PRICING EURODOLLAR TIME DEPOSIT FUTURES*

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Summary

This paper tests alternative binomial models for pricing CME-IMM Eurodollar time deposit futures contracts. These models are fitted to the initial term structure of interest rates and to volatilities estimated using ARCH methodology. The interest rate processes used in numerical pricing tests are the normal, the lognormal, the Cox-Ingersoll-Ross (CIR) square root, the CIR variable rate, and the constant elasticity of variance processes. The paper also compares price predictions for the square root process to exact prices calculated using the Cox-Ingersoll-Ross analytic model. Finally, since it studies short maturities for which predicted forward and futures prices differ very little for any of the model variants, the paper uses the difference between the forward price and the model's predicted futures price as a benchmark assessment of each model's prediction errors.

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Fixation du prix de contrats de dépôts à terme en EURODOLLARS*

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Résumé

Le présent exposé met à l'essai des modèles binômes alternatifs pour la fixation du prix de contrats de dépôts à terme en eurodollars CME-IMM. Ces modèles sont élaborés en fonction de la structure initiale des taux d'intérêt et des volatilités estimées à l'aide de la méthodologie ARCH. Les processus du taux d'intérêt utilisés dans les tests de fixation numérique du prix sont le normal, le lognormal, la racine carrée de Cox-Ingersoll-Ross (CIR), le taux de variable CIR et l'élasticité constante des processus de variance. Cet exposé compare également les prédictions de prix pour le processus de racine carrée aux prix exacts calculés à l'aide du modèle analytique Cox-Ingersoll-Ross. Enfin, puisqu'il se penche sur des maturités courtes pour lesquelles les prix des contrats à terme et des contrats à terme ferme prévus diffèrent très peu pour toute variante du modèle, cet exposé emploie la différence entre le prix à terme ferme et le prix des contrats à terme prévu par le modèle comme banc d'essai de chaque erreur de prédiction du modèle.

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PRICING EURODOLLAR TIME DEPOSIT FUTURES

While models for pricing options on interest sensitive securities have been extensively tested, there are few similar studies of futures contracts. Moreover, most of the available studies of futures contracts examine monthly data and report average pricing errors without taking the contracts' different maturities into account. Thus further study of interest sensitive futures contracts is warranted, and this paper reports tests of several such pricing models. It examines futures contracts on Eurodollar time deposits using weekly data, reporting both average pricing errors and their time series characteristics.

Groundwork for the theory of commodities futures pricing is developed in Black [1976], while properties of forward and futures contracts and prices are developed in Cox, Ingersoll, and Ross (CIR) [1981]. Jacobs and Jones [1980] are among the first authors to compare model predictions against observed futures prices. This approach, which has since been widely adopted, is discussed further in Black, Derman and Toy [1990], Hull and White [1990a], Ritchken and Sankarasubramanian [1990] and Jamshidian [1991].

Our approach is to compare predicted against actual prices of Eurodollar time deposit futures, using eight alternative models of the underlying interest rate process. Model predictions are compared against each other, against benchmark forward prices, and against observed futures prices. Our framework can account for delivery options in futures contracts, but we need not recognize this possibility here since our test data are for contracts without delivery options.

Our binomial models assume an arbitrage-free setting in which the basic inputs are the initial term structure of interest rates and an interest rate volatility. We use a flat volatility structure, but the conditional volatility can vary, depending on the process being investigated. To simplify numerical calculations, we transform the processes with changing volatilities using a procedure described by Hull and White [1990b]. In addition to the numerical models, we also test the CIR [1985] analytic model (whose solution assumes constant volatility).

Our numerical models take greater advantage of existing data than the analytic model does, and as a result give generally better predictions than the latter. The prediction errors are usually about the size of the benchmark forward - futures price difference, an encouraging result since for short maturities, forward and futures price data are empirically indistinguishable. Within the class of numerical models considered, no single interest rate process exhibits better overall explanatory power than other processes. Although the volatility estimates display large peaks, particularly in December, the models' price predictions are affected only minimally by the peaks.

I. Empirical approach

We investigate a family of interest rate processes by developing price predictions for each member of the family. The family studied contains the following processes: normal (Merton [1973], Vasicek [1977]), lognormal (Dothan [1978], Black and Scholes [1973], Brennan and Schwartz [1977]), square-root (Cox, Ingersoll and Ross [1985]), variable rate (Cox, Ingersoll, and Ross [1980]), and constant elasticity of variance (Cox [1975], Cox and Ross [1976]). We fit a discrete time version of each process to a binomial lattice which matches both the initial term structure of interest rates and a volatility measure. In all cases, the volatility estimates are determined from conditional variances obtained using an ARCH model.

To fit the data to the lattice, we use forward induction (as described in Jamshidian [1991] and in Black, Derman and Toy [1990]) to calculate future one period rates consistent with both the initial term structure and volatilities, assuming the existence of a martingale to do so. We then calculate futures prices using backward induction (cf. Cox, Ross, and Rubinstein [1979]) and the same martingale. Finally, we compare model prediction errors for different processes and different maturities.

I-A. Data

Our data are from two sources. For Eurodollar time deposit futures, we use the closing price for the first three contract maturities traded on the International Monetary Market (IMM) of the Chicago Mercantile Exchange, and convert IMM index data into futures prices. The contracts promise delivery of a Eurodollar time deposit with a maturity of three months, but are in fact settled in cash. Eurodollar time deposit futures are the most actively traded short term interest rate instruments and have maturities extending up to two years. Our futures price data consist of a total of 521 weekly observations and cover the period from January 6, 1982 to December 27, 1991.

Eurodollar time deposit rates are from Reuters. The interest rate series for 1, 30, 90, 180, and 360 day maturities extend from January 6, 1982 to December 27, 1991 while the 7-day series covers the period from January 2, 1980 to December 27, 1991. We use a longer series for the 7-day rates to maximize the number of observations in estimating the interest rate processes' parameters. The data are an average of bid and offer rates observed each Wednesday. If no rate was available on Wednesday we substituted the Thursday rate, or failing that, the Tuesday one for the same week. Table 1 reports descriptive statistics for the interest rate series.

I-B. Fitting the interest rate process

Let r be the short term riskless interest rate and let dZ represent an increment to a Wiener process. Chan, Karolyi, Longstaff and Sanders [1992] (CKLS) examine eight models nested within the stochastic differential equation

$$dr=(\alpha+\beta r)dt+\sigma r^{\lambda}dZ. \tag{1}$$

As CKLS observe, this unrestricted model nests models by Merton $(\beta = \lambda = 0)$, Vasicek $(\lambda = 0)$, Cox, Ingersoll and Ross square root (CIR SR) process $(\lambda = 1/2)$, Dothan $(\alpha = \beta = 0, \lambda = 1)$, geometric Brownian motion $(\alpha = 0, \lambda = 1)$, Brennan and Schwartz $(\lambda = 1)$, Cox, Ingersoll and Ross variable rate (CIR VR) model $(\alpha = \beta = 0, \lambda = 3/2)$, and the constant elasticity of variance (CEV) process $(\alpha = 0)$.

We face at least two problems when fitting these models to very short term interest rates. First and most important is the inadequacy of any model which attempts to describe the interest rate process' conditional variance as a function of only the interest rate level. To deal with this problem, we estimate the interest rate processes using an ARCH model (Engle [1982], Bollerslev [1986]) which allows the process conditional variance to change over time. Second, the interest rate data reveal outliers that seem to be concentrated near holidays, particularly in late December. To recognize this feature, we use an exogenous indicator variable for the December 20-31 observations.[1] We also set the value of this indicator variable to *1* for the week immediately following the July 4, 1986 holiday, when a large temporary interest rate jump occurred.

Let r_t be the 7-day interest rate from time t to $t+1$, I_t an indicator variable, and let ϵ_t have a conditional normal distribution with mean zero and variance σ_t^2. Corresponding to (1) in its unrestricted form, the system of equations to be estimated is

$$\begin{aligned} r_t - r_{t-1} &= \alpha + \beta r_{t-1} + \sigma_t r_{t-1}^{\lambda} \epsilon_t \\ \sigma_t^2 &= c + (1-b) u_{t-1}^2 + b \sigma_{t-1}^2 + \phi I_t \\ u_{t-1} &= \epsilon_{t-1} / r_{t-1}^{\lambda} , \end{aligned} \tag{2}$$

in which the time-dependent conditional variance is estimated as an integrated, generalized ARCH (IGARCH) process. The ARCH process, based on data known at time *t-1*, is standardized through division by r_{t-1}^{λ}, a transformation used by Hull and White [1990] and similar to one used (in an ARCH model) by Duan and Hung [1991].

Table 2, following CKLS, shows the results of comparing various pairs of nested models. Models without the interest rate in the variance or with only the square root of the interest rate, are rejected against the unrestricted model based on (1). Otherwise, the simpler model is retained in tests against models with extra parameters. In both these respects, our results with 7-day interest rates are similar to those of CKLS. For example, the Merton model is retained against the Vasicek model but both are rejected in favour of the unrestricted general model, as is the CIR SR model.

The Dothan model is retained against all other more complicated models, including the unrestricted model. The CIR VR model is also retained against either the CEV or the unrestricted model. The mean reversion parameter β does not improve the models' performance, implying that the 7-day interest rate data show no evidence of mean reversion. Thus we would expect either the Dothan or the CIR VR model to

[1]We ignore some outliers in early January. With the recursive nature of ARCH models, the increased conditional variance in December, tracked by the indicator variables, persists for some time.

perform best in price predictions. Nevertheless, for benchmarking purposes we also retain the other models in several of our subsequent tests.

Table 3 shows the estimate of λ in the unrestricted model (2) is *1.29.* Since the standard error is *0.46,* neither the Dothan value of *1.0* nor the CIR VR value of *1.5* for this parameter is inconsistent with the data. Table 3 shows the estimated parameters for the Dothan, CIR VR, CEV and unrestricted models. Table 4 gives some selected diagnostic test results for autocorrelation, ARCH model performance and remaining heteroscedasticity. Positive values of the runs test statistic are consistent with negative first order autocorrelation.[2] In other respects, the test statistics are satisfactory.

Along with the parameter estimates we use estimates of conditional variances; i.e., of $\sigma_t^2 r_{t-1}^{2\lambda}$, to obtain estimates of volatilities σ_t for each week in the sample period. We then use the latter estimates as inputs to the binomial lattice model of the next subsection.

II. Constructing the Lattice

To construct a lattice displaying the evolution of interest rates we require a term structure of zero-coupon bond yields and either a point estimate or a term structure of interest rate volatilities. Any necessary intermediate points on the yield curve are inferred by linear interpolation in a process equivalent to using geometric means of bond prices. Consistency of the lattice of future one period rates with the term structure and volatility estimates is obtained using forward induction (cf. Jamshidian [1991]; Black, Derman and Toy [1990]). The lattice is reconstructed for each week of data, thus accounting for changes in initial conditions.

Calculations are much easier to carry out if the volatilities are constant within the lattice, but some of the processes we investigate have changing volatility. To deal with this problem, we follow Hull and White [1990], transforming the processes using

[2]Discrete time data may not behave in diffusion fashion. Negative autocorrelation in asset prices subject to bid-ask spreads was analyzed by Roll (1984).

$$\phi(r) = \frac{r^{-\lambda+1}}{-\lambda+1} \; ; \; 0 \le \lambda \le 3/2 \; ; \tag{3}$$

where $\phi(r) \equiv ln\ r$ when $\lambda = 1$. The transformed version of (1) takes the form

$$d\phi = q(r,\ t)dt + \frac{\partial\phi}{\partial r} r^{\lambda} \sigma\, dz, \tag{4}$$

where the transformed drift term $q(r,\ t)$ is functionally dependent on r as well as on time. However since (3) gives

$$\frac{\partial\phi}{\partial r} = r^{-\gamma},$$

the volatility term in (4) is just σ, a constant independent of time; cf. Tian [1992].

To illustrate obtaining the volatility under the Hull-White transformation, consider the lognormal process, i.e., equation (3) with $\lambda = 1$. In the discrete time binomial approximation to (3), an initial interest rate $ln\ r$ either increases to $ln\ r_u$ or decreases to $ln\ r_d$. In this case the volatility is $(1/2)ln(r_u/r_d)$; cf. Black, Derman, and Toy [1990].

To illustrate how the lognormal process is then fitted to the data, suppose that the spot one and two period zero coupon yields are y_1 and y_2 respectively. In the binomial model the one period rate will be either r_u or r_d in the next period. Consistency means the rates must satisfy the condition

$$\frac{1}{2}(e^{-r_u} + e^{-r_d})e^{-y_1} = e^{-2y_2}. \tag{5}$$

(Note that we set the martingale probabilities to 0.5 and search for up and down rates which are consistent with the initial term structure. We could instead fix the up and down rates and search for martingale probabilities that produce prices consistent with the initial term structure as in Heath, Jarrow and Morton [1991]). Again for consistency, the two unknown rates and the exogenous volatility parameter (cf. section I) must satisfy

$$\frac{1}{2}\ln\left[\frac{r_u}{r_d}\right] = \sigma; \tag{6}$$

(cf. Hull and White [1990]). The rest of the lattice is obtained similarly.

The other processes are fitted to the initial term structure and volatility estimates in exactly the same fashion, although different processes require different transformations in the second equation. (Analogous to Tian's [1992] simplified binomial approximation, if for the CIR SR , CIR VR, and CEV processes the interest rate becomes negative at some point on the lattice, it is reset to zero.) Our lattices fit the two input term structures with accuracy of 10^{-8} in order to reduce approximation errors to a minimum.

III. Results

We predict futures prices by using the constructed lattice to evaluate futures contracts by backward induction, (cf. Cox, Ross, and Rubinstein [1979]). The only input to the lattice model from the short term interest rate equations is the estimated conditional volatility: we ignore the estimates of α and β in models with unrestricted parameters. We then calculate prediction errors as the difference between CME-IMM daily settlement Eurodollar futures prices and the futures prices estimated from the binomial lattice for the three nearest contract maturities (1 to 40 weeks). Table 5 presents descriptive statistics for the errors under each process as well as for the forward model, in which forward prices are inferred directly from the interest rate data.

Given the findings in Table 2, we would expect that the Merton, Vasicek and CIR SR models will not be as satisfactory as the others; and that models in which α and β are unrestricted will do no better than the equivalent models in which these parameters are suppressed. More specifically, Table 2 predicts the Vasicek model should not be an improvement on the Merton model, but neither should perform well relative to simple models such as Dothan and CIR VR. On the other hand, the Dothan model should do as well as the GBM or CEV models. The GBM model, in turn, should do as well as either the Brennan-Schwartz model or the CEV model. Similarly, the CIR VR model should perform as well as the CEV model.

Since the two preferred models, Dothan and CIR VR, can be nested only within the unrestricted model U, Table 2 does not distinguish between them. However Table 5, which begins with summary statistics for the data aggregated over all contract maturities, provides a direct comparison of the two. In Table 5 the mean squared errors for all models are virtually identical to the mean squared error for the forward rate as a predictor of the futures price. In summary, all models perform just as well when it comes to pricing Eurodollar time deposit futures.

Table 5 continues with summaries of the pricing errors broken down by contract maturity. We drop the Vasicek, GBM, Brennan-Schwartz and CEV models in these panels because they all have nested, simpler, counterparts that perform at least as well. Figures 1 and 2 show the pricing error patterns for this subgroup of models as a function of contract maturity and Table 6 reports the average pricing error statistics presented in these two figures. The mean squared errors all increase with contract maturity. The difference between forward and futures prices is used as a benchmark for comparison of the model prediction errors. It is difficult to discriminate between the various processes for the nearby contract with less than 13 weeks to maturity, but subtle differences arise for longer maturities. Overall, all models seem to overprice short lived contracts but their average pricing bias becomes progressively smaller with longer maturities.

In Figure 3, our numerical approach for the square-root process performs substantially better than the CIR closed form solution when the 7-day rate is used as the proxy for the instantaneous rate. By definition, any numerical approach must be less accurate than the corresponding exact solution which uses the same data. However, our numerical approach utilizes the whole term structure, and thus employs more information than the closed form solution using only a single rate of interest. The additional data improves the accuracy of the estimates to the point where the numerical model performs better than the analytic model which uses less information. The net improvement is obtained regardless of whether the proxy for the instantaneous rate in the closed form model is the 7-day rate or a longer term rate.

Figure 4 shows that the time series of conditional standard deviations for the Dothan model demonstrated great fluctuation during our sample period, with large peaks in December of each year. However, eliminating observations for December 20-31 had no sizeable impact on average pricing errors of the various interest rate processes considered in our study. Figure 5 shows the relationship (again for the Dothan model) between average pricing error and contract maturity for all observations as well as for a subsample excluding observations from December 20-31. There does not seem to be a significant late December increase in the price prediction error, thus providing further evidence that the model internalizes large changes in conditional volatility very well.

Table 7 compares theoretical prices of Eurodollar futures for maturities representative of our sample under the five processes examined in this paper. Futures prices are obtained by fitting the same term structures of interest rates and of volatilities to each process. The lattices match the two hypothetical input structures with an accuracy level of 10^{-8}. This experiment shows that model prices exhibit slight differences (in the order of 10^{-3}) for the maturities examined. We also observe a positive relationship between theoretical prices and the parameter λ of equation 1, which is consistent with theory and is useful for validation purposes.

The conditions under which our empirical work was carried out are quite different from those under which the experiment shown in Table 7 was conducted. Although in our empirical tests the same term structures of interest rates were used as an input for each process, the input volatilities were different for each model (cf. section I-B). Since volatility estimates conditional on each process were used to generate our empirical predictions, the small differences in pricing errors across models reported in this study support the widely held view that any one interest rate model can be made to provide accurate price predictions if it is properly calibrated.

IV. Conclusions

This paper has estimated prediction errors in futures pricing models based on a family of interest rate processes. Models based on the term structures of both interest rates and volatilities perform better than models using point estimates of these parameters. The lognormal, constant elasticity of variance, and variable rate processes provide the best statistical fit among all processes examined. However, when performance is judged by a model's ability to predict Eurodollar futures prices, none of the interest rate processes examined in this paper stands out as a clear winner. The models' predictions are all roughly equivalent to those of our forward price benchmark.

Even so, the models examined here tend to slightly overprice short lived contracts but this bias disappears beyond 13 weeks to maturity. In the short maturity data we consider, interest rate models with a small number of parameters, such as the Dothan model, perform as well as more complicated models that allow for mean reversion. The analytic CIR square root model does not perform well but its arbitrage-free numerical version provides substantially more accurate predictions.

Our evidence clearly shows the merits of the arbitrage-free framework compared to traditional models which use only a small fraction of all the information contained in the term structure of interest rates. Our evidence also shows that price predictions for interest rate instruments with maturities of less than one year are not sensitive to the process chosen to model the evolution of rates.

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Table 1 : Descriptive statistics for Eurodollar time deposit interest rates
521 weeks from 82/01/06 to 91/12/27

	1-day	7-day	30-day	90-day	180-day	360-day
mean	8.410	8.491	8.563	8.691	8.846	9.100
standard deviation	2.185	2.122	2.138	2.218	2.305	2.319
minimum	4.438	4.438	4.750	4.313	4.313	4.375
maximum	24.500	16.500	16.625	16.625	16.625	16.375

Table 2 : Likelihood ratio tests of eight interest rate models

Midweek 7-day eurodollar rates for 1980-1991. Sample size 625 weeks.

H_0	*L*	H_1	H_1:LRT	H_2	H_2:LRT	H_3	H_3:LRT
M	271.041	V	2.16 (0.14)			U	35.39 (0.00)
V	272.123					U	33.23 (0.00)
CIR SR	281.386					U	14.70 (0.00)
D	287.441	GBM	0.04 (0.84)	CEV	2.34 (0.31)	U	2.59 (0.46)
GBM	287.460	BS	0.01 (0.92)	CEV	2.30 (0.13)	U	2.56 (0.28)
BS	287.465					U	2.55 (0.11)
CIR VR	287.405	CEV	2.41 (0.23)			U	2.67 (0.45)
CEV	288.610					U	0.26 (0.61)
U	288.738						

Notes: M = Merton, V = Vasicek, CIR SR = Cox, Ingersoll, Ross square root, D = Dothan, GBM = geometric Brownian motion, BS = Brennan and Schwartz, CIR VR = Cox, Ingersoll, Ross variable rate, CEV = constant elasticity of variance, U = unrestricted. *L* is the log likelihood (excluding the constant), LRT is the likelihood ratio test statistic, for which the p-values in parentheses are for the chi-square distribution. H_0 is the null hypothesis under test against a more general model in each case.

Table 3 : Parameter estimates for selected models

$$\begin{aligned} r_t - r_{t-1} &= \alpha + \beta r_{t-1} + \sigma_t r_{t-1}^{\lambda} \epsilon_t \\ \sigma_t^2 &= c + (1-b) u_{t-1}^2 + b \sigma_{t-1}^2 + \phi I_t \\ u_{t-1} &= \epsilon_{t-1} / r_{t-1}^{\lambda} . \end{aligned} \qquad (8)$$

model	α	β	λ	c	b	φ
D	0.	0.	1.	0.037	0.228	0.053
				(0.011)	(0.100)	(0.025)
CIR VR	0.	0.	1.5	0.055	0.155	0.084
				(0.011)	(0.061)	(0.039)
CEV	0.	0.003	1.263	0.046	0.184	0.068
		(0.014)	(0.394)	(0.016)	(0.078)	(0.034)
U	-0.026	0.034	1.288	0.047	0.180	0.070
	(0.099)	(0.121)	(0.463)	(0.010)	(0.065)	(0.031)

Notes. D = Dothan, CIR VR = Cox, Ingersoll, Ross variable rate, CEV = constant elasticity of variance, U = unrestricted. I_t is an indicator variable for December 20-31. Standard errors, shown in parentheses, are robust [Bollerslev and Wooldridge (1988)].

Table 4 : Diagnostic test statistics for models in Table 3

model	Z	Q(10)	$Q^2(10)$	JSSB
D	3.79	19.09	6.87	1.23
	(0.000)	(0.039)	(0.738)	(0.298)
CIR VR	3.79	19.51	10.89	2.07
	(0.000)	(0.034)	(0.366)	(0.103)
CEV	4.35	19.44	8.93	1.76
	(0.000)	(0.035)	(0.539)	(0.154)
U	3.32	19.49	9.09	1.67
	(0.001)	(0.035)	(0.524)	(0.172)

Notes. D = Dothan, CIR VR = Cox, Ingersoll, Ross variable rate, CEV = constant elasticity of variance, U = unrestricted. p-values are shown in parentheses. Z is the runs test statistic, e.g. Lehmann (1976), with unit normal distribution aymptotically. Q(10) is the Ljung and Box (1974) portmanteau test statistic for 10 lags of the autocorrelation function of the standardised residuals and $Q^2(10)$ the equivalent for their squares. Both of these are assumed to have a chi-square distribution with 10 degrees of freedom under the null hypothesis. JSSB is the joint sign and size bias test of Engle and Ng (1991) for asymmetric response in the ARCH model performance; the null hypothesis distribution is F with 3 and 621 degrees of freedom.

Table 5 : Pricing errors from interest rate models and forward pricing model
82/06/01 to 91/12/27.

Panel	Contract maturity (observations)	model	mean	mse	minimum	maximum
A	all	Merton	-0.0070	0.0090	-1.30	0.55
	1563	Vasicek	-0.0069	0.0089	-1.29	0.55
		CIR SR	0.0609	0.0164	-1.32	0.55
		Dothan	0.0082	0.0032	-0.27	0.55
		GBM	0.0082	0.0032	-0.27	0.55
		BS	0.0082	0.0032	-0.27	0.55
		CIR VR	0.0686	0.0150	-0.19	0.56
		CEV	0.0674	0.0150	-0.43	0.56
		Forward	0.0038	0.0030	-0.26	0.54
B	shortest	Merton	0.0159	0.0019	-0.14	0.22
	521	CIR SR	0.0116	0.0071	-1.32	0.22
		Dothan	0.0184	0.0018	-0.10	0.22
		CIR VR	0.0189	0.0018	-0.10	0.22
		Forward	0.0106	0.0016	-0.13	0.23
C	second	Merton	-0.0064	0.0061	-0.57	0.30
	521	CIR SR	0.0517	0.0109	-0.33	0.38
		Dothan	0.0078	0.0028	-0.19	0.30
		CIR VR	0.0595	0.0105	-0.19	0.39
		Forward	0.0046	0.0026	-0.21	0.27
D	longest	Merton	-0.0307	0.0190	-1.30	0.55
	521	CIR SR	0.1194	0.0311	-0.20	0.55
		Dothan	-0.0016	0.0052	0.27	0.55
		CIR VR	0.1274	0.0328	-0.18	0.56
		Forward	-0.0037	0.0049	-0.26	0.54
E	all	Merton	-0.0031	0.0085	-1.30	0.55
	(exc Dec)	Vasicek	-0.0030	0.0084	-1.29	0.55
	1434	CIRSR	0.0687	0.0160	-0.79	0.56
		Dothan	0.0091	0.0033	-0.27	0.55
		GBM	0.0091	0.0033	-0.27	0.55
		Bren-Sch	0.0091	0.0033	-0.27	0.55
		CIRVR	0.0716	0.0154	-0.19	0.56
		CEV	0.0716	0.0153	-0.19	0.56
		Forward	0.0048	0.0031	-0.26	0.54

Note: mse = mean squared error

Figure 1

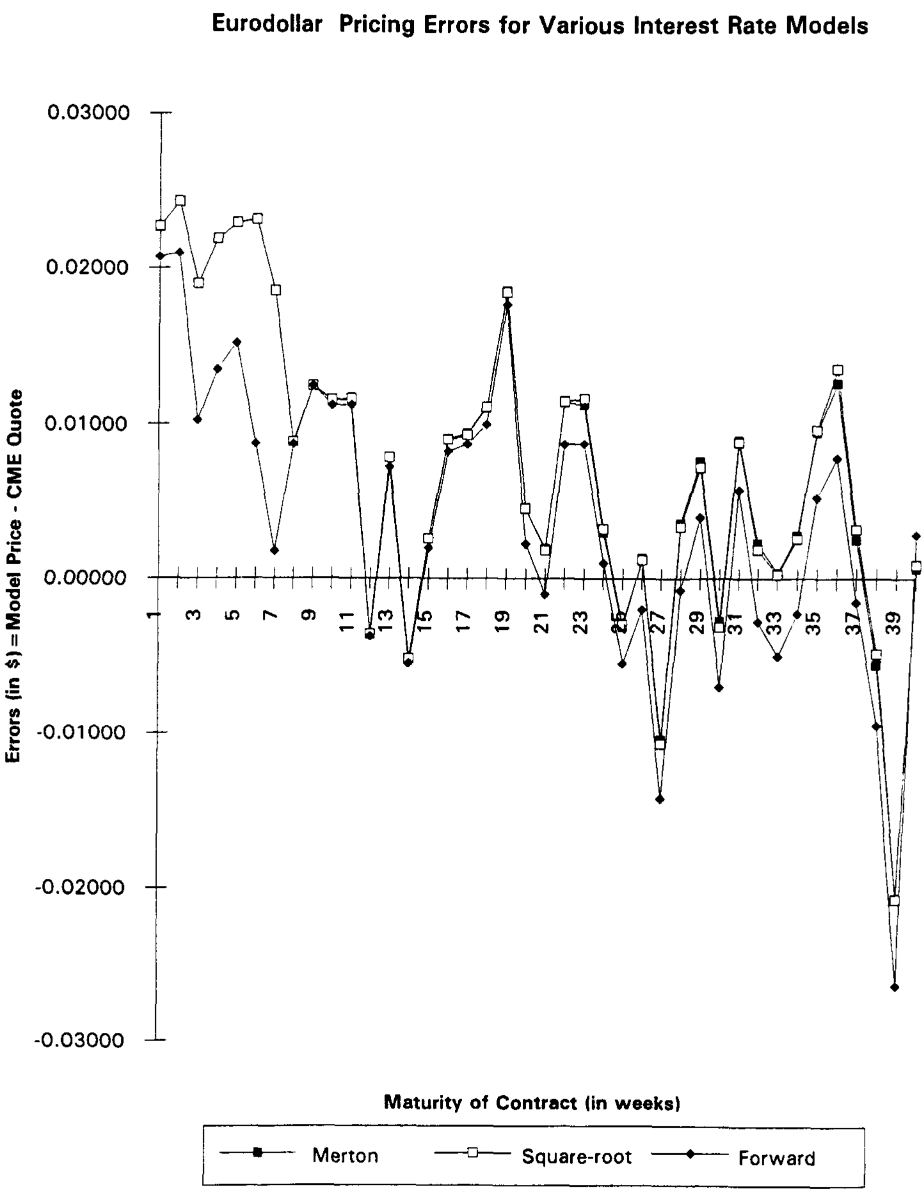

Eurodollar Pricing Errors for Various Interest Rate Models
0.03000
0.02000
0.01000
0.00000
-0.01000
-0.02000
-0.03000
Errors (in $) = Model Price - CME Quote
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
Maturity of Contract (in weeks)
Merton
Square-root
Forward

Figure 2

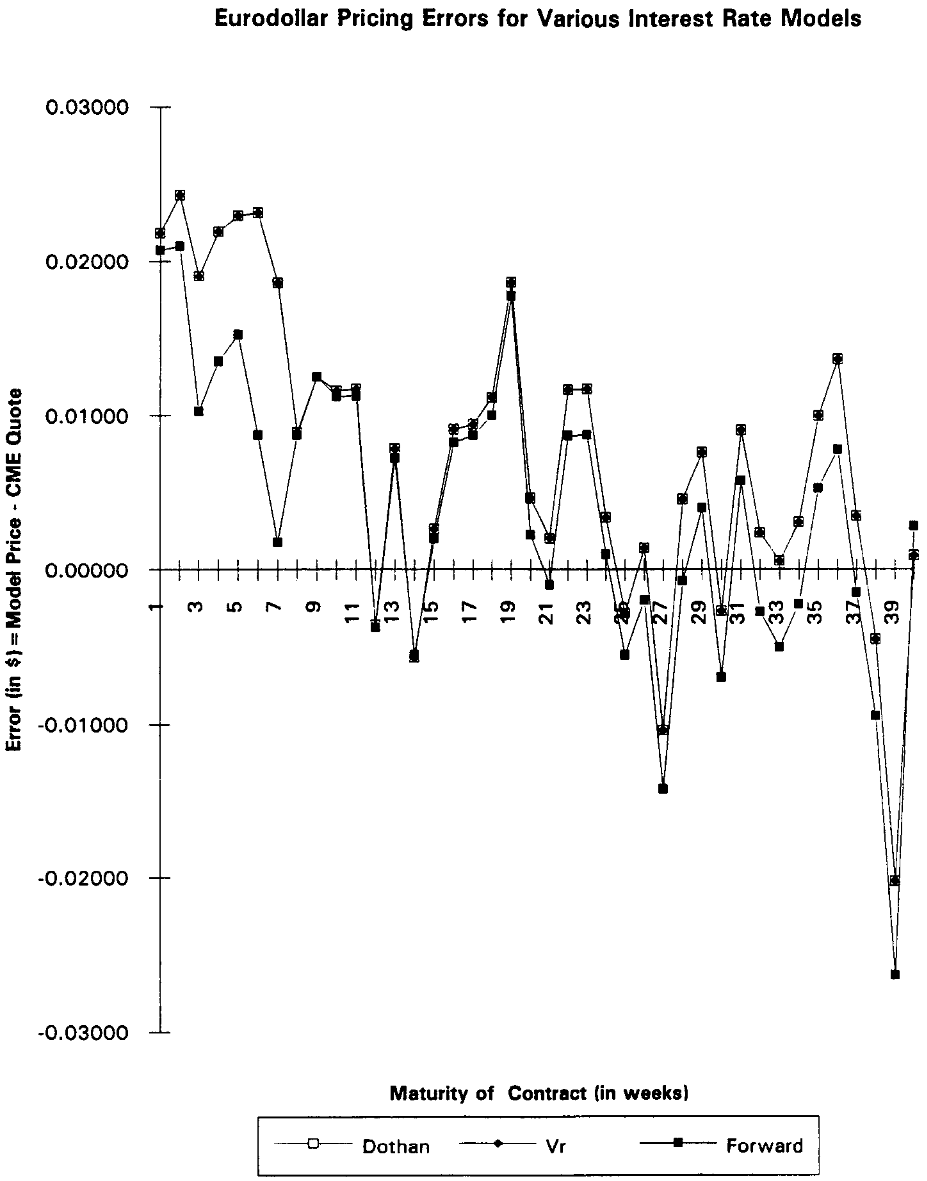

Eurodollar Pricing Errors for Various Interest Rate Models
0.03000
0.02000
0.01000
0.00000
-0.01000
-0.02000
-0.03000
Error (in $) = Model Price - CME Quote
1
3
5
7
9
11
13
15
17
19
21
23
27
29
31
33
35
37
39
Maturity of Contract (in weeks)
Dothan
Vr
Forward

Figure 3

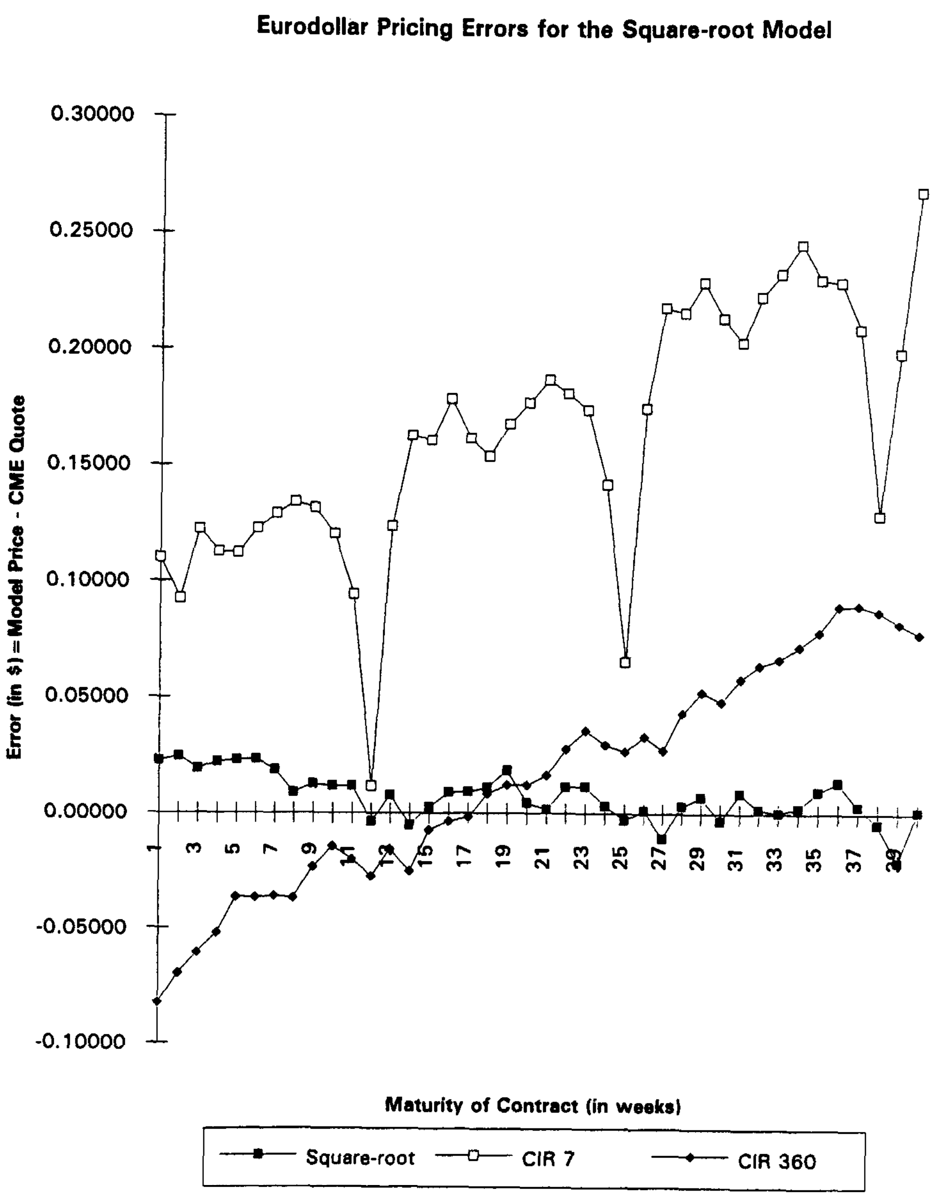

Eurodollar Pricing Errors for the Square-root Model
0.30000
0.25000
0.20000
0.15000
0.10000
0.05000
0.00000
-0.05000
-0.10000
Error (in $) = Model Price - CME Quote
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
Maturity of Contract (in weeks)
Square-root
CIR 7
CIR 360

Figure 4

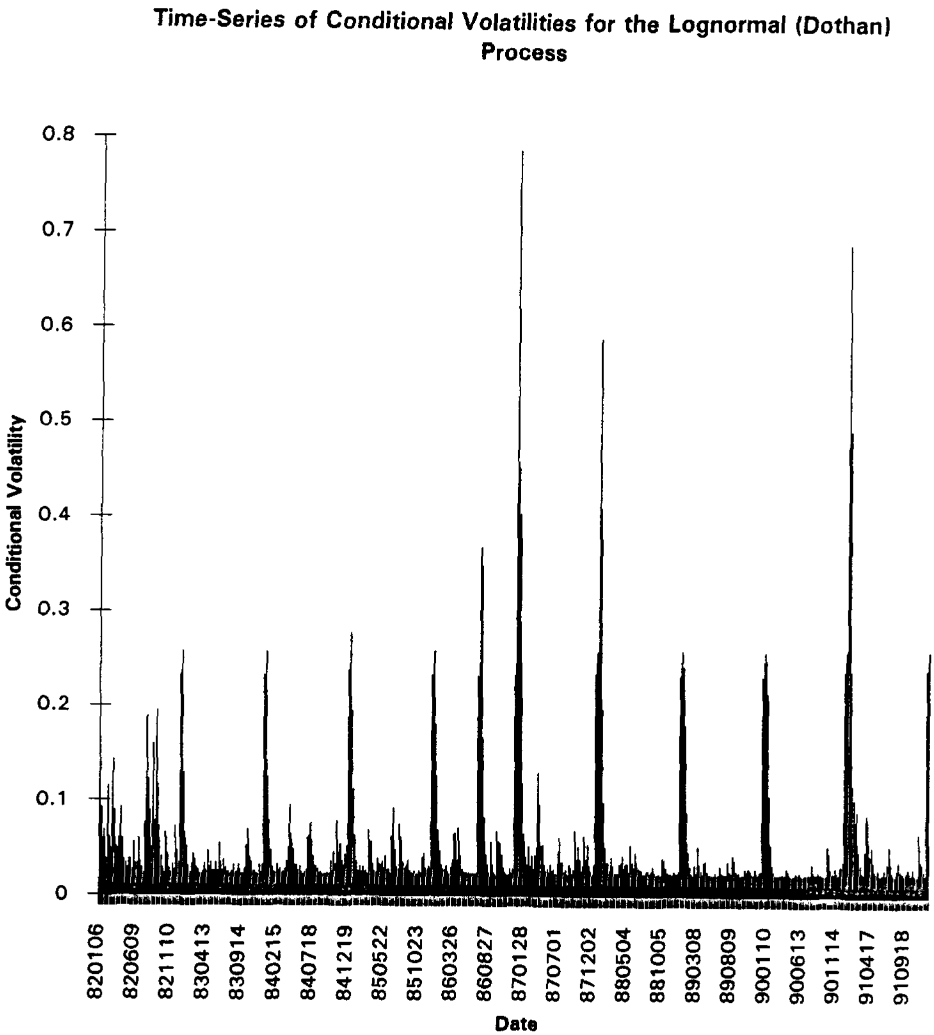
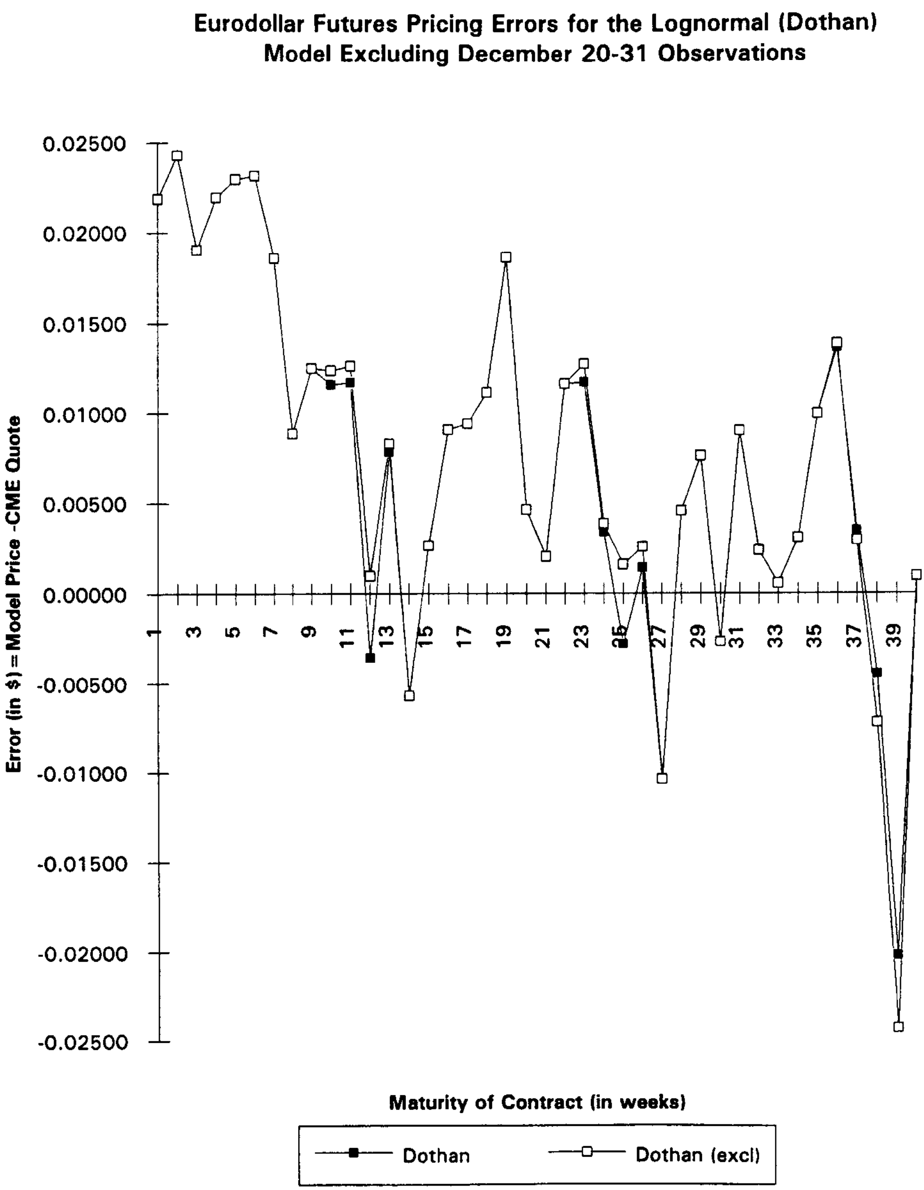

Time-Series of Conditional Volatilities for the Lognormal (Dothan) Process
Conditional Volatility
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
820106
820609
821110
830413
830914
840215
840718
841219
850522
851023
860326
860827
870128
870701
871202
880504
881005
890308
890809
900110
900613
901114
910417
910918
Date

Figure 5


Eurodollar Futures Pricing Errors for the Lognormal (Dothan) Model Excluding December 20-31 Observations
0.02500
0.02000
0.01500
0.01000
0.00500
0.00000
-0.00500
-0.01000
-0.01500
-0.02000
-0.02500
Error (in $) = Model Price -CME Quote
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
Maturity of Contract (in weeks)
Dothan
Dothan (excl)