in the first moments μ_{ij} and $V(\mu_{ij})$ of the underlying exponential-type distribution.
Therefore a "quasi-likelihood" approach has been developed without any reference to any exponential-type distribution. In this approach we need only the parameter ϕ and a variance function V. For instance ϕ and $V(\mu) = \mu$ generate the overdispersed Poisson model.

The quasi-likelihood (in fact quasi-loglikelihood) is now defined by

$$q\left[(x_{ij})_{i+j\le n}; \mu, (\alpha_i), (\beta_j), \phi\right] = \sum_{i+j\le n} \int_{x_{ij}}^{\mu_{ij}} \frac{x_{ij} - u}{\phi V(u)} du$$

Deviance is extended to "quasi-deviance":

$$D = -2\phi q \ \text{ or } D = 2 \sum_{i+j\le n} \int_{\mu_{ij}}^{x_{ij}} \frac{x_{ij} - u}{V(u)} du$$

but like deviance quasi-deviance cannot be used to compare models with different variance functions. For performing that we introduce the "extended quasi-likelihood" (Nelder et al., 1987) q^+ defined by

$$-2q^+ = \frac{D}{\phi} + \sum_{i+j\le n} Log\left[2\pi\phi V(x_{ij})\right]$$

This last function has to be minimized.

Remark: we have an alternative approach using Pearson residuals and "pseudo-likelihood" (Nelder et al., 1992).

To limit the size of optimal model searching, it is possible to introduce some parametric constraints on V and g functions, for instance power type :

$$V(\mu) = \mu^{\xi}$$

$$g(\mu) = \mu^{\alpha} \text{ with the convention } \lim_{\alpha\to+\infty} \mu^{\alpha} = Log\mu .$$

Varying on values of the real parameters ξ and α , we can find all standard GLM models presented in § 4.

Best model searching in this class is a non linear optimisation problem :

$$\min_{\alpha,\xi,\gamma,\phi} -2q^{+}(\alpha,\xi,\gamma,\phi)$$

This can be achieved in two steps:

Step 1: for (α,ξ) given, minimisation on γ and ϕ gives $\hat{\gamma}(\alpha,\xi)$ and $\hat{\phi}(\alpha,\xi)$

Step 2: minimisation of $-2q^{+}\left[\alpha,\xi,\hat{\gamma}(\alpha,\xi),\hat{\phi}(\alpha,\xi)\right]$.

7. CONCLUSION

Without passing a value judgment on the relevance of IAS standards at this current stage the aim of this paper is to provide a technical framework to funding principles.

Additional work will be necessary for a balance sheet approach on future discounted cash flows. One of the first fields should be the "correlation" analysis between future cash flows of gross reserves on the liability side and the reinsured reserves on the assets side. The copula theory could be a solution to this problem.

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