

# Investment Strategy for Defined Contribution Plans

Peter Ludvik  
Principal Actuary - Investment Strategy

Sedgwick Noble Lowndes  
Norfolk House  
Wellesley Road  
Croydon  
Surrey  
CR9 3EB  
Great Britain

Telephone: 081-666-8166  
Fax: 081-666-1458

## Summary

The aim of this paper is to quantify the impact of investment strategy on emerging benefits in a defined contribution pension plan.

In the first part of the paper the author examines the relationship between different asset class characteristics and the style of benefits provided by a typical defined contribution plan. In the second part of the paper the risk of benefit shortfall relative to a target benefit is assessed for three asset classes, namely equities, fixed interest and cash. In the final part of the paper several 'dynamic' asset mix investment strategies are examined to see whether they are superior to 'constant mix' investment strategies.

The analysis has been carried out by simulation, using a stochastic model for the equities, fixed interest, cash and salary inflation. The models are described briefly in an Appendix.

## **Stratégie d'investissement pour les régimes de retraite à cotisation déterminée**

Peter Ludvik  
Actuaire Principal - Investment Strategy

Sedgwick Noble Lowndes  
Norfolk House  
Wellesley Road  
Croydon  
Surrey  
CR9 3EB

Téléphone : 081-666-8166

Télécopie : 081-666-1458

### **Résumé**

L'objectif du présent article est de quantifier l'impact de la stratégie d'investissement sur les prestations d'un régime de retraite à cotisation déterminée.

Dans la première partie, l'auteur commence par examiner les relations entre les caractéristiques des différentes catégories d'avoirs et le type d'avantages fournis par un régime à cotisation déterminée type. Dans la deuxième partie, il évalue le risque de déficit des prestations par rapport aux prestations visées pour trois catégories d'avoirs, à savoir les actions, les valeurs à revenu fixe et l'argent liquide. Dans la troisième et dernière partie de l'étude, il examine plusieurs stratégies d'investissement à répartition "dynamique" des éléments d'actif pour déterminer si elles sont supérieures aux stratégies d'investissement à répartition constante.

L'analyse a été effectuée par simulation en utilisant des modèles stochastiques pour les actions, les valeurs à revenu fixe, l'argent liquide et l'inflation des salaires. Ces modèles sont décrits brièvement en appendice.

**Introduction**

This paper is a progress report of an investigation into investment strategies for Defined Contribution (DC) plans. The typical DC plan in the United Kingdom provides a pension and a cash lump sum benefit to a member from a predetermined retirement age. The level of benefit depends on the amount of the accumulated assets for the member at the time of his/her retirement. This amount also depends on the contributions paid by the member and the sponsor into the plan and the investment returns on the invested contributions. The investment return will depend on the investment strategy of the fund, namely the proportion of the fund held in equities, bonds, cash etc.. The choice of an investment strategy should be determined by the objectives of the members and the sponsor, their perception of risk and the standard for the measurement of risk. Full discussion of what might be the appropriate objectives and standards of risk measurement is beyond the scope of this paper. Therefore, for the current investigation I have assumed that the main objective of a DC plan, from both member's and sponsor's point of view, is to maximise the level of benefit at retirement, whilst keeping costs to predetermined level. For the measurement of risk I have adopted the shortfall (at 95% confidence level) of the down-side relative to the expected level of benefit.

It is hoped that fuller presentation of the results and the process in general will be made at the conference.

**DC Plan Model**

A relatively simple model has been adopted for the benefits and contributions of the DC plan. It is assumed that all annual contributions are paid at the beginning of a year and invested immediately according to an agreed investment strategy. Similarly, all income from investments is assumed to be received at the end of a year. Investments are rebalanced at annual intervals. The accumulated fund for a member after N years from present year t can be expressed as

$$\text{Fund}(t,N) = \text{Fund}(t,0) * \text{AF}(t,N) + \sum_{(i=0,N-1)} \text{Cont}(t+i) * \text{AF}(t+i,N-i) \tag{1}$$

$$\text{Cont}(t+i) = \text{Salary}(t) * \text{SI}(T,i) * \text{ContRate}(t+i) \tag{2}$$

**AF(u,m)** is the accumulation factor for the asset mix of the fund over the m years from year u. The actual calculation of this function would depend on the investment strategy adopted as discussed in the next section and the investment model adopted for the individual asset classes. A proprietary stochastic model for Equities, Bonds and Cash has been used (for a brief description and references see Appendix).

**Cont** is the amount of total annual contributions, calculated as a fraction (**ContRate**) of annual salary.

**ContRate** will be plan specific; for the purposes of this paper **ContRate** is constant; however, results are available for **ContRate** varying with age of member.

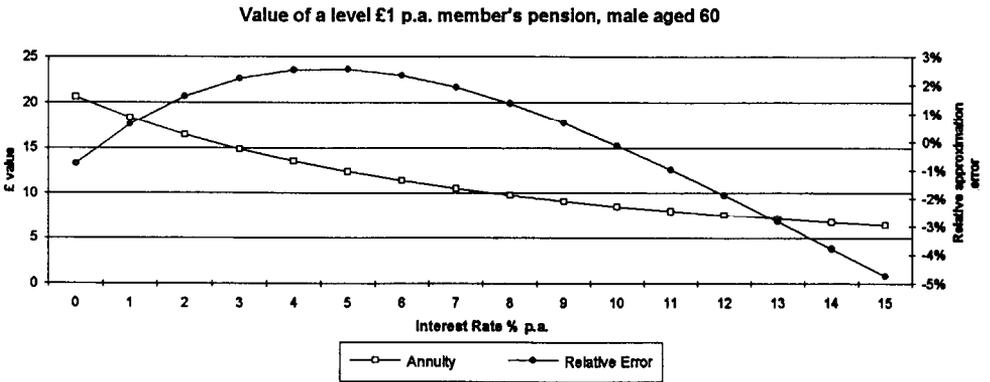
**Salary** is a variable representing member's salary progression due to merit, promotion etc.; for the purposes of this paper this will be constant.

**SI** is a stochastic variable representing salary increases due to general inflation (see Appendix).

The fund will have to be converted to a pension and/or cash benefit at retirement. The conversion factor **CF(u)** at time u is normally linked to a life office immediate annuity rates. This has been approximated by a function linked to a member's age and the redemption yield of a long term fixed interest bond at the time of the retirement. The formula used is

$$CF(u) = \text{age dependent constant} * PV(\text{yld}(u), 15, 11\%) \tag{2a}$$

where  $PV(\text{yld}(u), 15, 11\%)$  is the value of an 15 year bond with 11% coupon (payable half-yearly) at redemption yield at time u. The graph below shows how  $CF(u)$  depends on the long term interest rates and the relative error of the approximation:



The amount of pension benefit after N years from present time t can be therefore expressed as

$$Pension(t,N) = (Fund(t,N) - Cash) / CF(t+N) \tag{3}$$

To make the benefits more directly comparable with other types of pension arrangements, the pension benefit has been expressed as a fraction of the salary at retirement

$$FinalSalary(t,N) = Salary(t) * SI(t,N) \tag{4}$$

$$PensionFraction(t,N) = ((Fund(t,N) - Cash) / CF(t+N)) / FinalSalary(t,N) \tag{5}$$

As a further yardstick, a ratio of the DC pension with a target pension that would be provided from a defined benefit plan based on a simple formula  $\text{FinalSalary} * \text{Service} * \text{AccrualFraction}$ , has been considered. This can be written

$$\text{TargetRatio}(t,N) = \text{PensionFraction}(t,N) / (N * \text{AccrualFraction}) \tag{6}$$

If we assume that

$$\text{Cash} = 0 \tag{7}$$

$$\text{Salary}(t) = S \tag{8}$$

$$\text{ContRate}(t) = c \tag{9}$$

$$\text{Fund}(t,0) = K * \text{Salary}(t) \tag{10}$$

then the formula (5) becomes

$$\text{PensionFraction}(t,N) = \frac{K * \text{AF}(t,N) / \text{SI}(t,N) + c * \sum_{(i=0,N-1)} \text{AF}(t+i,N-i) / \text{SI}(t+i,N-i)}{\text{CF}(t+N)} \tag{11}$$

It is helpful to consider how this quantity behaves under the classical 'expected values' approach normally used in actuarial valuations. Assuming that

$$\text{AF}(t,N) = (1+r)^N \tag{12}$$

$$\text{SI}(t,N) = (1+j)^N \tag{13}$$

then (11) simplifies to

$$\text{PensionFraction}(t,N) = \frac{K * (1+R)^N + c * \sum_{(u=0,N-1)} (1+R)^{(N-u)}}{\text{ann}(r)} \tag{14}$$

where R is the 'real rate of return' relative to the salary inflation and  $\text{ann}(r)$  is the value of the conversion factor at rate r. It is clear that for a given level of K and c the PensionFraction is an increasing function of R and r. In the context of (11), the same holds true in the sense that  $\text{AF}(t+i,N-i) / \text{SI}(t+i,N-i)$  represents the total 'real return' on the fund between times t+i and t+N.

**Investment Strategy Model**

The class of investment strategies that have been investigated in this paper consists of two portfolios, called Initial and Target portfolio respectively, together with a procedure of how to switch assets from the Initial portfolio to the Target portfolio during the pre-retirement period. It is assumed that at entry to the DC plan, the member's asset are 100% in the Initial portfolio and at the time of retirement they are 100% in the Target portfolio. Normally the two portfolios would have fundamentally different asset mixes: for example, Initial portfolio might be 100% equities and the

Target portfolio might be 100% bonds. The switching procedure is defined by a function that specifies the proportion of the total fund held in the Target portfolio. If this function is a constant throughout, between 0% and 100% inclusive, we have a constant asset mix investment strategy.

Some providers of investment vehicles for DC plans have strategies that switch between the Initial and Target portfolio uniformly over the entire pre-retirement period, others have used a 10 year period prior to expected retirement date for switching.

The main results of this paper show that these long switching periods do not give protection against the 'real risk', namely inadequate pension provision at retirement. It is shown that what is conventionally thought of as a risky strategy with high equity content can have low 'real risk'. It will be shown that on our investment model assumptions the switching period should be much shorter, typically 3 to 5 years prior to expected retirement date.

There are two practical problems with high equity Initial portfolio and a shorter switching period. The member's expected retirement date is often not the normal retirement date as defined in the rules of the DC plan. If the member retires early, a short switching period could mean that the whole fund would be invested in a highly volatile asset at the time of retirement and ill-matched for the purchase of annuities. However, this problem can be mitigated by 'staggered vesting' or 'phased annuity'. This facility would enable the member to convert only a small part of the fund into an annuity at a time and to build up the income over a number of years, with the balance of income in the early years being provided from a cash sum. In a DC plan that does not provide this facility it might be possible for a member to take a transfer value on early retirement and invest the money in a personal pension contract that would allow this.

The second problem is one of perception: members do not like to see the value of their fund go down, particularly at the later stages, when the amount in the fund can be large compared with the normal amounts the member is used to handling in their personal financial transactions. This problem might be reduced if the value of the fund was expressed as a notional pension secured by the fund.

Let  $IW(k)$  and  $TW(k)$  denote the weighting of asset class  $k$  in the Initial portfolio and Target portfolio respectively,  $TP(t)$  denotes the proportion of the fund held in the Target portfolio at time  $t$  and  $Ret(t,k)$  denotes the total return on the asset class  $k$  at time  $t$ . Then we can define

$$IR(t) = \sum_{(k=1,M)} IW(k) * Ret(t,k) \quad (15)$$

$$TR(t) = \sum_{(k=1,M)} TW(k) * Ret(t,k) \quad (16)$$

$$R(t) = (1-TP(t)) * IR(t) + TP(t) * TR(t) \quad (17)$$

$$Inf(t) = SI(t,1) \quad (18)$$

IR, TR and R are the total returns at time t on the Initial portfolio, the Target portfolio and the total fund respectively and Inf(t) is the salary inflation in year t. Using this notation and assuming continuous rebalancing, the formula for the pension fraction (11) can be written as

$$\text{PensionFraction}(t,N) = (K * \prod_{(i=0,N-1)} R(t+i) / \text{Inf}(t+i) + c * \sum_{(i=0,N-1)} \prod_{(j=i,N-1)} R(t+j) / \text{Inf}(t+j)) / CF(t+N) \tag{19}$$

It is clear that algebraic expressions for the statistical properties of (19) would be very complex when a realistic model for R and Inf (see Appendix) is used and simulation methods are necessary to obtain numerical answers.

**Results of the Investigation**

Only a selection of results are required to capture the essence of the investigations. All the results shown are for males, expected retirement age 60, contribution rate 5%.

The tables below show the median pension fractions and the 'floor' shortfall for the 3 asset classes considered in this paper and one dynamic strategy which entails switching from 100% equities to 100% bonds over the last 5 years of membership. The 'floor' shortfall is defined here as a value such that only 5% of the results are worse than (Median - Shortfall) and represents a measure of downside volatility. All the results are expressed as a percentage of a final salary and are based on 1000 simulations.

**No accumulated fund or special payment at entry**

Asset Mix	Equities		Bonds		Cash		Dynamic	
	Median	Shortfall	Median	Shortfall	Median	Shortfall	Median	Shortfall
Entry Age								
25	41.0	22.8	25.7	11.3	20.4	11.0	38.0	20.4
30	31.4	17.0	21.1	8.1	17.0	8.4	28.7	13.5
35	23.0	11.2	16.9	6.2	14.2	6.4	21.3	8.8
40	16.4	7.7	13.1	4.5	11.3	4.9	15.5	6.3
45	11.2	4.8	9.4	2.8	8.4	3.1	10.5	3.7
50	6.7	2.5	6.0	1.5	5.6	1.6	6.3	1.8

The fund multiple of 1 times salary after 20 years corresponds to poor performance of equity fund (bottom 5%), about lower quartile performance of a bond fund and median performance of a cash fund.

**Accumulated fund after 20 years or special payment on late entry  
of 1 times salary**

Asset Mix Entry/Current Age	Equities		Bonds		Cash		Dynamic	
	Median	Shortfall	Median	Shortfall	Median	Shortfall	Median	Shortfall
45	30.1	14.4	23.1	8.5	19.6	9.1	28.1	12.1
50	22.3	9.6	18.6	5.6	17.0	6.5	20.8	7.3
55	15.9	5.6	14.6	3.1	13.7	3.4	15.5	3.9

The fund multiple of 2 times salary after 20 years corresponds to upper quartile performance of equity fund and top 5% performance for the bond fund. It is unlikely reached by the cash fund.

**Accumulated fund after 20 years or special payment on late entry  
of 2 times salary**

Asset Mix Entry/Current Age	Equities		Bonds		Cash		Dynamic	
	Median	Shortfall	Median	Shortfall	Median	Shortfall	Median	Shortfall
45	49.1	24.8	36.6	14.2	30.9	15.0	45.6	20.7
50	38.1	16.9	31.2	10.1	28.4	11.5	35.3	12.9
55	28.9	10.5	26.4	5.9	24.6	6.3	28.1	7.6

### Conclusions

It can be seen that if the predictability of the 'floor' level (e.g. the worst 5% percentile) is paramount, both Bonds and Cash strategies are superior to the Equity or Dynamic strategy. However, this comes at a price as the medians are considerably lower for Bonds and Cash strategies. In fact, the results show that the 'floor' for Equity strategy is between the lower quartile and median for Bond strategy and nearly the median performance for Cash strategy.

The Dynamic strategy chosen reduces the volatility of the 'floor' whilst preserving or improving the 'floor' level at practically all entry ages. However it would seem that to obtain more significant improvements more complex strategies may need to be followed.

One possible strategy is to use derivatives in conjunction with an Equity strategy to protect a 'floor' level on an annual basis relative either to a Cash index or Bond index. In the UK some investment managers have started to provide these vehicles and we would hope to investigate the impact of these strategies on DC plan target benefits in the future. One difficulty in modelling these is that the costs incurred in obtaining the

protection is not guaranteed by the investment manager and can vary from year to year; this may be very costly in adverse conditions.

Another area of research would be to investigate strategies that are self-modifying, for example increase allocation to Bond portfolio when the accumulated fund is ahead of target, or 'locking in' high redemption yields through the use of matched portfolios or purchase of deferred annuities when the bond/equity yield ratio make this advantageous.

## Appendix

### Stochastic Investment Model

Our investment model is an extension of a well known model originally published by Professor Wilkie in 1986 (3) and since then used in numerous papers. We have extended the model to include time series for all the major investment categories and national average earnings (NAE).

As a further refinement, we allow the error terms in the individual time series to follow multivariate normal distribution, rather than being independent and normally distributed. One of the reasons for this refinement is the need to improve the behaviour of the correlation between the time series at shorter duration (2).

At the centre of the model is the retail price index (RPI) series, and we model it as a first order auto-regressive conditionally heteroscedastic time series. The model for national average earnings (NAE) series is a multiple of the RPI plus a random component representing the gap between NAE and RPI.

The Equity yield and Cash series have two components. The first is linked to RPI and the second is a first order auto-regressive process about the long term mean.

The growth of Equity dividend model is a complex one, having four components: link to current level of RPI, lagged effect of RPI, link to the random shock in the Equity yield series and link to the random shock in its own development in the previous year. The series has mean reverting character in the long run but it is not a simple auto-regressive process.

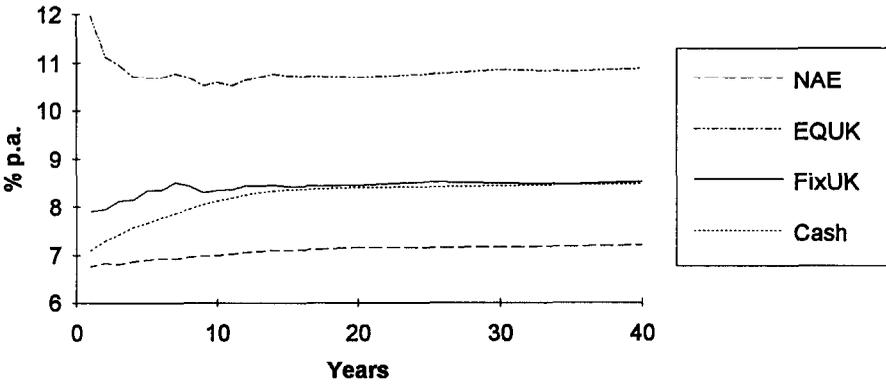
In the original model the Consol yield was modelled directly. As discussed in (2) this leads to potential problems in the short to medium term time horizon and our model for Consol yield is obtained by modelling the bond/equity yield ratio. The model for the Bond/Equity yield ratio has the same structure as the Equity yield model.

Rather than showing the formulae here (see (1) for a recent exposition), the graphs that follow illustrate the results obtained from a 1000 simulations of the model for the series used in this paper. They should give the reader some insight into the behaviour of the investment model and provide better understanding of the results presented in the paper.

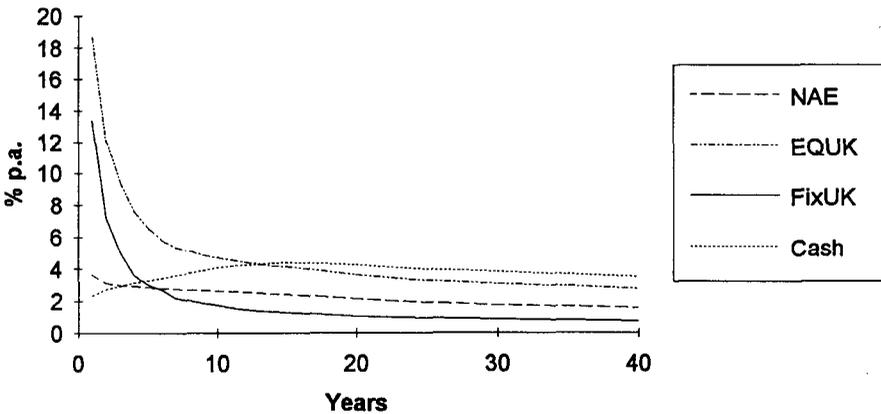
The correlation between the time series are also time dependant. As an illustration, the correlation of the simulated time series over 20 year horizon is

	NAE	Equity	Bonds	Cash
NAE	1.0	0.63	0.28	0.29
Equity		1.0	0.35	0.22
Bonds			1.0	0.12
Cash				1.0

**Log of Annualised Total Return**



**Standard Deviation of the Log of Annualised Returns**



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