On Stochastic Modeling of Inflation

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Summary

The aim of scientific modeling is to express the essential features of a real world phenomenon in the form of a model which should be as simple as possible. In many cases model adequacy is tested by its forecasting ability. On the other hand, in several applications the process itself and the resulting model may contain elements which are not forecastable. Abrupt changes in the inflation caused e.g. by the two oil crises in the middle of the seventies and in the beginning of the eighties are examples of such unpredictable events.

In recent years observed inflation series have been often described by an AR(1) model, i.e. the first order autoregression. During the last decades the inflation series also reflect the two oil crises. Besides these, the inflation series also reflect the effects of a number of political or labour market decisions made in the country in question. This means that the innovation series of the corresponding linear models can hardly be stationary. Therefore it is obvious that the innovations are generated by time dependent-distributions.

In principle, the innovations can be explained by various factors. In this study we have used an oil price index as an explanatory variable in the inflation model. Transfer function noise models are built for the inflation series of 11 countries. They can be considered as generalizations of AR(1) models. The estimated models can be used to generate realisations from the inflation models e.g. in studies concerning the solvency questions of insurance companies, in studies concerning investment returns of financial institutions etc. For the simulation purposes the oil price index in the transfer function noise model can be replaced by other external factors.

Sur la modélisation stochastique de l'inflation

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Résumé

Le but de la modélisation scientifique est d'exprimer les caractéristiques essentielles d'un phénomène ancré dans la réalité sous forme de modèle aussi simple que possible. Dans un grand nombre de cas, le caractère adéquat du modèle est testé en fonction de ses capacités de prévision. D'un autre côté, dans plusieurs applications, le processus-même ainsi que le modèle en résultant peuvent contenir des éléments n'étant pas prévisibles. Les changements abrupts dans l'inflation causés par exemple par les deux crises pétrolières survenues au milieu des années soixante-dix et au début des années quatre-vingt sont des exemples de ce type d'événements imprévisibles.

Récemment, les séries d'inflations observées ont souvent été décrites par un modèle AR(1), c'est-à-dire l'auto-régression de premier ordre. Au cours des dernières décades, les séries inflationnistes reflètent également les deux crises du pétrole. En plus de ces dernières, elles reflètent aussi les effets d'un certain nombre de décisions politiques ou de décisions affectant le marché du travail dans le pays concerné. Cela signifie que les séries innovatrices des modèles linéaires correspondants ne peuvent guère être stationnaires. Il est donc évident que les innovations sont produites par des distributions en fonction du temps.
En principe, les innovations peuvent être expliquées par divers facteurs. Dans la présente étude, nous avons utilisé l'indice du prix du pétrole comme variable explicative dans le modèle de l'inflation. Des modèles de transmittance ont été élaborés pour la série inflationniste de 11 pays. Ils peuvent être considérés comme des généralisations de modèles AR(1). Les modèles estimés peuvent être utilisés pour généraliser des réalisations à partir des modèles de l'inflation, par exemple dans des études portant sur des questions de solvabilité de compagnies d'assurance, dans des études touchant aux rendements d'investissement d'établissements financiers, etc. Dans un but de simulation, l'indice du prix du pétrole dans le modèle de transmittance peut être remplacé par d'autres facteurs externes.

1. Introduction

The main purpose of this article is to study and model the observed annual inflation series in 11 OECD countries. In this study the inflation is measured by the consumer price index. The 11 OECD countries are the United States(USA), Canada, Japan, France, Germany, Italy, the United Kingdom(UK), Denmark, Finland, Sweden, and Switzerland. The graphs of these 11 inflation series are given in Figure 1.

Often the purpose of time series modeling is the forecasting of the observed realisation. In fact, in many applications a time series model is tested by studying how accurate forecasts it produces. If the quality of the forecasts is poor, the model candidate should be rejected and other models from the same model family or completely different model types should be tried until a satisfactory result is reached. In this way model building is an iterative learning process.

Forecasting is not, however, the aim of this study. In spite of the fact that we are building time series models for the inflation series of the 11 countries, we will not try to use the resulting models for forecasting purposes. The purpose of these models is to provide us with such tools which could be used for simulation purposes, i.e. to generate simulated "inflation" series. Such artificial inflation series are needed e.g. in the simulation studies concerning the solvency issues of an insurance company, or more generally in modeling an insurance company.

In this article the behaviour of the observed inflation series are described by linear autoregressive moving average and transfer function noise models(see Box and Jenkins(1976)) with an oil price index as the only input of the transfer function model. This input series was selected because the oil price evidently has its effect on the inflation. The graph of the oil price index given in Figure 1 together with the 11 inflation series makes it clear that the two oil crises in the middle of the seventies and in the beginning of the eighties were realised as increased inflation. For this reason it is natural to study the statistical relation between the oil price and the inflation. For comparative purposes also univariate autoregressive moving average models will be built for the inflation series.

Besides the oil price there are a large number of other factors causing changes in inflation. These include political and economic decisions as well as salary agreements between trade unions in a country. History shows that certain events, such as the Korean war in the beginning of the fifties, have had their effect also on the inflation. Figure 1 clearly demonstrates that the Korean peak is apparent in all 11 inflation series. The Korean peaks do not have, however, any effect in the estimations of this study because they are located in the beginning of the data stretch and are therefore omitted from the estimations.

The effect of the other factors on inflation is, however, very complicated, and we do not attempt to describe their effect on the inflation directly in the final transfer function model. Their joint effect is described by the noise component in the transfer function model, or their effects are simply removed from the data before fitting models. It is possible that single peaks, which do not reflect the permanent generating system of the inflation phenomenon, can seriously disturb the modeling process and the resulting parameter estimates. Therefore such peaks should be taken into account in a proper way at least in those cases when there are natural causes for such peaks. One possibility is simply to smooth the peaks away from the series before modeling the data. This is done in this study for French and Finnish data, but before that the original unsmoothed data are used to build autoregressive moving average and transfer function noise models.

Univariate autoregressive models of order p, i.e. AR(p) models, or more generally autoregressive moving average models of order (p,q), i.e. ARMA(p,q) models, can be considered as special cases of transfer function noise models. These, on the other hand, are special cases of vector-valued autoregressive moving average models, the so called VARMA models.

One of the most difficult tasks in practically any modeling exercise is the identification stage. This includes the specification of the model family as well as the selection of variables of the model. For linear models the identification of the model consists of the estimation of the order of the model.

During the last twenty years a number of so called order selection criteria have been introduced in literature. These include the Akaike's AIC(Akaike(1974)) and the BIC criterion developed independently by Schwarz(1978) and Rissanen(1978). The BIC criterion is a consistent order selection criterion, but AIC is not. In this study we have used the BIC criterion in model selection.

An important reason for the development of the order determination criteria is that e.g. the least squares method does not lead to a consistent estimate of the integer-valued order of a model. This is because the more there are explanatory variables in the model, the smaller is the residual variance for a fixed number of observations. If, however, for each estimated parameter a positive penalty term, usually depending on the number of observations, is added in a criterion function, various order selection criteria can be obtained. Depending on the penalty term selected, some of the criteria are consistent, some not.

The article is organised as follows. In Section 2 some properties of linear autoregressive moving average and transfer function noise models are considered . Section 3 illustrates the identification of these models. Section 4 gives the estimation results and in Section 5 the estimated models are used to generate simulated inflation series. Finally Section 6 offers some concluding remarks.

2. Linear autoregressive moving average and transfer function noise models

It has been observed in practice that the family of autoregressive integrated moving average models of order (p,d,q), i.e. the family of ARIMA(p,d,q) models

$$\phi(B)(\nabla^d Y_t - \mu) = \theta(B)a_t \tag{2.1}$$

form a powerful collection to model a univariate time series $Y_1,Y_2,...,Y_n$. It is assumed that $\{a_t\}$ is a white noise porocess, i.e. a time series of independent random variables with mean zero and variance σ_a^2. B is the backshift operator such that $B^k Y_t = Y_{t-k}$ for all k. ∇ is the differencing operator such that $\nabla = 1 - B$, i.e. $\nabla Y_t = Y_{t-1} - Y_t$. It is assumed that $\{ \nabla^d Y_t \}$ is a stationary and invertible process. For stationarity it is necessary and sufficient that the polynomial

$$\phi(B) = 1 - \sum_{k=1}^{p} \phi_k B^k \tag{2.2}$$

has its zeroes outside the unit circle. Similarly for invertibility of the model it is required that the zeroes of the polynomial

$$\theta(B) = 1 - \sum_{j=1}^{q} \theta_j B^j \tag{2.3}$$

are outside the unit circle. Under the above assumptions $\mu = E\{\nabla^d Y_t\}$, the mean of the stationary process $\{\nabla^d Y_t\}$. Thus $\{\nabla^d Y_t\}$ is a stationary and invertible moving average process of order (p,q), i.e. an ARMA(p,q) process. If $d>0$, $\{Y_t\}$ is a non-stationary time-series which can be transformed into a stationary time series by applying the differencing operator for d times.

The above assumptions imply that $\nabla^d Y_t$ can be expressed as an infinite moving average model (MA(∞) model) in the form

$$\nabla^d Y_t - \mu = \psi(B) a_t, \tag{2.4}$$

where

$$\psi(B) = \phi^{-1}(B)\theta(B) = 1 - \sum_{k=0}^{\infty} \psi_k B^k. \tag{2.5}$$

Because of invertibility the model (2.1) can also be expressed as the infinite autoregessive model (AR(∞) model) in the form

$$\pi(B)(\nabla^d Y_t - \mu) = a_t, \tag{2.6}$$

where

$$\pi(B) = \theta^{-1}(B)\phi(B) = 1 - \sum_{k=1}^{\infty} \pi_k B^k. \tag{2.7}$$

The infinite moving average process (2.4) can be approximated as closely as wanted by a finite moving average model of order q', i.e. by a MA(q') model. Similarly the infinite autoregressive model can be approximated by a finite autoregression of order p', i.e. by an AR(p') model. It is possible that in a practical application the orders p' and q' can be rather large. In such cases a mixed ARMA(p,q) model as an approximation to $\pi(B)$ or $\psi(B)$ usually leads to rather low orders p and q.

The idea behind the model (2.1) is that the non-stationary time series $\{Y_t\}$ can be transformed into a stationary time-series by calculating the dth differences of the original series. After that the resulting stationary series $\{\nabla^d Y_t\}$ can be modeled by a stationary and invertible ARMA(p,q) model.

Stationarity cannot always be reached only by differencing . It is possible that before differencing it might be necessary to apply a non-linear transformation, e.g. the logarithmic transformation to the observed time-series in order to reach stationarity.

Besides univariate ARIMA models Box and Jenkins(1976) also propose a strategy for the building of transfer function noise models with one or several input processes. These models are of the form

$$Y_t - \mu = \sum_{k=1}^{m} \delta_k^{-1}(B)\omega_k(B)B^{b_k}(Z_{t,k} - \mu_k) + \phi^{-1}(B)\theta(B)a_t, \tag{2.8}$$

where

$$\omega_k(B) = 1 - \omega_{k1}B - \dots - \omega_{ku_k}B^{u_k}, k=1,\dots,m,$$

$$\delta_k(B) = 1 - \delta_{k1}B - \dots - \delta_{kr_k}B^{r_k}, k=1,\dots,m,$$

$$\phi(B) = 1 - \phi_1 B - \dots - \phi_p B^p,$$

$$\theta(B) = 1 - \theta_1 B - \dots - \theta_q B^q.$$

In (2.8) $\{a_t\}$ is a white noise series with zero mean and variance σ_a^2. The mean of the stationary input process is $E\{Z_{t,k}\} = \mu_k$, $k = 1,\dots,m$. It is assumed that the polynomials $\phi(B)$, $\theta(B)$, $\omega_k(B)$, $\delta_k(B)$, $k= 1,\dots,m$ have their zeroes outside the unit circle.

Often it is assumed that stationarity of the output and input series of the transfer function noise model defined by (2.8) can be reached by differencing and possibly by applying a non-linear transformation such as logarithms of the observed series before calculating the differences.

3. The identification of linear time series models

The identification of ARIMA(p,d,q) models or more generally transfer function noise models on the basis of the observed time series is a difficult task in practice. For ARIMA models the identification consists of the estimation of the integer parameters p,d and q. Box and Jenkins(1976)proposed as their identification method which is based on the behaviour of the estimated autocorrelation and partial autocorrelation functions.

As the first step of the identification process of ARIMA(p,d,q) model d, the degree of differencing is determined on the basis of the behaviour of the observed time series and the estimated correlations. In order to determine p and q one has to interpret the autocorrelations and partial autocorrelations of the observed series. Even for an experienced time-series analyst this can be a very difficult task, because the sampling variation in the estimated autocorrelations and partial autocorrelations usually makes it difficult to find typical patterns in the estimated correlation functions especially for small sample sizes.

Schwarz(1978) and Rissanen(1978) developed, independently of each other, from different starting points the order determination criterion which later has been known as the BIC criterion. In the case of ARMA(p,q) models the BIC criterion can be expressed in the form

$$\mathrm{BIC}(p,q) = n\log\hat{\sigma}_{p,q}^2 + (p+q)\log n,\ p = 1,2,\ldots,p^*,\ q = 1,2,\ldots,q^*, \tag{3.1}$$

where $\hat{\sigma}_{p,q}^2$ is the estimator of the residual variance. The order (p,q) of an ARMA model is estimated by minimising $\mathrm{BIC}(p,q)$ for $p = 1,2,\ldots,p^*$, $q = 1,2,\ldots,q^*$. It is clear that the minimisation of $\mathrm{BIC}(p,q)$ can be a computationally demanding task, because the minimisation problem consists of the estimation of $(p^*+1)(q^*+1)$ ARMA models. Naturally the order estimates also depend on p^* and q^*. The BIC criterion can be applied in an apparent way to estimate the order of a transfer function noise model. In the one input case the order of the transfer function noise model is estimated by minimising $\mathrm{BIC}(r,u,b,p,q)$ in the range of the parameters r,u,b,p,q.

Pukkila and Krishnaiah(1988) and Pukkila and Krishnaiah(1988a) developed a white noise test for univariate and multivariate time series. This test is based on order determination criteria such as BIC. The new test can be used also for the order selection of ARMA models. The resulting order selection method combines the identification idea of Box and Jenkins(1976) and the use of traditional order determination criteria. The articles Pukkila and Kallinen(1987), Koreisha and Pukkila(1988), Koreisha and Pukkila(1989), Pukkila(1989), Koreisha and Pukkila(1990), Koreisha and Pukkila(1990a), Pukkila, Koreisha and Kallinen(1990), Koreisha and Pukkila(1993) and Koreisha and Pukkila(1993a) demonstrate that this method usually outperforms the traditional order selection methods. The new method can also be used for the estimation of the degree of differencing.

The method is based on the idea that the model is expanded as long as the model transforms the data into white noise. The whiteness of the residuals is tested by applying an autoregressive order selection criterion in the estimated residual series. If the selected model is the AR(0) model, i.e. white noise, the model in question is considered to be adequate. The approach automatically guarantees that the resulting model is parsimonious. It also means that in the case of ARMA models it is not necessary to determine the maximum autoregressive and moving average order as it is necessary when traditional order determination criteria are applied. The method also avoids the estimation of overparametrized ARMA models. In this article, however, the traditional order determination criterion BIC will be applied in the selection of ARMA and transfer function noise models.

4. Inflation modeling

The effects of inflation can be seen in many areas of economy and in our everyday life. It has been apparent everywhere and it has guided the human behaviour. Salary agreements between the trade unions have often been built on the expected inflation. Similarly real estate prices have occasionally risen because of inflatory expectations. The rising prices have made it possible to hide real business results under the inflatory figures.

Several countries have defined low inflation as one of the most important goals for their economies. If the governments will achieve this goal, the future means a period of low inflation. This will have several important implications for the behaviour of people. Under low inflation the only real reason for salary increases will be the increase of productivity. People will not buy any more apartments on the basis of inflatory expectations, but because they will be needed for housing purposes. Low inflation means that the economy will in many ways become more transparent.

Inflation plays an important role in many areas of insurance. It is connected with salaries and other expenses of insurers. It is realised as claims inflation. Inflation also has its effects on technical reserves of the insurance companies. Naturally inflation is taken into account in the premium rates, but this usually takes place after a certain time lag. On the other hand, in most cases inflation is realised in claims amounts immediately. In practice it has been observed that there is a certain correlation between inflation and investment return.

For example in order to simulate the risk business of an insurance company a time-series model for inflation is needed. Let $I(t)$ be the value of the consumer price index at the year t. Wilkie (1984, 1986) proposed the AR(1) model

$$\log\left(\frac{I(t)}{I(t-1)}\right) = \mu + \phi\left(\log\left(\frac{I(t)}{I(t-1)}\right) - \mu\right) + \sigma_\varepsilon \varepsilon(t) \tag{4.1}$$

to describe the stochastic behaviour of the inflation process. In (4.1) the noise terms $\varepsilon(t)$ are assumed to form a sequence of independent and identically distributed random variables with mean zero and unit variance. Furthermore μ is the average rate of inflation in (4.1). When the AR(1) model was fitted in the UK data from the period 1919 to 1982 the estimates

$$\mu = 0.05,\ \phi = 0.6,\ \sigma_\varepsilon = 0.05$$

were obtained(Wilkie(1986)).

In order to build a transfer function noise model for the inflation series with oil price as the input series, an oil price index Z'_t is needed. This index was formed for the years 1952-1991 by combining the barrel price of oil in Saudi-Arabia(1952-1984) and the average spot price of oil in 1985-1991. The oil price index is formed such that its value is 100 at the year 1985. The graph of $Z_t = \log(Z'_t/Z'_{t-1}) = \log(Z'_t)\text{-}\log(Z'_{t-1})$ is given in Figure 1. The two oil crises can be seen as positive spikes in the graph for Z_t at the years 1974 and 1980.

Following Wilkie(1984, 1986) we estimated AR(1) models for $Y_t = \log(I(t)/I(t\text{-}1))$, where $I(t)$ is the consumer price index. This model(model a) is therefore of the form

$$\text{Model a:} \qquad Y_t = \mu + \frac{1}{1-\phi B} a_t. \tag{4.2}$$

Transfer function noise models of the form(Model b)

$$\text{Model b:} \quad Y_t = \mu + \frac{\omega}{1-\delta B} Z_t + \frac{1}{1-\phi B} a_t. \tag{4.3}$$

were also fitten in the data. Model b can be considered as a generalization of Model a. Both models have an AR(1) part, but Model 2 has also an additional part due to the input series Z_t. For each series also more general transfer function noise models(Model c)

$$\text{Model c:} \quad Y_t = \mu + \frac{\omega(B)}{\delta(B)} Z_t + \frac{1}{\phi(B)} a_t. \tag{4.4}$$

were identified and estimated by applying the BIC criterion for $p<5$, $r<3$ and $u<2$. The estimated orders for Models c are given in Table 1.

Table 1. The Schwarz-Rissanen order estimates for transfer function noise model with inflation as the output series and oil price index as the input series, u<2, r<3,p<5.

	u	r	p
USA	0	2	1
Canada	1	1	1
Japan	1	0	1
France	0	1	1
Germany	0	1	1
Italy	0	1	1
UK	1	1	1
Denmark	0	1	2
Finland	1	1	3
Sweden	0	1	2
Switzerland	1	0	1

Table 2 gives the least squares estimates for the parameters of Models 1, 2 and 3. The estimate of the parameter μ is the arithmetic mean of the output series. γ_ε is the skewness of the residual series.

The residual series of the models a, b and c are given correspondingly in Figure 2, Figure 3 and Figure 4 . It can be seen that the two oil crises are visible in Figure 2, i.e. in the residual series corresponding to the AR(1) models. This is natural because the simple AR(1) model cannot take the oil crises into account in a proper way. Table 2 and the graphs of the residual series clearly demonstrate that the residual series corresponding to the transfer function noise models have smaller residual variation than the simple AR(1) models. On the other hand it can be seen that also the transfer function noise models give rather high values for the residuals at the period of the first oil crisis especially in the case of Japan.

In the French inflation series there is a sharp peak in late fifties when the inflation jumped to about 15 %. In order to study its effect the peak was removed from the series by smoothing it away. The estimation results are given in Table 3. By comparing the estimation results for France in Tables 2 and 3, it can be observed that the high inflation peak has a dramatic effect especially in the value of the autoregressive parameter ϕ and naturally also in the residual variance.

Table 2. Least squares estimates for the parameters of the models a,b and c.

	Model	ϕ	μ_1	ϕ_2	ϕ_3	δ_1	δ_2	ω_0	ω_1	σ_ε	γ_ε
USA	a	0.042	0.810							0.017	0.733
	b	0.042	0.716			0.597		0.041		0.012	0.372
	c	0.042	0.873			0.635	-0.474	0.042		0.011	-0.117
Canada	a	0.044	0.868							0.015	-0.448
	b	0.044	0.788			0.834		0.029		0.013	-0.521
	c	0.044	0.816			0.476		0.025	0.026	0.012	-0.304
Japan	a	0.050	0.698							0.037	0.699
	b	0.050	0.524			0.617		0.074		0.030	0.663
	c	0.050	0.614					0.066	0.093	0.025	-0.167
France	a	0.056	0.581							0.030	1.715
	b	0.056	0.146			0.745		0.047		0.023	2.874
	c	0.056	0.146			0.745		0.047		0.023	2.874
Germany	a	0.029	0.788							0.010	-0.386
	b	0.029	0.789			0.621		0.021		0.008	-0.253
	c	0.029	0.789			0.621		0.021		0.008	-0.253
Italy	a	0.072	0.886							0.024	1.011
	b	0.072	0.587			0.846		0.059		0.016	0.019
	c	0.072	0.587			0.846		0.059		0.016	0.019
UK	a	0.065	0.791							0.029	0.904
	b	0.065	0.487			0.773		0.067		0.021	1.047
	c	0.065	0.498			0.567		0.057	0.048	0.019	0.210
Denmark	a	0.058	0.732							0.021	0.534
	b	0.058	0.682			0.290		0.037		0.018	-0.022
	c	0.058	0.613	-0.325		0.748		0.042		0.017	0.064
Finland	a	0.065	0.611							0.031	0.627
	b	0.065	0.405			0.651		0.057		0.027	1.346
	c	0.065	0.273	-0.244	-0.229	0.627		0.041	0.026	0.021	0.293
Sweden	a	0.059	0.670							0.021	0.027
	b	0.059	0.377			0.850		0.027		0.017	-0.323
	c	0.059	0.178	-0.319		0.874		0.027		0.015	0.068
Switzerland	a	0.033	0.703							0.015	-0.271
	b	0.033	0.684			0.520		0.034		0.012	-0.269
	c	0.033	0.637					0.033	0.028	0.012	0.262

The Finnish inflation series was studied similarly. However, in the Finnish inflation series there are several peaks corresponding to the years 1951, 1956, 1957, 1964 and 1968. In the period from 1952 to 1955 the Finnish economy was strictly regulated. In those years prices and salaries did not increase at all on the average. In 1956 there was a general strike lasting for about three weeks in the first quarter of the year. Salaries were increased as a result of that strike. In the following year the Finnish Mark was devalued by 39 %. The purchase tax system was introduced in Finland in 1964, and there is a sharp peak in the inflation series in 1964. In October 1967 there was again a devaluation of 31 %. In 1977 and 1978 the Finnish Mark was devalued three times. The joint effect of these devaluations was a little less than 20 %. When the "outliers" corresponding to the years 1951, 1956, 1957, 1964 and 1968 were smoothed, the estimation was repeated for the models 1, 2 and 3. The results are given in Table 3.

Table 3. Least squares estimates for the parameters of the models a,b and c corresponding to the smoothed inflation series.

	Model	μ	ϕ_1	ϕ_2	ϕ_3	δ_1	δ_2	ω_0	ω_1	σ_ε	γ_ε
France	a	0.053	0.824							0.019	0.366
	b	0.053	0.464			0.806		0.042		0.013	-0.302
	c	0.053	0.464			0.806		0.042		0.011	-0.302
Finland	a	0.058	0.825							0.022	0.254
	b	0.058	0.416			0.744		0.055		0.015	0.128
	c	0.058	0.489	0.118	-0.230	0.702		0.046	0.022	0.013	-0.612

Also in the Finnish case the value of the autoregressive parameter is larger for the smoothed inflation series. This is natural because "the outliers", the jumps in inflation, decrease the first order autocorrelation of the inflation series.

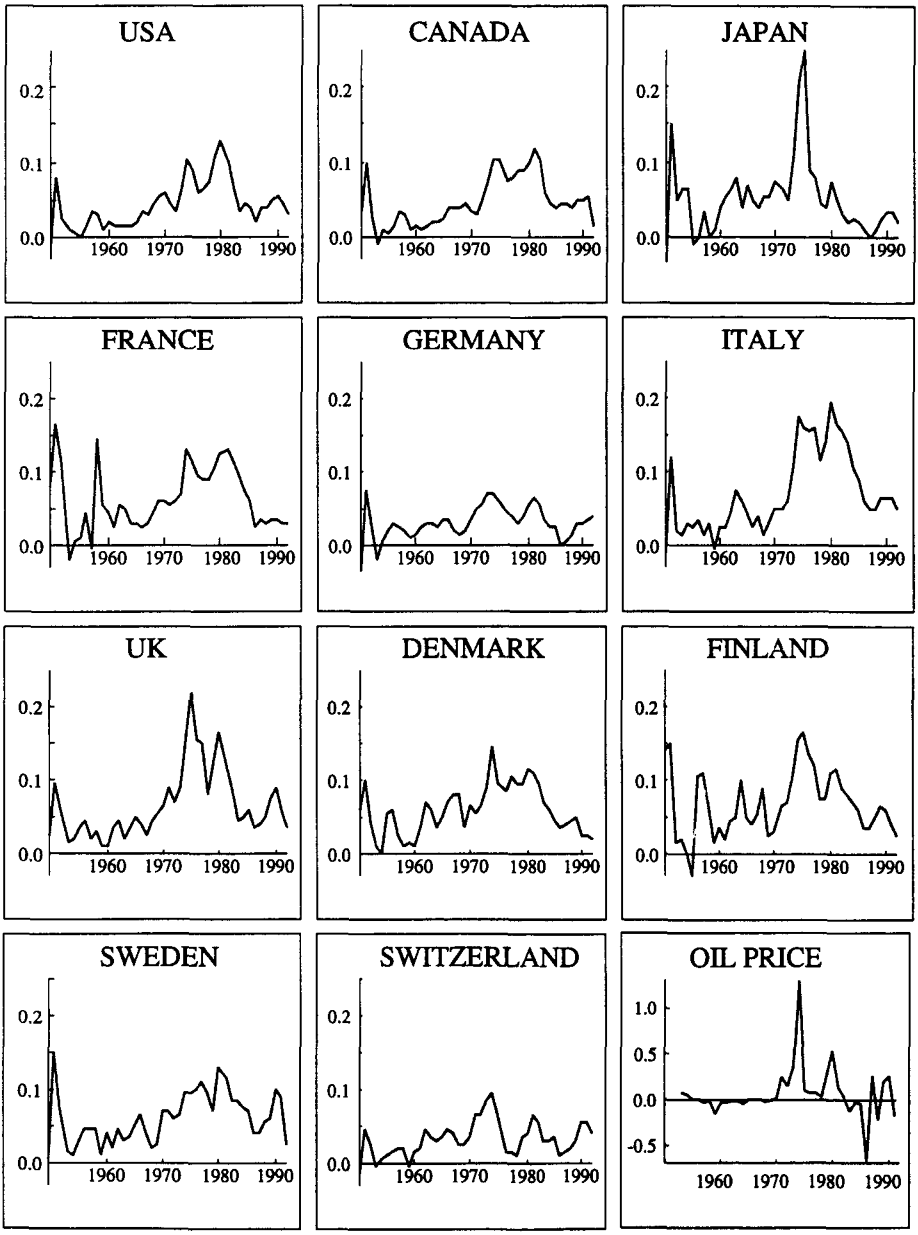

Figure 1. The graphs of the 11 inflation series and the graph of the logarithmic differences of the oil price index.

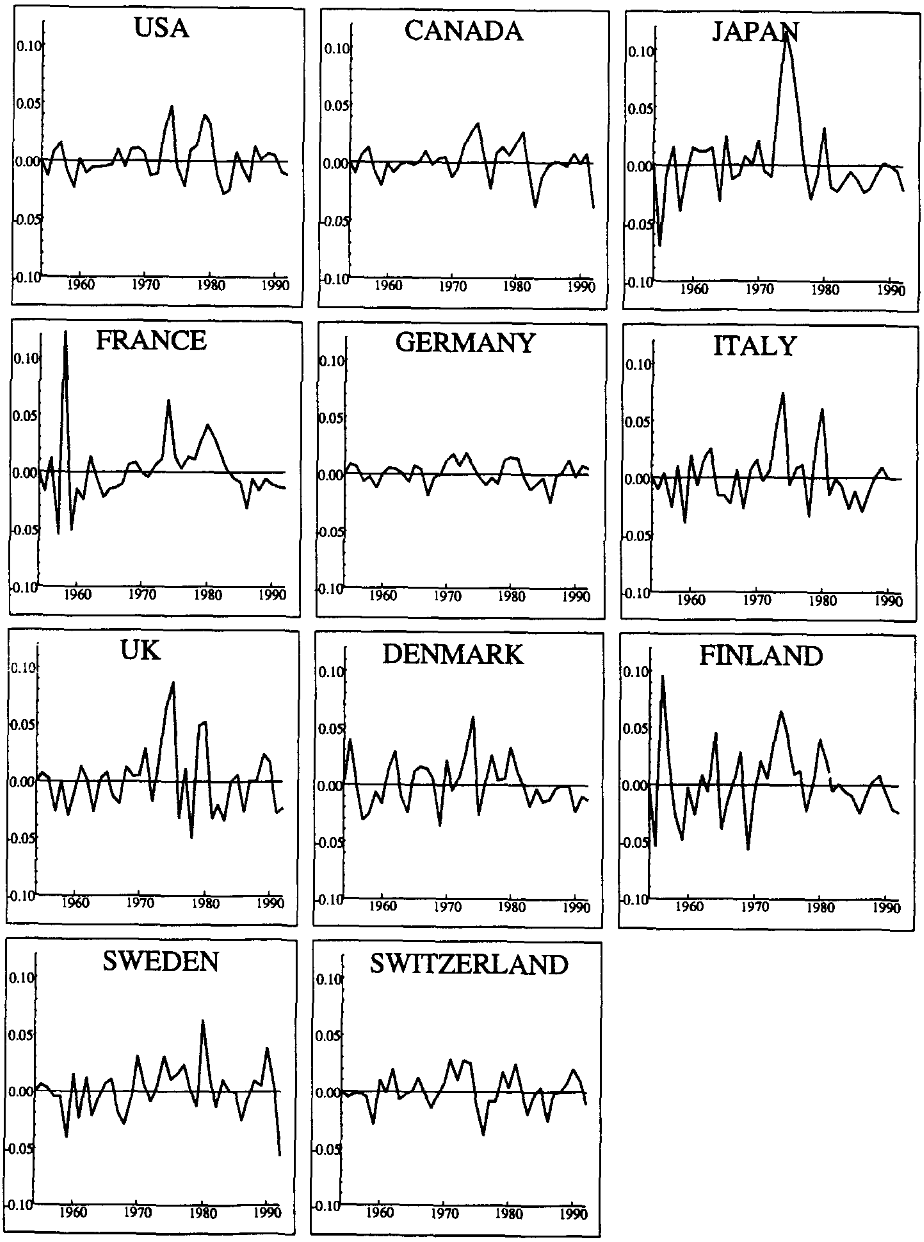

Figure 2. The graphs of the residual series corresponding to the AR(1) models for the inflation series.

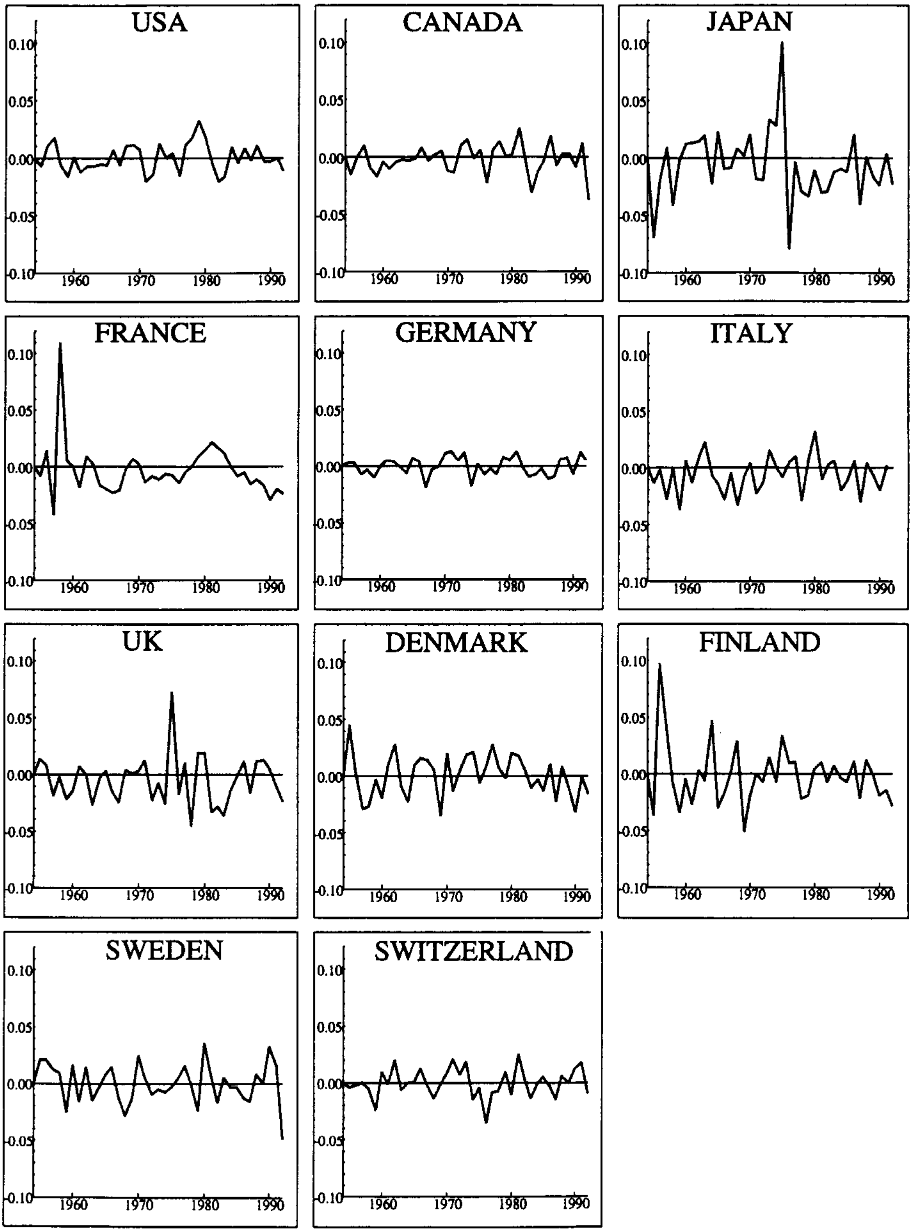

Figure 3. The graphs of the residual series of the transfer function noise models for the inflation series with *u=0, r=1, p=1.*

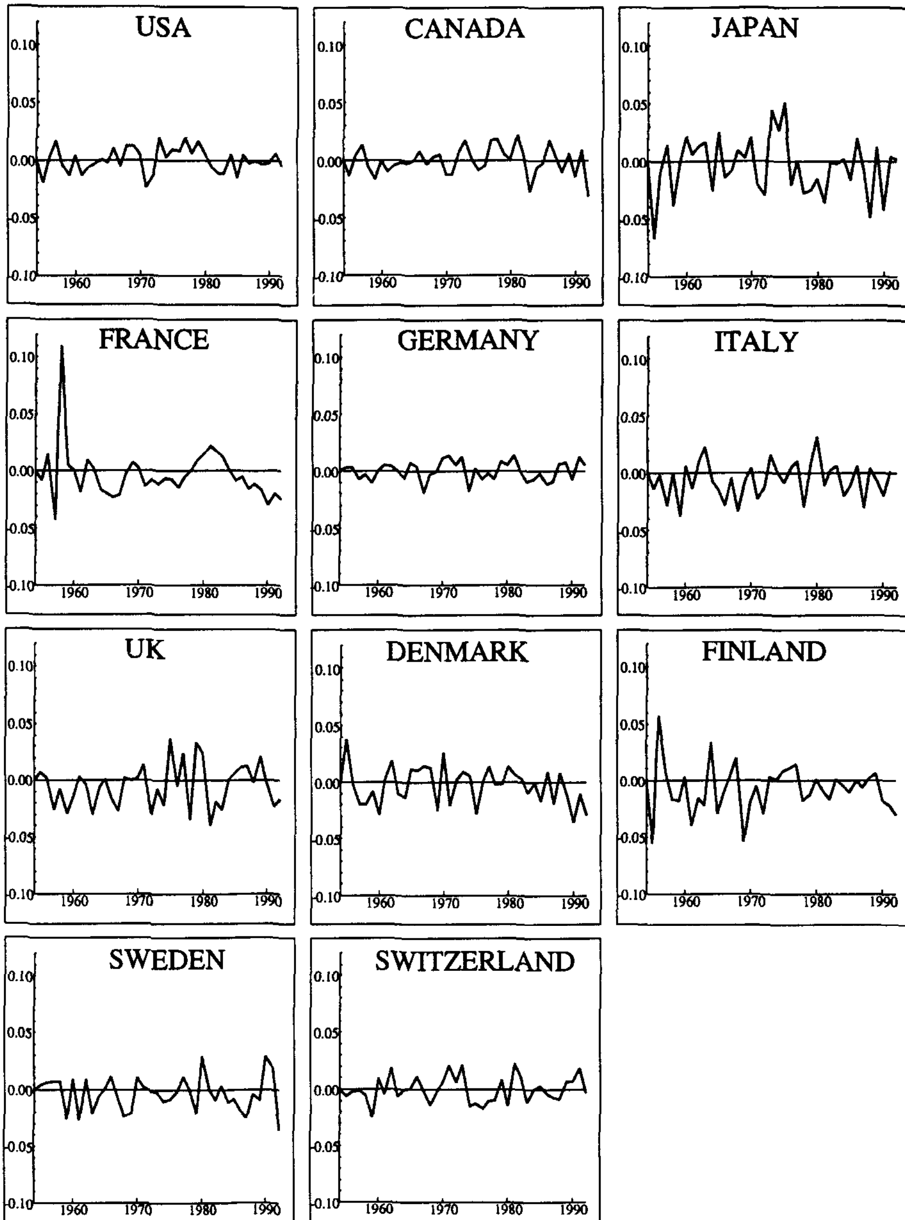

Figure 4. The graphs of the residual series of the transfer function noise models for the inflation series when u, r, p were selected using the BIC criterion.

5. Simulated inflation series

In this section examples are given on the simulated series using the models and parameter values corresponding to the estimation results from the Finnish and German data and the corresponding parameter estimates. The performance of the models a, b and c as inflation series generators is illustrated in Figures 5 and 6. The inflation series of length 50 were generated from these models. The realisations in Figure 5 were generated by applying the parameter estimates of Table 2 corresponding to the Finnish estimation results. The random shocks a_t were generated from the normal distribution. For all models a, b and c the same random shocks a_t were applied. In Figure 5 the graph c2 was generated by using the Finnish estimation results from Table 3.

In the graphs b, c, and c2 there is a shock corresponding to an oil crisis at t=30. The size of this shock corresponds to the value of the 1974 oil crisis. In addition, there is also an extra shock in the series c2 at the period t=15. This shock corresponds e.g. to the devaluation shocks whose effect does not last a long time. This shock raised the inflation by about 10 % for one year.

By comparing the simulated inflation series a, b, c and c2 with the real inflation series in Figure 1 it seems that c2 resembles a real inflation series. It is reasonably smooth, but it also contains that kind of variation which can be observed in practice. For this reason the Finnish model c of Table 3 was selected as the Finnish inflation generator. Similarly model c of Table 2 was selected as the German inflation generator. In Figure 6 the graphs of six realisations are given from the Finnish and German inflation models.

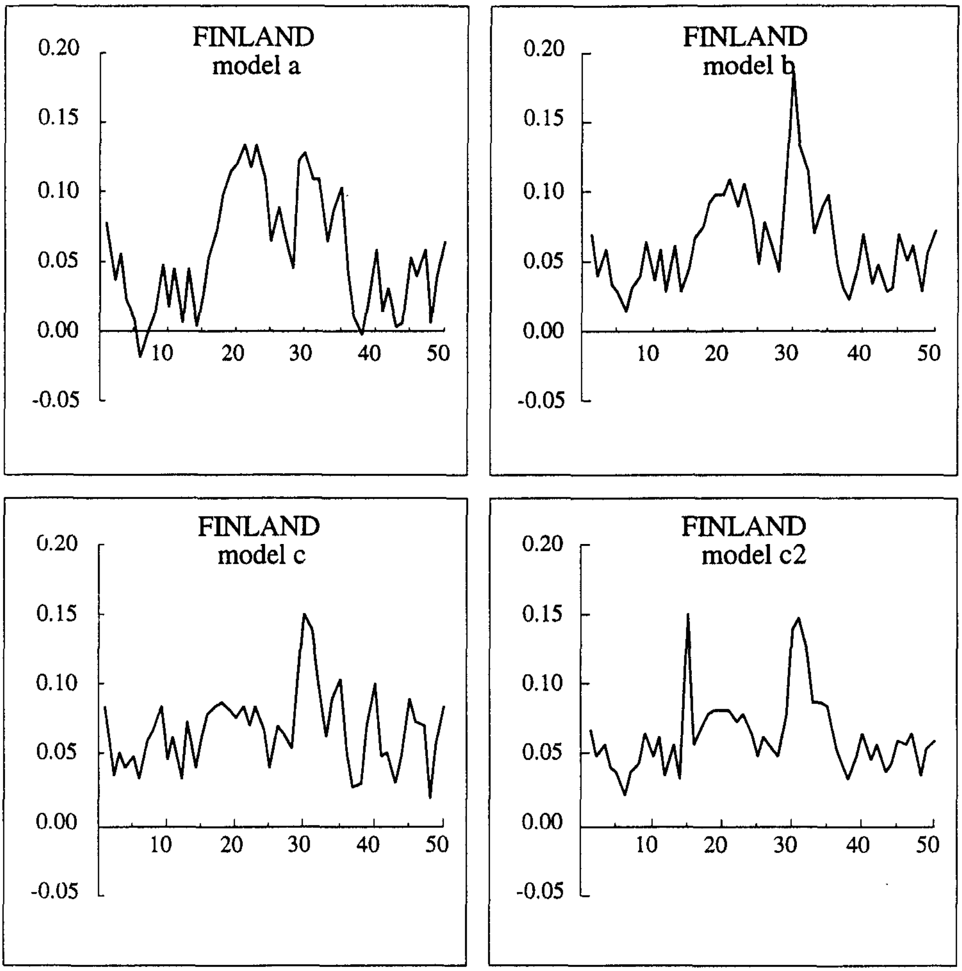

Figure 5. Realisations of length 50 from the Finnish inflation models a, b and c using parameters from Table 2 and a realisation (c2) from the model c using the estimates from Table 3.

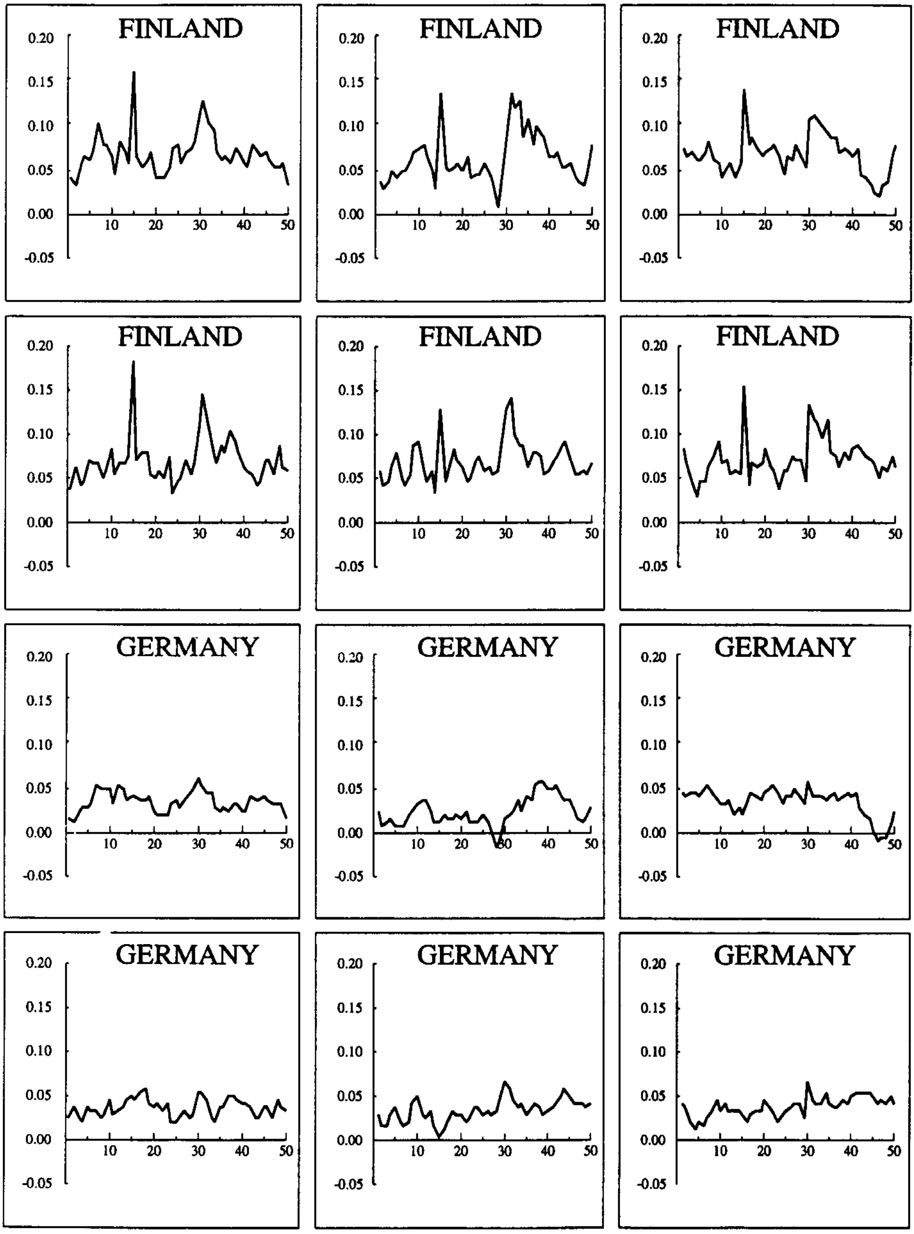

Figure 6. Six realisations of length 50 from the Finnish and German inflation models using the model c and the parameter values correspondigly from Tables 3 and 2.

6. Discussion

In this study a model for inflation was developed. The suggested model is a transfer function noise model with one input series. The noise component of the model is an autoregressive model whose order varies from country to country. Besides outer shocks having effect on the inflation through the transfer function it is necessary to introduce additive shocks in the data as was done in the Finnish case.

A conclusion that can be made on the basis of this study is that simple autoregressive models do not generate such realisations which behave similarly as the observed inflation series. Transfer function noise models seem to form a better alternative as inflation generators.

In the present study the level of inflation was kept constant through the whole observation period. However, the empirical observations indicate that more realistic results might be obtained if the constant term of the model would depend on time. Therefore future studies on the generating mechanism of inflation will be necessary.

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