# "Market Endogenous Solvency and Dividend Policy"

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**Summary**

The value of insurance companies to the shareholders can be expressed in book values in the annual accounts, but it can also be assessed by evaluating the future expected cash flows to the owners of the firm. The paradoxical situation occurs that the shareholders' value approach guides many articles in modern insurance theory, but that dividend policy is not an issue which is addressed. This is probably due to the fact that Miller and Modigliani showed that in perfect capital markets dividend policy is irrelevant to the value of a company. Because insurance claims can be considered to be a peculiar form of corporate debt, the dividend irrelevance theorem of Miller and Modigliani might hold.

We, however, assume in this article that the market for insurance coverage and the market for the concomitant insurance debt is far from perfect. Clients of insurance companies are not able to analyze the solvency of insurance companies perfectly, implying that the price of insurance will be determined by insurance market conditions and not as much by the solvency of the insurers which offer the insurance cover. The lack of responsiveness of clients implies that the amount of equity as well as dividend policy may affect the value of the insurance company to the shareholders. We therefore try to analyze the optimum level of equity for some given dividend distributions with a computerized search routine as well as the results of alternative dividend policies with simulation techniques.

**« Solvabilité endogène du marché et politique de dividende »**

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**Résumé**

La valeur que représentent les compagnies d'assurances pour leurs actionnaires peut être exprimée en termes de valeur nette comptable, telle qu'elle est présentée dans les rapports annuels ; cependant, elle peut également être évaluée en déterminant les flux futurs de l'encaisse dont disposeront les propriétaires de la société. Il est paradoxal de constater que c'est l'approche des actionnaires qui guide la plupart des articles de la théorie moderne de l'assurance, et que la politique de dividende n'est pas prise en considération. Ceci est probablement dû au fait que Miller et Modigliani ont pu démontrer que dans les marchés de capitaux parfaits la politique des dividendes est sans rapport avec la valeur d'une entreprise. Le fait que les demandes d'indemnisation peuvent être considérées comme une forme de dette pour une société est susceptible de valider le théorème de Miller et Modigliani.

Nous supposons cependant, dans le présent article que le marché de la couverture d'assurance et le marché de la dette d'assurance correspondante sont loin d'être parfaits. Les clients des compagnies d'assurance ne sont pas en mesure d'effectuer une analyse parfaite de la solvabilité des compagnies d'assurance, ce qui implique que le coût de l'assurance est déterminé non tant par la solvabilité des assureurs que par les conditions particulières du marché de l'assurance. Le manque de réactions des clients semble indiquer que le montant des capitaux propres ainsi que la politique de dividende influencent la perception de la valeur d'une compagnie d'assurances aux yeux de ses actionnaires. Nous nous efforçons donc d'analyser le niveau optimum de fonds propres correspondant à divers dividendes distribués à l'aide d'un programme de recherche informatisé, et les résultats de diverses politiques de dividende à l'aide de techniques de simulation.

## 1. Introduction

The value of insurance companies to the shareholders can be expressed in book values in the annual accounts, but it can also be assessed by evaluating the future expected cash flows to the owners of the firm. The paradoxical situation occurs that the shareholders' value approach guides many articles in modern insurance theory (Fairley, 1979; Cummins, 1991; Doherty and Garven, 1986; Garven, 1992), but that dividend policy is not an issue which is addressed. This is probably due to the fact that Miller and Modigliani (1961) showed that in perfect capital markets dividend policy is irrelevant to the value of a company. Because insurance claims can be considered to be a peculiar form of corporate debt, the dividend irrelevance theorem of Modigliani and Miller might hold.

We, however, assume in this article that the market for insurance coverage and the market for the concomitant insurance debt is far from perfect. Clients of insurance companies are not able to analyze the solvency of insurance companies perfectly, implying that the price of insurance will be determined by insurance market conditions and not as much by the solvency of the insurers which offer the insurance cover. The lack of responsiveness of clients implies that the amount of equity and dividend policy may affect the value of the insurance company to the shareholders. In section 2 the value of an insurance company is analyzed in a multiperiod setting. In section 3 optimum levels of equity under different levels of premiums and risk are presented. Section 4 gives simulation results for various dividend policies. Section 5 presents some implications.

## 2. Value to the owners of an insurance company

This paper is based on ideas developed by Borch (1985a, 1985b). These are an extension and a simplification of an earlier publication of Borch (1972), who attributes the original idea to DeFinetti. In his 1985 papers Borch assumed that in an efficient capital market the shareholders fix the optimum amount of equity. Such an optimum amount of equity can be arranged by the shareholders if the company pays out all end of period equity above the optimum as dividends. This is the case when insurance results are favourable. When insurance operations reduce the optimum amount of equity, shareholders will furnish additional capital in order to attain the optimum again. This is only done if not all equity is lost. When, however, the losses exceed both premiums and equity together, the insurance company will stop its operations, because of insolvency.

*Assumptions* All amounts are measured at the end of each period. This refers to equity, premiums, claims and (positive or negative) dividends. This is not to say that for example premiums are paid at the end of the period. Premiums might be received during the period and premiums and equity might even be invested to create investment income. However, premiums, equity and the concomitant investment income are assumed to result into monetary amounts which can be measured with certainty at the end of the period and stochastic interest are therefore absent. The only stochastic variable is the total amount of claims to be paid at the end of the period.

The present (time 0, beginning of period 1) value of expected dividend-payments in period 1 is then:

$$(1) \qquad E_0(D_1) = \int_0^{P+Z} (1+r)^{-1} \, . \, (P-x) \, f(x) \, dx$$

where: $E_0$ = the expectations operator at time 0
$D_1$ = the amount of dividends received or paid at the end of period 1
$P$ = the amount of premiums received
$Z$ = the amount of equity
$x$ = the stochastic total claim amount
$f(.)$ = a density function
$r$ = the interest rate appropriate for the insurer considered.

The probability that the insurer will remain solvent can be found from the distribution function of x,(F(x)) and is indicated by F(P+Z). The probability of ruin R(P+Z) therefore equals 1-F(P+Z).

If the operations of the insurer do not change in scale nor in character, expected dividends during period 2 will equal the expected dividends during period 1: $E_1(D_2)=E_0(D_1)$. The invariance of scale and of character of operations and the behavioral assumption that the shareholders will invest the optimum amount of equity imply that the probability of ruin will be constant too. Nevertheless ruin threatens each period. The expected present (time = 0) value of dividends to be received at the end of period 2 therefore equals $(1+r)^{-1}.F(P+Z).E_1(D_2)$, which is $(1+r)^{-1}.F(P+Z).E_0(D_1)$.

*Present Value* Summing all present values of expected dividends gives the present value ($V_0$) of the company to risk neutral shareholders:

$$(2) \qquad V_0 = E_0(D_1) + E_0(D_1).(1+r)^{-1}.F(P+Z) + E_0(D_1).\{(1+r)^{-1}.F(P+Z)\}^2 + \ldots$$

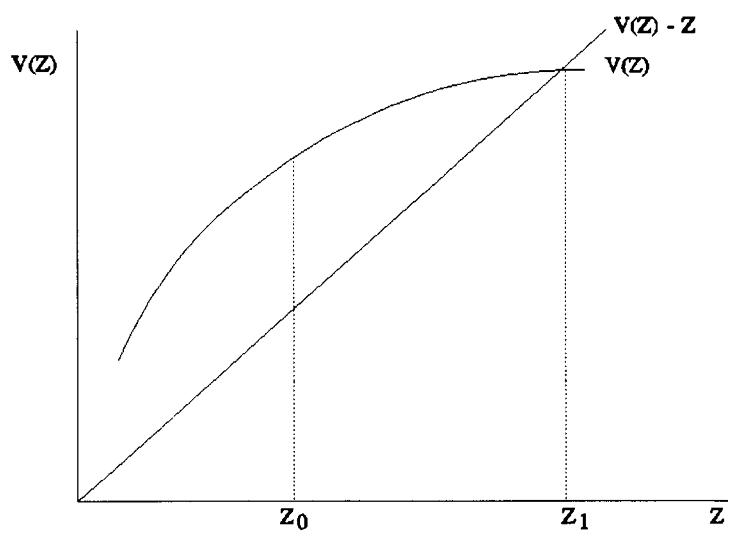
Because $(1+r)^{-1}.F(P+Z) < 1$ equation 2 can be rewritten as (See also Borch, 1985a, p. 205, equation 17 and Borch, 1985b, p. 12, eq. 8):

$$(3) \qquad V_0 = E_0(D_1)/\{1-(1+r)^{-1}.F(P+Z)\}$$

Equation 3 is also implicit in Scott (1976, p.38, eq. 1) and Aase (1990). Von Eije (1991b) also used this equation in deriving the optimum level of reinsurance cover.

*Trade-off Between Limited Liability Losses and Solvency Gains* The present value of the company $V_0$ is a function of the amount of equity Z. Expected end of period dividends $E_0(D_1)$ diminish if equity increases; the value of limited liability reduces because shareholders pay more losses before they accept the company to be insolvent. This means that if equity becomes infinitely large, the value of the option becomes zero and all expected claims will be paid[1]. The probability of ruin $R(P+Z)$ will be reduced too. If $E_0(D_1)$ is positive, a reduction in the probability of ruin will (ceteris paribus) increase the value of value of the company to the shareholders. Through increasing Z, a trade-off develops between value reduction via a less valuable limited liability option and value increases via solvency improvements. Value $V(Z)$ is thus a function of equity, (See Exhibit 1).

**Exhibit 1 The value of the insurance company as function of equity**




*Maximizing Value or Goodwill?* By putting the derivative of $V(Z)$ to zero the maximum value of $V(Z)$ is found when $V(Z)$ equals Z (at $Z=Z_1$ in Figure 1). Moreover, the function $V(Z)$ can be shown to be concave to the left of $Z_1$. The

[1] This implies that an increase in equity reduces the profit margin measured as the difference between premiums and expected payable claims.

maximum value will be found if company value equals the amount of equity invested in the company. Borch (1985b) then suggests that shareholders strive for a maximum company value, implying $Z_1$ to be the relevant optimum amount of equity.

Von Eije (1991a), however, mentioned that both value and equity are measured in monetary units. The investment of one additional unit of equity is therefore only considered if it creates at least one additional unit of company value. In Figure 1 net present value will be maximized when Z equals $Z_0$, i.e., in the point where $dV(Z)/dZ = 1$. Increasing the amount of equity above $Z_0$ will result in negative monetary returns. Above $Z_0$ one additional dollar invested in equity will result into an increase in market value of less than one monetary unit. Therefore, in stead of maximizing the value of the company, owners will be inclined to maximize the value of goodwill. This is the difference between company value and equity invested. If goodwill exists, the value of the company to the shareholders is higher than the amount of equity invested. Goodwill may, however, not be reaped in a competitive insurance market, because new entrants would be eager to use this money machine by investing equity and reaping net present value. Finding net present value and maximizing goodwill therefore implies that some barriers to entry exist (Carter, 1979, p. 167 ff.).

## 3. Optimum equity

As can be seen from equation 3 information is needed on expected dividends after taking into account limited liability, as well as on the interest rates (assumed to be equal to 6%) and the probability of ruin. However, the value of limited liability and the probability of ruin depend on the risk distribution, on the amount of equity invested and on the amount of premiums generated. A gamma distribution is frequently adequate in modelling the distribution of the total claim amounts (Beard, Pentikäinen and Pesonen, 1984):

(4) $f(x) = \{\lambda^m/\Gamma(m)\}.x^{m-1}.e^{-\lambda x}$ for $x \geq 0$, $\lambda > 0$ and $m > 0$

In order to reduce the number of calculations a low value for the parameter m of the gamma distribution (m=4) was chosen. We chose different values of lambda in order to consider the effects of changes in the gamma distribution. If lambda increases, the amount of expected losses ($m/\lambda$) and the variance ($m/\lambda^2$) diminish. For each Gamma distribution values in the interval (0, 10) which differed a fraction of 0.001 were found at each of which the probability of exceeding the value and the expectation of the truncated distribution above the value chosen was calculated[2]. We then searched that amount of equity which maximized the value of the goodwill of the shareholders (calculated according to equation 3) for given premium amounts. The optimum amounts of equity at various given premium amounts are presented in Table 1.

Table 1 indicates that for given premiums (e.g for P=3.5) and relatively high risk levels (small $\lambda$) the optimum amount of equity is zero. If risk and expected profits are at a medium level (for P=3.5 if $\lambda$ = 1.5) the optimum amount of equity becomes large. It then becomes worthwhile to protect goodwill and to discard with the value of the limited liability option. If, finally, risk is further reduced and profits increase ($\lambda$ at higher levels), the optimum amount of equity will diminish gradually.

It may be noticed that levels of equity which maximize shareholder's goodwill are not necessarily in accordance with solvency regulations of non-life insurance companies. Primarily the EC solvency regulations demand an amount of equity of

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[2] $$\int_a^\infty f(x)\,dx = \sum_{j=0}^{m-1} \exp(-\lambda a).(\lambda a)^j/j!$$

and

$$\int_a^\infty x\,f(x)\,dx = \sum_{j=0}^{m} \exp(-\lambda a).m.(\lambda)^{j-1}.(a)^j/j!$$

**Table 1 The amounts of equity ($Z_0$) which maximize goodwill to the shareholders for differences in risk ($1/\lambda$) and premiums (P).**

| $\lambda$ | P=2.5 | P=3.0 | P=3.5 | P=4.0 | P=4.5 |
|---|---|---|---|---|---|
| 1.0 | 0 | 0 | 0 | 0 | 0 |
| 1.5 | 0 | 0 | 3.2* | 3.5* | 3.4* |
| 2.0 | 2.1* | 2.6* | **2.5*** | 2.3* | 2.0* |
| 2.5 | 2.1* | 1.9* | 1.6* | 1.3* | 0.9* |
| 3.0 | 1.6* | 1.3* | 0.9* | 0.5 | 0.1 |

* indicates that the optimum level of equity satisfies the EC solvency regulations.

roughly 1/6 of premiums written (Berkouwer, 1992). Where insurers' shareholders prefer to use their limited liability rights above solvency (North-West part of Table 1), these rules will be more relevant to the clients, than in situations where shareholders prefer solvency and goodwill too (South-Eastern part of Table 1). In the latter situation insurers' value is not based on limited liability and clients may not need any protection, implying that the solvency rules could be refined by taking the insolvency risk explicitly into account.

The focal example is the situation in which lambda equals 2 (meaning that the expected claims equal 2) and in which the premiums equal 3.5. This generated an optimum amount of equity equal to 2.5 and an amount of goodwill of 21.8, implying that the value of the company equals 24.3. In this situation the annual probability of ruin becomes 0.00219. Comparing these results to those which were considered to be relevant by Borch, we find that the value of the company value is maximized if the amount of equity equals the value of the company, i.e. when the probability of ruin equals zero. The amount of equity which maximizes company value therefore equals for our focal example to $(P-E(L))/r = (3.5-2.0)/0.06 = 25$. It can thus be seen that the maximizing of the value of the company is very costly to the shareholders in this situation. They have to provide (25-2.5) = 22.5 additional equity in order to gain (25-24.3) = 0.7 in value, with a negative return of 97%.

## 4. Alternative dividend policies

Section 2 and 3 presented shareholders who were prepared to optimize solvency directly at the end of each year. In principle many alternative dividend strategies can be formulated. The following alternative groups of shareholders are analyzed:

- The first group are the direct optimizers by using stationary dividend policies. These shareholders show the behaviour presented in section 2. They directly distribute profits and directly refill losses up to the desired level of equity. For this group simulation results are given for starting values of equity which maximized the value according to Borch ($Z_1$ = 25) as well goodwill ($Z_0$ = 2.51).
- The second group consist of greedy shareholders. The greedy ones are assumed to accept in first instance the amount of equity available, but they will never refill it in case of losses and if the insurance company generates a profit, then these profits will be distributed.
- An intermediate, but more realistic alternative form the group of slow adaptors. They do not pay negative "dividends" if the amount of equity falls below the optimum, that is if $Z < Z_0$, in some year, but they accept that a proportion of future profits, $\alpha(P - x)$ is added to the equity until $Z = Z_0$. Further the slow adaptor's group accept a maximum ruin probability $R_T^{max}$

over a time horizon of T periods, which is smaller than the probability of ruin of the first group[3].

For these different forms of behaviour the present value of the dividend flows to the shareholders was calculated. The calculations were based on 100,000 random numbers. These numbers were applied to the Gamma distribution of our focal example, i.e. to the insurance company which was able to levy 3.5 premiums and to pay on average 2 in claims. From the random numbers 100,000 years of profits or losses were attributed to hundred companies with each a maximum possible life of thousand years. For the different forms of dividend policy the dividends paid to the shareholders were derived. After taking the present value and after summation we found for each of these companies the total value of the company based on shareholders' dividends. In Table 2 the average value of the results generated for the shareholders in 100 companies is shown, as well as the variance of the 100 values generated under each dividend policy and the number of companies which were ruined during a period of 10, 100 and 1000 years.

We must conclude that the simulation procedure seems to underestimate value. For the Borch results (first row) the estimated value over 100 companies was 24.8, while the theoretical value should be equal to 25. For the goodwill maximizing results we found a value of 23.8 in stead of the direct calculated optimum of 24.3. This effect is attributable to translation of random numbers to the claims generated with the Gamma distribution. Nevertheless the general simulation results are in line with the theoretical expectations.

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[3] At time t and equity $Z_t$ the ruin probability for period t is $R(Z_t + P) = R_t$. Given $R_t$ the ruin probability over T periods (T > 1) is $\{1 - (1 - R_t)^T\} = R_{t,T}$. When $R_{t,T} > R_T^{max}$ additional equity is provided at the amount of $\alpha(P - x)$ only when $(P - x) > 0$ with $0 \leq \alpha \leq 1$. In the simulation study we have taken $\alpha = 0.2$, $T = 10$ and $R_{T=10}^{max} = 0.00219$, which is the one year probability of ruin $R(Z_0 + P) = R(2.5+3.5)$ of our focal example from Table 1.

**Table 2 The simulation results for 100 companies under different forms of dividend policy.**

| assumptions | initial equity | average value | variance | ruined in 10 years | ruined in 100 years | ruined in 1000 years |
|---|---|---|---|---|---|---|
| Borch | 25 | 24.8 | 7.5 | 0 | 0 | 0 |
| Optimizers | 2.5 | 23.8 | 27.7 | 3 | 20 | 100 |
| Greedy | 2.5 | 22.0 | 38.0 | 6 | 91 | 100 |
| Slow adaptors | 2.5 | 18.0 | 14.6 | 3 | 3 | 3 |

It should be noted that the relatively small difference in expected value between value maximizers and goodwill maximizers is not reflected in small differences in variance which is much higher for the goodwill maximizing shareholders. This large variance is in line with the relatively large number of goodwill maximizing companies which breaks down during the first ten years of operating. This can be attributed to the fact that goodwill maximizing shareholders do not try to protect themselves from ruin with all means: they only aim at maximizing goodwill. However, if goodwill is optimized all companies become bankrupt during the period of 1000 years, while in the situation where shareholders maximize value directly no company will ever fail. Table 2 also shows that direct optimizing equity is the best dividend policy in terms of expected value.

## 5. Concluding implications

In an imperfect insurance market the value of a company to its shareholders depends on the characteristics of the cash flow distribution, on the amount of equity invested and on the relevant interest rate. In a multiperiod analysis the amount of equity affects both the value of the limited liability option as well as the present value of expected future profits. It was then shown that shareholders normally prefer to maximize goodwill, and not company value. Optimum levels of equity may conflict with the regulations of the European Community, which may be adequate in protecting the clients, but which may force too much equity to be invested in low risk insurers. Dividend policy which directly optimizes goodwill is shown to be better than to be greedy with respect to value as well as to insolvency risk. Under a more realistic dividend policy shareholders are prepared to offer some profits for the improvement of the solvency of the insurer but not to refill losses. This reduces value, but because it also reduces the probability of ruin it might preferred by risk averse shareholders as well as other (risk averse) stakeholders.

In our opinion this paper generates four general implications.

1. Risk neutral shareholders will maximize goodwill if the insurance market is imperfect, i.e. if clients do not react to solvency and if barriers to entry exist.
2. If solvency regulators would have knowledge of the aggregate risk profile of insurers, they might be able to discriminate between shareholders who prefer the survival of the insurance company and those who do not and to require only large amounts of equity to be invested by the latter group.
3. Managers of insurance companies should know the aggregate risk profile too, in order to satisfy shareholders' needs.
4. Realistic dividend policies diminish expected value to risk neutral shareholders, but may be preferred by risk avers shareholders as well as other risk averse stakeholders, like clients, managers and regulators.

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