Comparison of Time Series and Interest Rate Models to Forecasts of the German Inflation Rate

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Abstract

The subject of this study is the representation and the comparison of univariate time series and interest rate models with reference to the one-step-ahead forecasts of the inflation rate. The analysis is based on the monthly seasonally adjusted price index for the living standard of all private households in West Germany in the periods 1/1963 to 12/1994 in-sample and 1/1995 to 1/1999 out-of-sample. With the comparison of an ARMA(1,1) time series model, an ARMA(1,1) interest rate model and a naïve interest rate model in-sample a construction dependent superiority of the time series model is determined. Within the out-of-sample period for none of the specified models is a better forecast performance statistically provable. This is attributed to the variability of monthly changes of the historical West German inflation rate falling over time. Altogether, the economically founded interest rate models — in particular the naïve interest rate model, which can be regarded as an approximation of the subtler ARMA interest rate model — seem to become more attractive out-of-sample.

1. Introduction

Forecasting the inflation rate plays an important role in many economic questions. Above all there are financial equilibrium models, which seek to explain market prices of risky financial investments. Thus Friend / Landskroner / Losq (1976) extend the basic version of the Capital Asset Pricing Model (CAPM) based on nominal sizes in the light of inflation expectations. In the context of the Arbitrage Pricing Theory (APT) for instance, Chen / Roll / Ross (1986), McElroy / Burmeister (1988) as well as Young / Berry / Harvey / Page (1991) point out that under different macroeconomic factors the expected inflation rate in particular represents an important explanation-factor of capital market returns. A problem in testing these models is that the overall expected inflation rate is latent. Therefore, it has to be modelled in a suitable and transparent manner. Derived forecasts from such an inflation model can then be used as proxies for the expected inflation rate.

A similar problem exists in the empirical analysis of inflation hedge abilities of different forms of investments. Thus Fama / Schwert (1977) examine whether short-term nominal yields of American shares, securities at fixed interest and real estates are independent of changes in the inflation expectation. The methodology of this investigation was refined in numerous other empirical studies. Especially in the real estate literature, hedging potentials of different kinds of real estates form a standard question. Thus for instance Gyourko / Linneman (1988), Hartzell / Hekman / Miles (1987) and Miles / Mahoney (1997) examine the inflation protection characteristics of US-American real estates of different locations and types of use. The question whether the yields of investing in different forms of real estate companies (e.g. REITS, real estate funds) protect against expected inflation risks has been investigated for different countries among others by Liu / Mei (1992), Glascock / Davidson (1995), Liu / Hartzell / Hoesli (1997) and Maurer / Sebastian (1999) on the basis of inflation forecasts.

Also in the popular actuarial stochastic investment models univariate forecasts of the future inflation rate play an important role due to the recursive structure of these models. See for example the stochastic investment model of Wilkie (1986, 1995) for Great Britain, the analogous works of Carter (1991), Harris (1995a, 1995b) and Sherris / Tedesco / Zehnwirth (1999) for Australia, in all of which inflation represents the driving force, as well as Claasen (1993) and Thomson (1996) for South Africa and Pentikäinen et al. (1994) for different industrialized countries. These stochastic investment models are used as a basic tool in the asset management and the asset liability management of insurance companies.

Apart from expert questionings (like the Livingston Survey), time series and interest rate models are most commonly used for the generation of inflation expectations. However, survey-based inflation forecasts have considerable disadvantages in the context of their specific applications to West German financial markets. Firstly, there is no sufficiently long history of such inflation forecasts available for Germany. Secondly, expert expectations for one point in time cannot be updated because of the inherent lack of transparency in the forecasting mechanism. For these reasons throughout the remainder of this paper only the representation and the comparison of time series and interest rate models are focused on.

2. Time series models

The aim of the extrapolation of time series models is the prognosis of the future inflation rate based on its past development. Explicitly, a stochastic model for the observed process is assumed, on whose validity the forecast quality depends. Therefore, the ARIMA model family of the Box / Jenkins (1970) generation is frequently taken into consideration. When using ARIMA(p,d,q) models, the inflation rate $R^I(t)$ is specified by the following process (t=1,2,...)

$$\nabla^d R^I(t) = \mu + \sum_{i=1}^{p} \varphi_i \left[\nabla^d R^I(t-i) - \mu\right] + U(t) - \sum_{j=1}^{q} \theta_j U(t-j), \tag{1}$$

where ∇^d means the backwards difference operator of order d (d=0,1,2,...), $\nabla^d = (1-B)^d$, whereby B represents the backshift operator, $B^i R^I(t) := R^I(t-i)$,

$R^I(t)$ are the inflation rates in each period t (with $E(\nabla^d R^I(t)) = \mu$),

φ_i, θ_j are unknown process parameters which have to be estimated (i=1,..,p; j=1,..,q),

$U(t)$ represent from each other and of the delayed $\nabla^d R^I(t-i)$, i=1,2,..., independent and identically $(0,\sigma^2)$-distributed error terms.

An ARIMA(p, d, q) model for $R^I(t)$ defines an ARMA(p, q) model for $\nabla^d R^I(t)$. The d-th order difference of the origin series is assumed to be weakly stationary. With the weakly stationary AR(p) process component the present value of the explained variables is described by a long-term average value μ and the deviations from this value in the last p periods. Autocorrelations between the error terms of the AR process, that is $\tilde{U}(t) = U(t) - \sum_{j=1}^{q} \theta_j U(t-j)$, can be captured by the MA(q) process component.

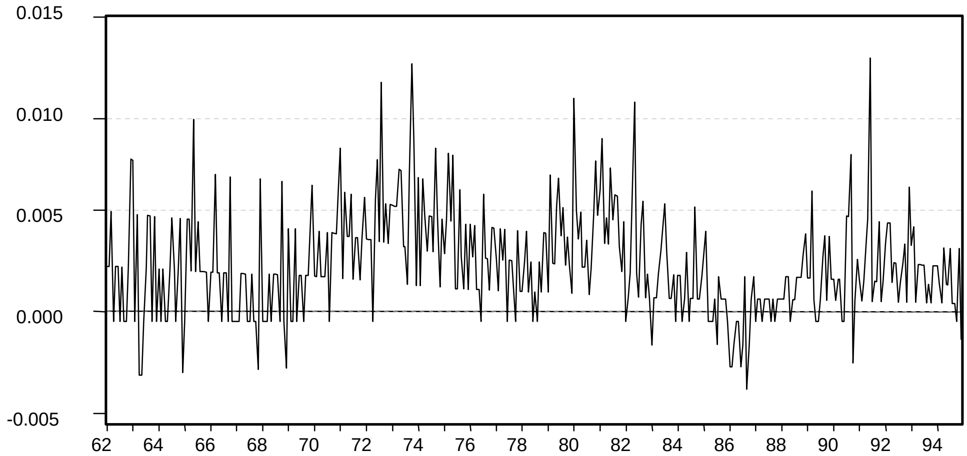
The monthly inflation rate is determined as the change of the price index Q(t) for the living standard over the appropriate period. For the following studies the continuously compounded inflation rate

$$R^I(t) = \ln Q(t) - \ln Q(t-1) = \ln[Q(t)/Q(t-1)] \tag{2}$$

is used. The following figure shows the development of this rate calculated on the basis of the price index of the living standard of all private households in West Germany (1991=100), seasonally adjusted by the German Federal Bank with the Census X-11 procedure.

Figure 1

Monthly seasonally adjusted inflation rate from 1/1962 to 12/1994



M = 0.002826, SD = 0.002583, AC(1) = 0.293, AC(2) = 0.280, AC(3) = 0.263, AC(4) = 0.236, AC(12) = 0.228

(*Source of Data*: Statistical supplements to the monthly reports of the German Federal Bank, 4, Seasonally adjusted economic data)

The average monthly inflation rate M during the observation period from 1/1962 to 12/1994 amounts to 0.2826 % with a standard deviation SD of 0.2583 %. Significant positive autocorrelations at lag one, AC(1), to four, AC(4), and twelve, AC(12), point to positive dependencies of the inflation rate on its previous development, which are reproduceable by ARMA(p,q) modelling.[1] For the purpose of forecasting, not only the fit to the historical time series is considered but also the principle of parsimonously parameterizing. Under different ARMA(p,q) processes ($p \leq 12$, $q \leq 2$) for the German monthly inflation rate in the estimating period from 1/1962 to 12/1994 the most favorable value of the Bayes information criterion (BIC) results for an ARMA(1,1) process.[2] Thus in the following, this time series model is used. Its estimation by the method of non-linear least squares results in the following process (with estimated standard deviations of estimated coefficients in parentheses)

$$R^{I}(t) = \underset{(0.0007)}{0.0028} + \underset{(0.0155)}{0.9763}\, R^{I}(t-1) + U^{ZR}(t) - \underset{(0.0355)}{0.8656}\, U^{ZR}(t-1). \qquad (3)$$

For the process being I(0), that is d = 0, it is necessary that all eigenvalues of the AR section have modulus less than 1. To see whether $\hat{\varphi}_1$ is significantly smaller than 1, an appropriate unit root test in consideration of the MA(1) component has to be chosen. The unit root test on the basis of an AR(12) approximation of the ARMA(1,1) process concludes rejecting the non-stationarity of the time series.[3] Intuitively, an exploding inflation rate over time is indeed not economically convincing in a stable economic system such as the West German.[4]

The estimated autocorrelations of the empirical ARMA(1,1) residuals are with AC(1) = 0.016, AC(2) = -0.018, AC(3) = -0.020, AC(4) = -0.069 and AC(12) = 0.005 not significantly different from zero. Accordingly, the monthly West German inflation rate, calculated from the seasonally adjusted price index for the living standard of all private households, can be illustrated in the past sufficiently well by an ARMA(1,1) process. This agrees with the results of inflation investigations in other countries. Thus ARMA(p,q) model specifications of low order can be found for example in Wilkie (1986) and in Speed (1997) for the U. K., in Metz / Ort (1993) for Switzerland as well as in Fama (1975), in Ibbotson / Sinquefield (1976) and in Caporale / Jung (1997) for the USA.

In the literature for empirical inflation modelling, the consideration of conditional heteroscedasticity has aquired special meaning. As examples one can find an AR(4) - ARCH(1,1) model in Engle / Kraft (1983) and an AR(4) - GARCH(1,1) model in Bollerslev (1986) for the USA, a revised ARCH model in Wilkie (1995) for Great Britain as well as AR(12) - GARCH(1,1) respectively – GARCH(2,2) models for different industrial countries in Grier / Perry (1998). However, with AC(1) = 0.044, AC(2) = 0.041, AC(3) = 0.015, AC(4) = 0.032 and AC(12) =-0.025 none of the estimated correlations of the squared residuals from the above model (3) are significantly different from zero. So for the West German inflation rate no GARCH effects are indicated.

3. Interest rate models

While univariate time series models explain the inflation development essentially according to the information available in its past rates, multivariate methods additionally use the close mutual relations of financial time series. A simple popular system for the prognosis of the inflation rate is based on the classic Fisher hypothesis concerning the connection of nominal interest $R^G(t)$, real interest $R^G_{real}(t)$ and inflation $R^I(t)$. The Fisher hypothesis has attained popularity in the following formalization going back on Fama (1975)[5]

$$R^G(t) = E[R^G_{real}(t)] + E[R^I(t)]. \tag{4}$$

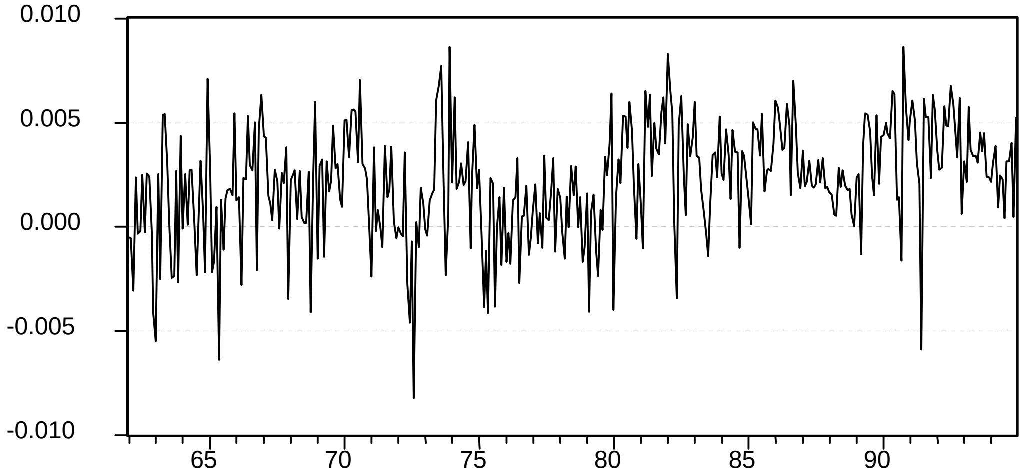
So the expected inflation rate is implicitly described as the nominal interest rate minus the expected real interest rate. The short term nominal interest rate at the beginning of period t is a well-known risk less size. At the same time the inflation rate of the period is still unknown and has therefore to be forecasted. Under the assumption that no effect proceeds from inflation on the expected real interest rate, a simultaneous equation bias is excluded so that the expected inflation rate for the period t is directly derivable from (4). It is then possible to specify the expected real interest rate by a suitable model and to derive the expected inflation by subtraction from the nominal interest rate. Forecasts of inflation rates from interest rate models implicitly assume that real interest rates are easily forecastable (because of persistent behaviour for example) so that inflation rates are more accurately predictable from past interest rates than from their own history. In the basic version of Fama (1975) it is assumed that real interest rates are constant. With the validity of this hypothesis inflation forecasts can be derived directly from current nominal interest rates, and changes of inflation expectations correspond exactly to the changes of nominal interest rates. If real interest rates are not constant, they have to be specified by a suitable model. In the context of so-called naïve interest rate models the expected real interest rate is estimated by the moving average of the last n observed real interest rates. The idea is to thereby illustrate the persistence of the real interest rates and their slow modification over time. So the resulting inflation forecast gives

$$\hat{R}^I_{\text{naïve}}(t) = R^G(t) - \left[\frac{1}{n}\sum_{i=1}^{n} R^G_{real}(t-i)\right] \tag{5}$$

Fama / Gibbons (1984) vary n from 1 to 24. For US American monthly data they conclude that with the selection of different factors from 6 to 24 inflation forecast characteristics hardly differ, but that $n = 12$ minimizes the one-step-ahead forecast mean squared error.

For the nominal interest rate $R^G(t)$ the monthly money market rate published by the German Federal Bank is used in the following (transformed into their continuous monthly equivalents). The historical monthly real interest rate calculates itself as $R^G_{real}(t) = R^G(t) - R^I(t)$. The following line graph shows that the assumption of its constancy does not hold.

Figure 2
Monthly real interest rates from 1/1962 to 12/1994



M = 0.002145, SD = 0.002673, AC(1) = 0.324, AC(2) = 0.280, AC(3) = 0.246, AC(4) = 0.191, AC(12) = 0.133

(*Source of Data*: Statistical sections of the monthly reports of the German Federal Bank)

Specifying a naïve interest rate model we use the factor n = 12 without further statistical analysis due to the used monthly frequency. Furthermore it seems natural to forecast real interest rates by an adequate ARIMA time series model because of the continuously significant positive autocorrelations in the observed series. For monthly US American real interest rates from 1953 to 1977 Fama / Gibbons (1982) specify an ARIMA(0,1,1) process for example. However, the instationarity of German real interest rates cannot be confirmed. Instead, using the Bayes information criterion (BIC) an ARMA(1,1) process is most favourable in the ARMA(p,q) model class with p ≤ 12, q ≤ 2 for the estimation period from 1/1962 to 12/1994. Non-linear least squares estimation of this process equation results in the following process (with estimated standard deviations of the estimated coefficients in parentheses)[6]

$$R^{G}_{real}(t) = \underset{(0.0004)}{0.0023} + \underset{(0.0297)}{0.93}\ R^{G}_{real}(t-1) + U^{Zi}(t) - \underset{(0.0535)}{0.78}\ U^{Zi}(t-1). \qquad (6)$$

The autocorrelations of the residuals are, with estimated values of AC(1) = 0.037, AC(2) = 0.003, AC(3) = -0.013, AC(4) = -0.070 to AC(12) = -0.010 with the exception of AC(8) = 0.119 not significantly different from zero. Therefore, the explanation of the real interest rates by an ARMA(1,1) process seems to be sufficiently accurate. The inflation process can be derived as

$$R^{I}(t) = R^{G}(t) - \left[\, 0.0023 + 0.93\ R^{G}_{real}(t-1) + U^{Zi}(t) - 0.78\ U^{Zi}(t-1) \,\right]. \qquad (7)$$

When using interest rate models to forecast inflation, the financial literature has concentrated on the short run (with mixed results). Nevertheless, the Fisher equation is also exploitable for the long run although several difficulties arise. The longer the maturity of the underlying bond, the lower constant real interest rates are. Long-term nominal interest yields attainable on the market are no longer a well-known and risk less size, so that their data generating process also has to be modelled stochastically. However, a more extensive examination of the resulting problems is beyond the scope of this paper.

4. Comparison of one-step ahead forecasts

4.a. In-sample analysis

In the following, the univariate ARMA(1,1) time series model (3), the naïve interest rate model (5) and the ARMA(1,1) interest rate model (7) are compared for the estimation period 1/1963 to 12/1994 concerning the one-step ahead inflation forecasts.[7] The in-sample comparison applied here is based on a deviation analysis of the one-step-ahead forecast values of the different models from the actual inflation rates. Purely graphic comparisons are unclear due to the multiplicity of observations. Therefore, their information can be bundled in different ways. One possibility suggested by Fama / Gibbons (1984) is to regress the actual inflation rate on the respective inflation point forecasts. **Table 1** contains six regression equations estimated by the least squares method for West German data. With equations (1) to (3) linear single regressions are reported, whose quality is to be judged as follows:

(i) The nearer the regression coefficient α for the constant term is to zero and the nearer the regression coefficient of the forecast variable is to one, the better the respective forecast model adapts to the actual inflation generating process on average, so that the explanatory power of the model is high.

(ii) The residuals should be uncorrelated and should indicate only a small standard deviation σ_u, so that no evidence against uncorrelated and identically distributed error terms and the validity of the statistical tests arises.

For regressions (1) to (3) of the **table 1** condition (ii) holds equally well. The coefficients of determination R^2, which rise with removing mean squared error (MSE), also hardly differ, being best however for the time series model. In addition, only for the univariate time series model the null hypotheses $\alpha = 0$ respectively $\beta = 1$ are not rejected by parameter t-tests. For

Table 1

In-sample regressions from 1/1963 to 12/1994 (T = 384) of the monthly German inflation rate on the one-step-ahead forecasts of the

ARMA(1,1) time series model $\hat{R}^{I}_{ZR}(t) = 0{,}0028 + 0{,}9763\, R^{I}(t-1) - 0{,}8656\, U^{ZR}(t-1)$

naïve interest rate model $\hat{R}^{I}_{naïve}(t) = R^{G}(t) - \frac{1}{12}\sum_{i=1}^{12} R^{G}_{real}(t-i)$

ARMA(1,1) interest rate model $\hat{R}^{I}_{Zi}(t) = R^{G}(t) - \left[0{,}0023 + 0{,}93\, R^{G}_{real}(t-1) - 0{,}78\, U^{Zi}(t-1)\right]$

No.	Estimated effects of						Autocorrelations				
	α	$R^{I}_{ZR}(t)$	$R^{I}_{naïve}(t)$	$R^{I}_{Zi}(t)$	R^2	S_U	AC(1)	AC(2)	AC(3)	AC(4)	AC(12)
(1)	-0.00003 (0.0003)	1.0078 (0.1003)			0.21	0.0023	0.002 (0.001)	-0.007 (0.002)	-0.021 (0.194)	-0.049 (1.118)	0.002 (15.697)
(2)	0.0012 (0.0002)		0.5663 (0.0619)		0.18	0.0023	0.087 (2.935)	0.058 (4.219)	0.027 (4.508)	-0.014 (4.812)	-0.040 (15.545)
(3)	0.0010 (0.0002)			0.6306 (0.0684)	0.18	0.0023	0.030 (0.341)	0.019 (0.475)	-0.000 (0.475)	-0.031 (0.858)	0.031 (20.038)
(4)	0.0001 (0.0003)	0.7082 (0.1557)	0.2363 (0.0944)		0.22	0.0023	-0.003 (0.005)	-0.019 (0.150)	-0.039 (0.755)	-0.071 (2.725)	-0.022 (13.859)
(5)	0.0001 (0.0003)	0.7216 (0.1741)		0.2343 (0.1167)	0.22	0.0023	-0.018 (0.125)	-0.027 (0.399)	-0.042 (1.099)	-0.070 (3.033)	-0.050 (16.406)
(6)	0.0011 (0.0002)		0.2547 (0.1935)	0.3636 (0.2141)	0.19	0.0023	0.048 (0.873)	0.029 (1.1924)	0.005 (1.202)	-0.030 (1.565)	0.009 (15.384)

Notes: Columns 2 to 5 show the parameter estimators of the regression equations. Estimated standard deviations are indicated in parentheses. R^2 indicates the coefficient of determination and S_u is the estimated standard error of the residuals. AC(i) in columns 8 to 12 are the estimated autocorrelations of the residuals to lag i. The Ljung / Box Q-statistic for testing the null hypothesis that up to lag i no autocorrelation exists is reported in parentheses. Under the null the test statistic is asymptotically χ^2-distributed with i degrees of freedom.

the two interest rate models the estimated coefficients are significantly different from the required values. To that extent the time series model has the most favourable characteristics among the three models. This contradicts Fama / Gibbons (1984), who observe a superiority of a naïve and an ARIMA(0,1,1) interest rate model over an ARIMA(0,1,1) time series model in a comparable study for the US American inflation in the period from 1953 to 1977. Equations (4) to (6) in **table 1** describe two-fold regressions. The aim of these multiple regressions is to compare directly the explanatory power of the inflation forecasts. Assuming independence between the different inflation models, the coefficient estimators concerning the different one-step forecasts can be interpreted as marginal explanation contributions. Equations (4) and (5) contain pair-wise comparisons of the time series model with the naïve and with the ARMA interest rate model. It is shown that the respective interest rate model as well as the time series model have significant positive marginal explanatory power for the inflation process. So it can be concluded that the time series model ignores forecast information which is contained in the interest rate models and vice versa. However, the inflation coefficient from the time series model is clearly higher than the one from the respective interest rate model. To that extent the time series model dominates both interest rate models. In equation (6) a comparison of the two interest rate models is made. But due to the high standard deviations no inferences can be derived. This is attributable particularly to the high collinearity of the two inflation forecasts — with an estimated correlation of 0.95.

Altogether the in-sample regression study points out that under the three alternative forecast models only the univariate time series model sufficiently explains the development of the historical inflation rate. As far as the interest rate models are concerned, the above results indicate that the naïve interest model is already a good approximation for the substantially more subtle ARMA interest rate model. In fact, this is precisely the intention behind the use of naïve interest rate models: The AR(∞) representation of an invertible ARMA(1,1) process is approximated by an equivalently weighted moving average of a certain number of past observations.

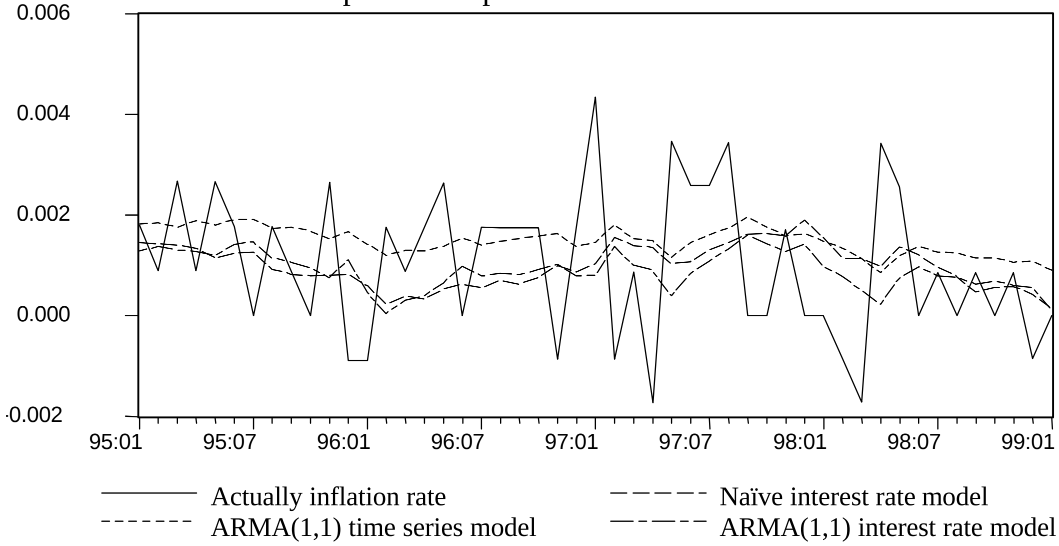
4.b. Out-of-sample analysis

The favourable in-sample characteristics of the time series model are based on the BIC model selection principle on the one hand and on the chosen model evaluation method on the other. Thereby model selection and evaluation access the same target criterion and the same data basis. A part of the BIC selection criterion is the minimization of the mean squared error, which is also minimized by the least squares estimation in the course of model evaluation. In contrast, the two fitted interest rate models are not — at least not directly — based on a statistical selection criterion for the optimization of adjustment quality and parsimony, but on an economic hypothesis of the forecasting ability of the inflation rate from real interest rates.

For a comparative out-of-sample study of the three forecast models specified above the 49 months from 1/1995 to 1/1999 are used. The following figure shows the respective one-step-ahead forecasts as well as the seasonally adjusted inflation rates.

Figure 3

Out-of-sample one-step-ahead forecasts from 1/1995 to 1/1999



In the out-of-sample comparison the adjustment of the forecast values to such data is examined, which themselves have not been used during the model estimation. Again it appears that the two interest rate models generate very similar inflation forecasts so that the accurate approximation of the ARMA interest rate model by the naïve interest rate model is corroborated. In the examined out-of-sample period the time series model generates higher forecast values than the two interest rate models. In all models the actual volatility of the seasonally adjusted inflation rates is not expressed.

Due to the systematic bias of the above regression based evaluation criteria in favour of the time series model now one-step-ahead forecast errors are observed. For the purpose of forecasting this criterion is particularly valuable. In **table 2** the mean one-step-ahead forecast errors (MFE) and the appropriate mean squared forecast errors (MSFE) of the out-of-sample period are compiled. For comparison the in-sample forecast values are also reported.

Table 2

Comparison of monthly inflation rate forecasts

Forecast model	MFE	RMSFE	AC(1)	AC(2)	AC(3)	AC(4)	AC(12)
In-sample one-step-ahead forecasts from 1/1963 to 12/1994 (T = 384)							
ARMA(1,1) time series model	-0.000004 (0.0023)	0.00229	0.002 (0.002)	-0.006 (0.016)	-0.020 (0.178)	-0.048 (1.069)	0.002 (15.757)
Naïve interest rate model	-0.000050 (0.0025)	0.00248	0.144* (7.979*)	-0.096 (11.566*)	0.052 (12.619*)	-0.006 (12.634*)	-0.092 (21.210*)
ARMA(1,1) interest rate model	-0.000006 (0.0024)	0.00242	0.027 (0.274)	0.009 (0.304)	-0.015 (0.390)	-0.054 (1.527)	-0.112 (16.004)
Out-of-sample one-step-ahead forecasts from 1/1995 to 1/1999 (T = 49)							
ARMA(1,1) time series model	-0.00045 (0.00158)	0.00153	-0.088 (0.407)	-0.126 (1.244)	-0.021 (1.280)	-0.224 (4.180)	-0.175 (22.278*)
Naïve interest rate model	-0.00004 (0.00152)	0.00150	-0.170 (0.014)	-0.048 (0.137)	0.038 (0.215)	-0.144 (2.307)	-0.178 (18.982)
ARMA(1,1) interest rate model	0.00013 (0.00152)	0.00151	-0.077 (0.308)	-0.109 (0.937)	-0.009 (0.941)	-0.220 (3.639)	-0.175 (21.211*)

Notes: MFE indicates the mean forecast error (with estimated standard deviations in parentheses), RMSFE is the root of the mean squared forecast error. The AC(i) are the estimated autocorrelations of the residuals to lag i. With * marked values are significantly different from zero at the 5% level (with an absolute value larger than $1.96 / T^{0.5}$). The Ljung / Box Q-statistic for testing the null hypothesis that up to lag i no autocorrelation exists is reported in parentheses. Under the null hypothesis it is asymptotically χ^2-distributed with i degrees of freedom. Values marked with * are at the 5% significance level within the rejection region.

In both sample periods the MFE is not significantly different from zero whereby the estimated standard deviations do not differ in dimension within the same period. In-sample autocorrelations of the one-step-ahead forecast errors significantly differ from zero for the naïve interest rate model. This points to the fact that the model neglects information systematically. Furthermore, the time series model indicates the smallest MSFE in-sample. This once again repeats the regression results: Due to the highest R^2 within the single regressions in **table 1** the mean squared error (MSE) is necessarily smallest for the time series model. The MSE in turn represents a lower bound for the MSFE, which is exactly achieved, if the regression parameters reach the theoretical values of 0 and 1 – which applies at at best only for the time series model. Out-of-sample the MSFE of the three forecast models have the same dimension and the uncorrelatedness of the forecast errors at low lags cannot be rejected. A statistical inference concerning the superiority of one of the models out-of-sample can therefore not be derived from the forecast errors. Due to the constant term in the ARMA process, the time series model necessarily has a MFE of zero in the in-sample-period. This does not apply out-of-sample. Accordingly, it is remarkable that the MFE of the naïve interest rate model in the period 1/1963 to 12/1994 is around more than ten times as large as that of the time series model, out-of-sample the case is almost diametrically the opposite. Altogether, with the transition from the in-the-sample to the out-of-sample period a reduction of the forecast quality of the time series model relative to the interest rate models turns up. Thereby, especially the naïve interest rate model gains in attractiveness.

Finally in **table 3** a sub sample comparison of the one-step-ahead forecast error is employed in-sample to receive potential further explanation factors of the out-of-sample forecast performance. Therefore, the period from 1/1963 to 12/1994 is divided into four sub periods á 96 points of observation. It is shown again that the MFE is not significantly different from zero for all sub periods with the MFE of the time series model being smallest in absolute value in all periods. Out-of-sample the MFE is absolutely largest for the time series model. Altogether the in-sample results of the total period reaffirm themselves in the sub periods. The estimated standard deviations and the MSFE have the same dimensions within a sub period, which points to the fact that the forecast qualities of the different inflation models do not clearly differ. Slight advantages of the time series model concerning the MSFE can again be observed in all sub periods. However, as already discussed, these are in-sample construction dependent to a large extent.

Table 3

Sub sample comparisons of monthly one-step-ahead inflation forecasts

Forecast model	MFE	RMSFE	AC(1)	AC(2)	AC(3)	AC(4)	AC(12)
Sub period from 1/1963 to 12/1970 (T = 96)							
ARMA(1,1) time series model	-0.00004 (0.0026)	0.0026	-0.127 (1.589)	0.013 (1.607)	-0.004 (1.609)	-0.098 (2.588)	0.082 (12.32)
Naïve interest rate model	-0.00023 (0.0027)	0.0027	-0.039 (0.151)	0.045 (0.355)	0.013 (0.372)	-0.086 (1.130)	0.051 (9.202)
ARMA(1,1) interest rate model	0.00005 (0.0027)	0.0027	-0.116 (1.340)	0.015 (1.362)	-0.015 (1.385)	-0.101 (2.432)	0.103 (12.87)
Sub period from 1/1971 to 12/1978 (T = 96)							
ARMA(1,1) time series model	0.00017 (0.0023)	0.0023	-0.093 (0.862)	-0.004 (0.864)	-0.067 (1.312)	-0.147 (3.518)	-0.026 (17.65)
Naïve interest rate model	0.00020 (0.0027)	0.0027	0.223* (4.919*)	0.218* (9.663*)	0.110 (10.882*)	-0.011 (10.894*)	-0.143 (23.85*)
ARMA(1,1) interest rate model	0.00052 (0.0025)	0.0026	0.025 (0.063)	0.063 (0.460)	-0.024 (0.517)	-0.117 (1.920)	-0.078 (13.99)
Sub period from 1/1979 to 12/1986 (T = 96)							
ARMA(1,1) time series model	-0.00017 (0.0023)	0.0023	0.214* (4.540*)	-0.020 (4.578)	0.071 (5.092)	0.032 (5.198)	-0.027 (10.53)
Naïve interest rate model	-0.00024 (0.0024)	0.0024	0.228* (5.152*)	-0.010 (5.162)	0.040 (5.320)	-0.031 (5.416)	-0.108 (10.58)
ARMA(1,1) interest rate model	-0.00034 (0.0023)	0.0024	0.119 (1.398)	-0.122 (2.822)	-0.015 (2.905)	-0.079 (3.540)	-0.074 (9.755)
Sub period from 1/1987 to 12/1994 (T = 96)							
ARMA(1,1) time series model	0.000 01 (0.0020)	0.0020	0.009 (0.080)	-0.060 (0.369)	-0.199 (4.395)	-0.031 (4.496)	-0.015 (24.53*)
Naïve interest rate model	0.000 08 (0.0020)	0.0020	0.096 (0.914)	0.022 (0.961)	-0.110 (2.195)	0.094 (3.100)	-0.048 (17.14)
ARMA(1,1) interest rate model	-0.000 25 (0.0020)	0.0021	0.027 (0.071)	-0.041 (0.241)	-0.176 (3.362)	0.040 (3.524)	-0.011 (22.88*)

Notes: MFE indicates the mean forecast error (with estimated standard deviations in parentheses), RMSFE is the root of the mean squared forecast error. The AC(i) are the estimated autocorrelations of the residuals to lag i. With * marked values are significantly different from zero at the 5% level (with an absolute value larger than 1.96 / $T^{0.5}$). The Ljung / Box Q-statistic for testing the null hypothesis that up to lag i no autocorrelation exists is reported in parentheses. Under the null hypothesis it is asymptotically χ^2-distributed with i degrees of freedom. Values marked with * are at the 5% significance level within the rejection region.

Apart from this, it is interesting to observe that the estimated standard deviations of the mean forecast errors (MFE) and of the mean squared forecast errors (MSFE) decrease continuously over time. Also in the period from 1/1995 to 1/1999 this trend carries on. A reason for this may be that the variability of the historical seasonally adjusted inflation rate from one month to the next has sunk in time, so that inflation has become increasingly easy to forecast. This hypothesis is supported by the estimated standard deviations of the monthly changes of the inflation rate — with continually decreasing values of 0.0036, 0.0032, 0.0028 and 0.0026 for the in-sample sub periods and 0.0021 out-of-sample. For model comparison this has the disadvantage that the different forecast models can hardly be evaluated concerning their forecast qualities. We think that this is exactly why out-of-sample no statistical inferences of the relative forecasting abilities are possible. To that extent it seems to be of subordinate importance that the evaluation period contains only 49 monthly observations.

5. Conclusion and further research

Since the eighties at the latest, time series models have been evaluated as effective forecasting tools. Additionally, for inflation forecasting models which are based on the economical Fisher hypothesis have gained attention. For the analysed West German financial data a superiority of univariate time series forecasts can be construction-dependently confirmed in-sample, but not in the out-of-sample period. Because of the lack of variability in the present West German inflation rates, it is not statistically provable for the one-step-ahead forecast errors that the interest rate forecast models become more attractive. It may be supposed that particularly in the long run, i.e. for more than one-step-ahead inflation forecasts, interest rate models are favourable out-of-sample. But due to the low variability in the present West German inflation time series, a more extensive study must be undertaken by others.

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Endnotes

1 All tests in this study are based on an a priori level of significance of 5%.

2 For all estimations and tests in this study the software package EViews3.1 is used.

3 For problems of the approximation quality according to a large MA-parameter in the ARMA(1,1) model viz. Schwert (1989), Pantula (1991).

4 In the literature — in particular in the context of cointegration analysis — the inflation rate is also found to follow an I(1) process, viz. for example Nelson / Schwert (1977), Fama / Gibbons (1984), Barsky (1987), Crowder / Hoffmann (1996), Freeman (1998) for the USA and Carter (1991) for Australia.

5 The approach applies to discrete growth sizes X_t/X_{t-1}-1 approximately, with the use of the continuous equivalents ($\Delta \ln X_t$) it holds exactly.

6 The unit root test on the basis of an AR(12) approximation of the ARMA(1,1) process shows that the estimated AR coefficient is significantly smaller than one thus that the real interest follows a stable process. For problems of the AR approximation of an ARMA process viz. also note 3.

7 For the naïve interest rate model with factor n = 12 the first one-step-ahead forecast value exists for January 1963. Therefore the in-sample-period is uniformly only regarded here starting from 1/1963.