"The Demand for Equity and Reinsurance by Non-life Insurance Companies if Consumers React and Interest Rates Change"

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Summary

This paper presents a model in which insurance cover is demanded by the clients of non-life insurance companies. The clients remunerate insurers at competitive market prices if the capacity of the insurer equals the demand for cover. Clients are, however, prepared to reward individual insurers for holding capacity in excess of insurance cover demanded, because this will increase the probability that their claims will be paid in full. Nevertheless, insurers do not offer an unlimited amount of capacity, because the generation of capacity from equity and reinsurance cover is costly. For plausible pricing relationships, the present value of expected losses is then shown to be relevant in determining reinsurance and equity demanded by primary insurers. This implies that interest rate elasticities exist for equity and reinsurance cover which contain the Macaulay-duration of expected losses. The model then indicates that insurers tend to substitute equity for reinsurance cover if interest rates rise, though the impact will be less in companies which insure long tail business.

« Demande de capitaux propres et de réassurance des compagnies d'assurance autre que l'assurance-vie en cas de réaction des clients et d'évolution des taux d'intérêt. »

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Résumé

On présente ici un modèle selon lequel une couverture est sollicitée par les clients de compagnies d'assurance autre que l'assurance-vie. Les clients rémunèrent les assureurs à des tarifs alignés sur le marché si les capacités de ceux-ci correspondent à la demande de couverture. Ils sont toutefois prêts à dépasser ces tarifs pour les assureurs dont les capacités dépassent la couverture demandée, étant donné que cette condition augmente la probabilité d'un paiement intégral des demandes d'indemnité. Cela étant, les assureurs ne disposent pas de capacités illimités, la création de capacité par l'accroissement des capitaux propres et par la réassurance étant coûteuse. Pour établir des correspondances de prix plausibles, la valeur actuelle des pertes prévues est pertinente pour déterminer les demandes de capitaux propres et de réassurance des assureurs directs. Ceci implique qu'il existe des élasticités de taux d'intérêt pour les capitaux propres et la réassurance où intervient la duration Macaulay des pertes prévues. Le modèle indique donc que les assureurs tendent à substituer les capitaux propres à la couverture par réassurance lorsque les taux d'intérêt montent, mais que cette tendance est atténué dans le cas des compagnies qui assurent des entreprises pour le long terme.

1. Introduction

The insurance industry is confronted with an increasing number of bankruptcy announcements [Swiss Re, 1992, p. 3; Best's Rating Monitor 1992, p. 3]. Insolvencies in the insurance industry are mainly caused by unexpected large amounts of claims, which consume all surplus. Insurance companies therefore try to reduce the probability of becoming insolvent by accumulating equity or by reducing the volatility of claims by reinsuring, or -in other words- by creating an adequate level of solvency. The larger the amount of equity invested in the primary insurer and the larger the quantity of reinsurance cover obtained, the better a primary insurance company is able to offer insurance cover to its clients and the better it can survive. In this article we will go into the demand for equity and reinsurance cover by individual insurance companies. We present a model in which insurance cover is demanded by clients who are prepared to reward improvements of solvency. An unlimited amount of solvency is, however, not offered by primary insurers, because of equity and reinsurance costs.

In section 2 we indicate that reinsurance cover may be relevant if clients of insurance companies are able to perceive insolvency risks. Because we want to model the effects of solvency improvements explicitly, we present a cash flow model which incorporates such effects in section 3. It is then shown in section 4 that the present value of expected losses, the interest rate and the quantity of insurance demanded are relevant in determining both the quantities of reinsurance and of equity demanded. Section 5 shows that insurers tend to substitute equity based solvency for reinsurance cover if interest rates rise, though such effects will be less in companies which insure long tail business. In section 6 we discuss some further implications and testability problems. Section 7 present a summary and the conclusions.

2. Financial paradigms and solvency improvements

According to the CAPM [Sharpe, 1964] only a reduction in systematic risk would be valuable to shareholders of a primary insurance company. If there would exist systematic risk in the insurance portfolio, a reduction of this risk by reinsurance might be considered. In efficient markets a transfer of systematic risk to the reinsurer can, however, not be made at prices below market prices. Therefore, in the absence of bankruptcy costs[1], the value of the ceding company cannot be improved by reinsurance. Nevertheless, perfect capital markets do not imply perfect insurance markets. In order to be able to analyze the value of reinsurance to the shareholders, it might therefore be useful to analyze the insurance market reactions to solvency improvements. Doherty and Tinic [1981] showed that reinsurance is irrelevant if clients do not react to improved solvency. If, however, clients are able to evaluate the solvency of insurance companies correctly, or if a positive relationship exists between actual and perceived solvency [Tapiero, Kahane and Jacque, 1986], they may react to improved solvency. Taking out more reinsurance cover may then also allow the ceding insurer to charge higher premiums to its clients. Such "demand induced reinsurance" may benefit the owners of the ceding company too. In our approach we will therefore explicitly consider the pricing effects of additional capacity generated by reinsurance or equity.

In order to do so, we first considered using the option pricing paradigm of Black and Scholes [1973] to model these relationships. Such models are recently used in the analysis of (mainly non-life) insurance companies [Doherty and Garven, 1986; Cummins, 1988; Cummins 1991; Garven, 1992]. Interesting features of these models are that they combine both an explicit measure of total risk as well as capital structure in a consistent framework. Additional reinsurance cover would reduce risk

[1] If bankruptcy costs [Baxter,1967; Warner, 1977] are relevant then an increase in solvency may in some circumstances affect shareholders' or shareholders clients' value positively [Von Eije, 1989; Hoerger, Sloan & Hassan, 1990].

and additional equity would reduce leverage. We, nevertheless, discarded the use of the option pricing theory in this case, because it models the relevant relationships with given cash flow characteristics. This makes this theory less versatile in incorporating changes in cash flow characteristics, like those generated by improvements in solvency. Moreover, leverage in the option pricing theory is based on market values. The market value of shares incorporates the gains of improved solvency, based amongst others upon both increases of reinsurance cover as well as of the amount of equity. It will therefore be clear that shareholder market value will not be equal to the amounts of equity which are relevant in determining solvency[2].

The best thing to do would then be making the relation between the equity and shareholders' value explicit. This can, under very general assumptions on the total claim distribution, be done with the discounted cash flow approach of e.g. Borch [1985], Aase [1990] and Von Eije [1991]. Their approach, however, assumes the dividends (defined as the difference between premiums and total claims) to be given, implying that also this approach is inconsistent with our main idea of modelling the clients' reactions to solvency improvements explicitly.

We finally decided to utilize the traditional neoclassical production function paradigm in combination with a cash flow model. We consider the relevant cash flows of a primary insurance company like premiums, reinsurance premiums, investment income and costs to be direct disbursable flows in contradiction to the insurance and reinsurance claims, for which we take the expected present value of the future flows. We then assume that the clients of the primary insurer are able to correctly evaluate the capacity of the primary insurer and that they pay more for the cover of a company with excess capacity then for a company with a capacity shortage. Capacity can be produced by two production factors "equity" and

[2] It should be noted that the amount of equity relevant in determining the level of solvency will not necessarily be equal to the book value of equity presented in the annual accounts of the insurer.

"reinsurance cover", which are costly to the primary insurer. Profit maximization then generates the optimum levels of equity and reinsurance quantities[3].

3. The cash flow model used

In our model the quantity of insurance cover demanded is given. In order to sell insurance cover, insurance companies need capacity. Excess capacity exists if the capacity is larger than the quantity of insurance cover demanded, while vice versa a capacity shortage would exist. If the consumers perceive the ratio between the available capacity C and the total quantity of insurance sold by the insurance company q_v to be the relevant solvency measure, we may define the price of insurance p_v as follows:

$$(1) \qquad p_v = p_0 \cdot (C/q_v)^\tau \qquad \tau \geq 0$$

In equation 1, p_0 can be considered to be the competitive price for insurance cover in situations where capacity is fully utilized, i.e. where $q_v = C$. If the primary insurer increases capacity, she will according to equation 1 be rewarded with a higher price. The value of τ indicates how excess capacity (measured by the ratio C/q_v) is translated into higher prices. It can be considered to represent the appreciation of excess capacity by the clients. The special situation with $\tau = 0$ indicates that excess capacity is not rewarded; it would in fact then be slack.

We now assume capacity C to be generated with a Cobb-Douglas production function which transforms the quantities of reinsurance q_h and equity invested E into capacity:

$$(2) \qquad C = A \cdot q_h^\alpha \cdot E^\beta \qquad A > 0, \alpha \geq 0, \beta \geq 0$$

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[3] In our analysis we distinguish between quantities and prices of equity and reinsurance, though normally the amount of reinsurance premiums is considered to indicate the level of cover.

In equation 2, α and β indicate the relative importance of the two production factors, while A represents an efficiency parameter. In order to overcome diminsion problems we assume constant returns to scale, i.e. $\alpha + \beta = 1$.

Very frequently, reinsurance cover is measured as a flow of reinsurance premiums. As mentioned before, we want to discern between the quantities of reinsurance invested in the capacity generating process and the price of reinsurance cover. We therefore define the quantity of reinsurance as a ratio times the quantity of insurance demanded. The ratio which in our opinion is most relevant is the (present value) of expected claims to be paid by the reinsurer divided by the (present value) of total expected claims:

$$(3) \qquad q_h = q_v \cdot \{E(S_h)/E(S)\}^{\psi} \qquad \psi > 0$$

The constant ψ indicates that there is not necessarily a linear relationship between increases in the present value of expected claims and increases in the present value of expected recoverable claims, implying that non-proportional reinsurance cover might exist. We, moreover, explicitly consider in equation 3 the possibility that not all claims are paid directly. Because we measure all amounts to be received or to be paid at the beginning of the period, we must discount future claims. The equivalent amount to be paid at the beginning of the period can be found by discounting future claims. We assume systematic risk factors to be absent and consider (re)insurers to be able to determine the expected present values for given interest rates r. E(S) therefore equals $\Sigma\{E(S_t)/(1+r)^t\}$ and $E(S_h)$ is defined equivalently as $\Sigma\{E(S_{ht})/(1+r)^t)\}$. So E(S) and $E(S_h)$ are the present values of all the expected claims to be paid by the primary insurance company and the reinsurance company respectively.

In return for accepting risks, the primary insurance company receives premiums. The premiums received, P, depend on the present value of the expected claim-payments:

(4) $P = d \cdot E(S)$ $d > 1$

Normally, the amount of premiums received will be larger than the present value of expected claims, i.e. $d > 1$. We are now able to define a relationship between the total amount of premiums received and the generated price and quantity of insurance[4].

(5) $p_v = P/q_v$

The primary insurance company may reinsure part of its risks for capacity reasons. Because the reinsurance company expects reinsurance to be offered at a profit, the reinsurance premium H will be above the present value of the expected claims to be paid by the reinsurer:

(6) $H = b \cdot E(S_h)$ $b > 1$

The reinsurance premiums are the outlays of the primary insurance company for capacity generated by reinsurance. In using reinsurance, the primary insurer has to pay reinsurance premiums and afterwards he is rewarded with a possible reduction in claims. The expected costs of reinsurance are the difference between the reinsurance premiums paid and claims expected to be recovered. Reinsurance would therefore (because $b>1$) be a direct waste to the primary insurance company if it would not be rewarded with increases in capacity which induce clients to pay higher prices. Irrespective of these capacity effects the reinsurance price must now be defined. We could define it as a price related to net expected reinsurance costs or

[4] Equations 1-5 imply that both capacity and the quantity of insurance demanded are measured in monetary units. If $C=q_v$ (or if $\tau = 1$) we find the following relation between capacity and expected claims: $C = (d/p_0).E(S)$. If capacity is larger than expected claims, the technical profit ratio d will be larger than the competitive market price at full capacity utilization p_0.

as a price related to reinsurance premiums. We have chosen to do the latter in order to retain the symmetry between the price definition in insurance and in reinsurance:

(7) $$p_h = H/q_h$$

Total capacity is a function of both reinsurance and equity, and not only reinsurance is costly, but equity is too. We assume the price of equity to be dependent on the interest rate level r.

(8) $$p_E = a \cdot r \qquad a > 0$$

Total costs of equity equal $p_E \cdot E$. The price of equity can be seen as the shadow costs of allocating equity in its best alternative use.

Equity and reinsurance, however, do not only generate capacity, but also cash flows. The cash flows arising from reinsurance cover diminish the present value of the total claim amount with $E(S_h)$. The available (investable) funds, which include E, can be invested and thus generate investment income. We determine this investment income I as follows[5]:

(9) $$I = f \cdot r \cdot (E + P - H) \qquad 0 < f < a$$

In this equation we assume that the insurer will not be able to structurally beat the financial market ($f < a$). This means that some transaction costs must be made in obtaining and consecutively reinvesting the amount of equity. Nevertheless, the insurer may still be able to outperform non-financial institutions, because the cash flow residual (P-H) is frequently obtained in an imperfect insurance market at zero or below capital market costs.

[5] Positive or negative interest income on insurance and reinsurance claim reserves should not be taken into account, because interest flows are taken into account in the present value calculations of E(S) and $E(S_h)$.

Apart from all costs mentioned above, an insurance company also has other expenses such as administration costs, costs of acquiring clients, etcetera. We assume these expenses X to be related to the quantity of insurance (the amount of business) q_v. The expenses are assumed to be paid at the beginning of the year.

$$(10) \quad X = e \cdot q_v \qquad e > 0$$

The expected profit E(W) in a given year is a function of the above-mentioned variables.

$$\begin{aligned}(11) \quad E(W) &= P - H - p_E \cdot E + E(S_h) - E(S) + I - X \\ &= p_v \cdot q_v - b \cdot q_h^{1/\psi} . q_v^{(\psi-1)/\psi} \cdot p_v/d - a \cdot r \cdot E + q_h^{1/\psi} \cdot q_v^{(\psi-1)/\psi} \cdot p_v/d \\ &\quad - p_v \cdot q_v/d + f \cdot r \cdot (E + p_v \cdot q_v - b \cdot q_h^{1/\psi} \cdot q_v^{(\psi-1)/\psi} \cdot p_v/d) - e \cdot q_v\end{aligned}$$

In the next section we will look for the levels of equity and reinsurance which optimize the expected profit given a fixed capacity level.

4. Optimum levels of equity and reinsurance cover

Where Fairley [1979] analyzed the fair price of equity, we analyze the optimal levels of equity (and reinsurance) demanded given the prices of these capacity generators. We use the Lagrange's method by maximizing the expected profits E(W) subject to the capacity restriction $C = A \cdot q_h^{\alpha} \cdot E^{\beta}$. In equation 11 we substitute equation 1 for p_v, with equation 2 substituted for C in equation 1. The Lagrangian is:

$$
\begin{aligned}
L = \;& (p_0 \cdot A^{\tau} \cdot q_h^{\alpha\tau} \cdot E^{\beta\tau} \cdot q_v^{-\tau}) \cdot \\
& (q_v - b \cdot q_h^{1/\psi} \cdot q_v^{(\psi-1)/\psi}/d + q_h^{1/\psi} \cdot q_v^{(\psi-1)/\psi}/d - q_v/d + f \cdot r \cdot q_v - f \cdot r \cdot b \cdot q_h^{1/\psi} \cdot q_v^{(\psi-1)/\psi}/d) \\
& - e \cdot q_v + (f - a) \cdot r \cdot E - \lambda \cdot (A \cdot q_h^{\alpha} \cdot E^{\beta} - C)
\end{aligned}
$$

We want to determine the optimal q_h and E, so we have to take partial derivatives of L to q_h and E:

$$
\begin{aligned}
\partial L/\partial q_h = \;& (\alpha \cdot \tau/q_h) \cdot p_0 \cdot A^{\tau} \cdot q_h^{\alpha\tau} \cdot E^{\beta\tau} \cdot q_v^{-\tau} \cdot \\
& (q_v - b \cdot q_h^{1/\psi} \cdot q_v^{(\psi-1)/\psi}/d + q_h^{1/\psi} \cdot q_v^{(\psi-1)/\psi}/d - q_v/d + f \cdot r \cdot q_v - f \cdot r \cdot b \cdot q_h^{1/\psi} \cdot q_v^{(\psi-1)/\psi}/d) \\
& + p_0 \cdot A^{\tau} \cdot q_h^{\alpha\tau} \cdot E^{\beta\tau} \cdot q_v^{-\tau} \cdot (- b \cdot q_h^{(1-\psi)/\psi} \cdot q_v^{(\psi-1)/\psi}/(d \cdot \psi) \\
& + q_h^{(1-\psi)/\psi} \cdot q_v^{(\psi-1)/\psi}/(d \cdot \psi) \\
& - f \cdot r \cdot b \cdot q_h^{(1-\psi)/\psi} \cdot q_v^{(\psi-1)/\psi}/(d \cdot \psi)) - \lambda \cdot (\alpha/q_h) \cdot A \cdot q_h^{\alpha} \cdot E^{\beta} = 0
\end{aligned}
$$

$$
\begin{aligned}
\partial L/\partial E = \;& (\beta \cdot \tau/E) \cdot p_0 \cdot A^{\tau} \cdot q_h^{\alpha\tau} \cdot E^{\beta\tau} \cdot q_v^{-\tau} \cdot \\
& (q_v - b \cdot q_h^{1/\psi} \cdot q_v^{(\psi-1)/\psi}/d + q_h^{1/\psi} \cdot q_v^{(\psi-1)/\psi}/d - q_v/d + f \cdot r \cdot q_v - f \cdot r \cdot b \cdot q_h^{1/\psi} \cdot q_v^{(\psi-1)/\psi}/d) \\
& + (f - a) \cdot r - \lambda \cdot (\beta/E) \cdot A \cdot q_h^{\alpha} \cdot E^{\beta} = 0
\end{aligned}
$$

$$
\partial L/\partial \lambda = \quad A \cdot q_h^{\alpha} \cdot E^{\beta} - C = 0
$$

Rewriting gives:

$$
\begin{aligned}
(12) \quad & (q_h/E) \cdot p_0 \cdot A^{\tau} \cdot q_h^{\alpha\tau} \cdot E^{\beta\tau} \cdot q_v^{-\tau} \cdot (q_v/q_h)^{(\psi-1)/\psi} \\
& = (\alpha \cdot \psi/\beta) \cdot d \cdot (f - a) \cdot r/(1 - b - f \cdot r \cdot b)
\end{aligned}
$$

The left side of this equation is positive in case equity and reinsurance are acquired, so there only exist a solution to this equation if (f - a) and (1 - b - $f \cdot r \cdot b$) have the same sign (the other factors on the right side are positive). In our opinion this is not too restrictive, because both terms will normally be negative, unless the primary insurer is able to structurally beat the capital market. The latter situation is already discarded with in equation 9, where $f < a$.

From equations (2) and (12) and using equation 1 for C (that is $C = (p_v/p_0)^{1/\tau} \cdot q_v$) we can deduce equations for E and q_h:

$$(13) \quad E = A^{-1/(\alpha \cdot \psi + \beta)} \cdot [E(S) \cdot \{\beta/(\alpha \cdot \psi)\} \cdot (f \cdot r \cdot b + b - 1)/\{(a - f) \cdot r\}]^{\alpha \cdot \psi/(\alpha \cdot \psi + \beta)} \cdot C^{1/(\alpha \cdot \psi + \beta)} \cdot q_v^{-\alpha/(\alpha \cdot \psi + \beta)}$$

$$(14) \quad q_h = A^{-\psi/(\alpha \cdot \psi + \beta)} \cdot [E(S) \cdot \{\beta/(\alpha \cdot \psi)\} \cdot (f \cdot r \cdot b + b - 1)/\{(a - f) \cdot r\}]^{-\beta \cdot \psi/(\alpha \cdot \psi + \beta)} \cdot C^{\psi/(\alpha \cdot \psi + \beta)} \cdot q_v^{\beta/(\alpha \cdot \psi + \beta)}$$

Though these equations indicate relationships between various variables and both equity and reinsurance cover, the economic meaning of the relationships is not immediately clear because of interdependencies between the determining variables. We therefore analyzed the expansion path equation under proportional reinsurance cover conditions (i.e. for $\psi = 1$). We then find:

$$(15) \quad E/q_h = \{E(S)/q_v\} \cdot (\beta/\alpha) \cdot (f \cdot r \cdot b + b - 1)/\{(a - f) \cdot r\}$$

In this equation we are left with three relevant variables, namely the expected amount of claims E(S), the demand for insurance cover q_v and the interest rate r. Because of the positive right hand side of this equation, we conclude that for exogenously given levels of insurance demand an increase in the present value of expected insurance claims increases the demand for equity relative to reinsurance.

5. Effects of interest rate changes

In equations (13) and (14) the relevant elements (besides the parameters and the variables related to excess capacity C and q_v) are the interest rate as well as the expected present value of the claims, which is also determined by the interest rate. This means that interest rate changes would both affect equity and reinsurance demand. We are therefore interested in the interest rate elasticities of demand of E and q_h. To calculate these elasticities we first take the logarithm on both sides of equations (13) and (14):

$$\begin{aligned}(13') \quad \ln E = {} & \{\alpha\cdot\psi/(\alpha\cdot\psi+\beta)\}\cdot \\ & \{\ln\beta - \ln\alpha + \ln E(S) + \ln[(f\cdot r\cdot b + b - 1)/(a - f)] - \ln r - \ln\psi\} \\ & - \{1/(\alpha\cdot\psi+\beta)\}\cdot \ln A + \{1/(\alpha\cdot\psi+\beta)\}\cdot \ln C \\ & - \{\alpha/(\alpha\cdot\psi+\beta)\}\cdot \ln q_v\end{aligned}$$

$$\begin{aligned}(14') \quad \ln q_h = {} & \{\beta\cdot\psi/(\alpha\cdot\psi+\beta)\}\cdot \\ & \{\ln\alpha - \ln\beta - \ln E(S) - \ln[(f\cdot r\cdot b + b - 1)/(a - f)] + \ln r + \ln\psi\} \\ & - \{\psi/(\alpha\cdot\psi+\beta)\}\cdot \ln A + \{\psi/(\alpha\cdot\psi+\beta)\}\cdot \ln C \\ & + \{\beta/(\alpha\cdot\psi+\beta)\}\cdot \ln q_v\end{aligned}$$

We do not have to fear division by zero, because by assumption $f\cdot r\cdot b + b \neq 1$ and $f \neq a$. Taking the derivative of both equations to lnr, we get:

$$\begin{aligned}(16) \quad \epsilon_r^E & = \partial\ln E/\partial\ln r \\ & = \{-\alpha\cdot\psi/(\alpha\cdot\psi+\beta)\}\cdot\{1 + D\cdot r/(1 + r) - f\cdot b/(f\cdot r\cdot b + b - 1)\} \\ (17) \quad \epsilon_r^{qh} & = \partial\ln q_h/\partial\ln r \\ & = \{\beta\cdot\psi/(\alpha\cdot\psi+\beta)\}\cdot\{1 + D\cdot r/(1 + r) - f\cdot b/(f\cdot r\cdot b + b - 1)\}\end{aligned}$$

where D is the Macaulay-duration of the run-off process of the expected claims[6].

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[6] The Macaulay-duration in general equals $D = \{\Sigma t\cdot K_t/(1 + r)^t\}/\{\Sigma K_t/(1 + r)^t\}$ where K_t is equal to the total cash flow of the process in year t. In this situation $K_t = E(S_t)$. It may, moreover, be noted that equation 16 applies to reinsurance quantities; if we would calculate the interest elasticity of the reinsurance premiums paid, we would find:

So the interest rate elasticities of demand of equity and reinsurance quantities depend on the duration of the expected claims. We expect the sign of the right hand part of equations 16 and 17 to be positive respectively negative (except for situations in which interest rates are very high or the duration extremely large). This implies that interest rate increases tend to induce insurers to substitute equity for reinsurance cover. Because D is higher in case a larger part of the claims is paid in later years, the elasticities will normally be smaller for long tail business than what they are for short tail business. So the reactions to interest rate changes will then be less for long tail business than for short tail business. However, in countries where interest rates are high, an increase in these rates may induce insurers which are confronted with long run-off processes (high Macaulay-durations) to reduce their equity demand in favour of reinsurance cover.

6. Further implications

The model can be considered to contribute not only to the literature on de determinants of the demand for equity and reinsurance cover by primary insurance companies, but also to that on insurance capacity[7] [Stone, 1973; Doherty, 1980; Nielson, 1984; Nielson and Grace, 1988; Eden and Kahane, 1988]. Compared to the traditional actuarial literature where capacity is based on exogenous acceptable probabilities of ruin[8], solvency is endogenous in our model. Compared to the

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$\epsilon_r^H = \partial \ln H/\partial \ln r = D_h \cdot r/(1+r)$, where D_h = the Macaulay-duration of the reinsurance claims with $K_t = E(S_{ht})$.

[7] A distinction can be made between market capacity and individual (insurance company) capacity. Our implications focus on individual capacity.

[8] One example is Stone [1973, p. 236], who defines capacity as the leeway between the maximum and the actual exposure ratio (i.e the ratio of the standard deviation to the mean value of the loss) of the portfolio, where the maximum exposure ratio is determined by a stability constraint. According to Stone, the insurance industry can be described as maximizing profit subject to a survival and a stability constraint. The survival

incremental approach of Doherty, who defines capacity as the maximum acceptable weight which can be assigned to an incremental policy whilst maintaining the existing performance level[9], we consider capacity to be fully utilized if the level of solvency in relationship to the demand for insurance cover is such that the price of insurance cover of the individual company equals the competitive market price of that cover. We are, finally, in line with the research of Nielson and Grace [1988], who take reinsurance cover explicitly into account.

Our theoretical work, moreover, discards with traditional substitution relationships between equity and reinsurance premiums paid[10] because we distinguish between the quantities of reinsurance demanded and the relevant price of the quantity demanded.

If we want to test our theory in practice, we will have to deal with problems of measurability first. An important problem is that normally only the total amount of ceded premiums H is measurable and not quantity q_h and price p_h. Moreover, the amount of equity is not available except for book values. In general book values may be very dangerous to use because of window dressing by the primary insurer. Nevertheless, we think that reinsurers who work internationally may be able to test some of our theoretical results. These results indicate that the reactions to interest

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constraint implies a probability of less than P_1 that losses and expenses in a given time period will exceed the total of new premium income and capital funds, and the stability constraint implies a probability below P_2 that the combined ratio (loss ratio + expense ratio) will exceed some target value by X percentage points in a year. The problem then arises in determining P_1, P_2 and X.

[9] Doherty uses the Sharpe measure $S = (\mu_p - r_f)/\sigma_p$ as measure of performance, where μ_p is the expected return on equity, σ_p is the standard deviation and r_f is the risk free rate of return. In this approach, optimum capacity would be found if μ_p equals the expected rate of return on the market portfolio. This is the situation where the Sharpe measure for the individual primary insurance company equals the slope of the Capital Market Line. In general, S will be smaller than the CML-slope. Individual primary insurance companies may then use different values of S.

[10] These relationships, which are somewhat intuitive [Carter, 1983], can also be found in the non-life solvency regulations of the European Community [Berkouwer, 1992].

rate changes differ between countries with extremely high interest rates and those with more normal interest rates, while also long tail insurers will react differently from short tail insurance companies.

7. Conclusions

In this article we analyzed the demand for equity and reinsurance cover by primary insurance companies. It was shown that paradigms of financial theory may not be helpful in analyzing both equity and reinsurance demand. We therefore resorted to a cash flow model. In this model capacity is produced by equity and reinsurance quantities. It is then assumed that clients are prepared to reward individual insurers for holding capacity in excess of insurance cover demanded, because this will increase the probability that their claims will be paid in full. Nevertheless, insurers do not offer an unlimited amount of capacity, because the generation of capacity from equity and reinsurance cover is costly. For plausible pricing relationships, the present value of expected losses as well as the interest rate and the quantity of insurance demanded are relevant in determining reinsurance and equity demanded by primary insurers. We then analyzed the impact of interest rate changes. It was shown that interest rate elasticities exist for equity and reinsurance cover which contain the Macaulay-duration of expected losses. The model then indicates that insurers tend to substitute equity for reinsurance cover if interest rates rise, though the impact will be less in companies which insure long tail business.

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