RISK TOLERANCE OF INSURANCE COMPANIES

Matthew S. Easley, FSA
Stephen A. J. Sedlak, FSA
Nationwide Life Insurance
One Nationwide Plaza
Columbus, Ohio 43216
U.S.A.
Phone: (614) 249-5228 or 6742
FAX: (614) 249-0725

Summary

All too often, the risk aversion of insurance companies is ignored in pricing and other important decisions. However, an insurer's risk aversion can have important consequences for the company and those with whom it has dealings. In this paper, the risk aversion of a life insurance company is estimated on a quantitative basis. Based on these results, a representative utility curve for the company is derived. Some examples are given which demonstrate the effects of recognizing a company's aversion to risk.

Tolérance des risques des compagnies d'assurance

Matthew S. Easley, FSA
Stepehn A. J. Sedlak, FSA
Nationwide Life Insurance
One Nationwide Plaza
Columbus, Ohio 43216
Etats-Unis
Téléphone : (614) 249-5228 ou 6742
Télécopie : (614) 249-0725

Résumé

Trop souvent, l'aversion pour les risques d'une compagnie d'assurances est ignorée lors de la détermination des prix et lors de la prise d'autres décisions importantes. Toutefois, l'aversion d'un assureur pour les risques peut avoir des conséquences importante pour la compagnie et pour ceux qui font affaire avec elle. Cet article estime l'aversion d'une compagnie d'assurances pour les risques de manière quantitative. Sur la base de ces résultats, une courbe exprimant l'utilité est établie pour cette compagnie. Certains exemples illustrent les effets de la prise en compte de l'aversion qu'a une compagnie pour les risques.

SECTION I - DETERMINING A COMPANY'S RISK AVERSION

An experiment was designed to quantify the attitudes toward risk in our own company. This experiment involved asking each group of management responsible for our seven major areas of business to react to seven risk scenarios. This kind of experiment and the considerations involved were discussed in our previous paper on risk based pricing (1). A copy of the script that was used along with the responses are attached.

Each risk scenario was described as a one time project with only two different possible outcomes, one a loss and the other a gain. The participants were given the amount of the possible loss and its probability. They were then asked to choose the amount of gain which they would want in order to make the project worthwhile in their opinion. In this regard they were provided the "breakeven" gain (that which would give the project an expected gain of zero).

The risk scenarios were put in the context of having $700 million of surplus to invest and support the company. First, each individual was asked to make their own determination. The values were then placed in front of the entire group which was asked to discuss them and to come up with a group consensus for each scenario. The discussions varied substantially by group. We acted only as facilitators and the groups themselves determined what they wished to discuss.

The results are summarized in the following table:

Loss		Breakeven Gain	Desired Gain	
Amount	Probability		Mean	Median
10	20%	2.5	3.57	3.57
50	20	12.5	20.00	19.50
100	20	25.0	80.00	45.00
100	10	11.1	31.07	24.00
100	50	100.0	NO	NO
250	20	62.5	NO	NO
500	10	55.5	NO	NO

Assuming that these results were the point of indifference (where the gains expected by management were sufficient to offset the potential losses), we have seven equations of the form:

$$\mathbf{P(Loss)\ U(Loss) + P(Gain)\ U(Gain) = 0}$$

where P(Loss) and P(Gain) are the probabilities of loss and gain respectively and U(Loss) and U(Gain) are the utilities of loss and gain. When no amount of gain was considered acceptable, we set the gain to infinity and changed the equation to an inequality relationship.

It is interesting to note that the fact that there are risks that the company does not want to take for any price means that the positive end of the utility function must be bounded. This is a characteristic of many utility functions. One of the more interesting of these utility functions is the exponential function which can be given in the following form:

$$U(x) = 1 - e^{-rx}$$

Having selected this utility function, we used a least squares fit of the data to arrive at a value for r of .0083. For this purpose, we assumed we had a good fit if the inequalities used for the last three scenarios were met. The "correct" gains using this value of r would have been 2.6, 16.6, 47.0 and 18.7 with the last three projects being refused. These values can be compared with those in the table of results.

Alternatively, if we normalized our outcomes by expressing them as fractions of surplus (instead of millions), the value of r becomes 5.82. Normalizing the outcomes by dividing by the level of surplus is based on the idea that the ability to bear risk is proportional to surplus. This is designed to cause the r factor to be more durable across time and more comparable across various enterprise companies. In addition, it is more realistic in that it accounts for phenomena such as the willingness to undertake limited high risk ventures.

It is worth noting that the handling of the data was sensitive. Use of the mean responses from the seven groups instead of the median would have given a value for r of 9.03 instead of 5.82. The use of median results decreased the impact of extreme values. An alternative method of fitting the results would have been to minimize the absolute deviations. This would have produced a value for r = 5.48.

The exponential utility function has several useful features. It and all of it's derivatives are continuous for all real values. This avoids the need to recognize points of discontinuity in calculations. The first derivative is positive which is indicative of the "more is better" characteristic of how economic wealth is perceived. Also, the second derivative is negative for all values which is indicative of risk averse behavior, with r as a measure of the aversion.

Second, the exponential utility function is also bounded as the value of x becomes arbitrarily large, but has no lower bound with decreasing values of x. This is consistent with the existence of a risk that one would refuse to take at any price. In the example above, if we denote the probability of loss by q, the smallest loss which our utility function tells us we would reject is ln(q)/r. Thus, for the values of q used in our experiment (.1, .2 and .5), the limit for acceptable losses would be -278, -194, and -84 respectively. However, this would not be a good model for individuals or institutions where the attitude is that there is a fair price for any risk. Those interested in an unbounded model may wish to explore other utility functions.

To see a third feature of the exponential function, we define the expected utility value (EV) as a value such that its utility equals the expected value of the utilities of a random variable X. Symbolically:

$$U(EV_x) = E[U(X)]$$

For any exponential utility function of the form $U(x) = a - b\cdot e^{-rx}$ we have

$$EV_x = -\ln[\frac{a-E[U(x)]}{b}]/r = -\ln(E[e^{rx}])/r$$

The exponential function has the useful property that the EV which results from the sum of a number of independent random variables is equal to the sum of the EV for each of them. If we define S as the sum of n independent random variables, $x_1, x_2, \ldots, x_n$

$$E[U(S)] = E[a-b\, e^{-rs}] = a-b\, E[e^{-rs}] = a-b\, M_s(-r)$$

where $M_s(-r)$ is the moment generating function of S. Since the random variables of S are independent, its moment generating function can be factored into the product of the moment generating functions of $x_1, x_2, \ldots, x_n$(2). Thus:

$$E[U(S)] = a-b\,M_{x1}(-r) \cdots M_{x_n}(-r) = a-b\,E[e^{-rx_1}]\,E[e^{-rx_2}] \cdots E[e^{-rx}$$

Substituting in the above expression for EV gives:

$$EV_s = -\ln(E[e^{-rx_1}] \cdots E[e^{-rx_n}])/r = -\ln E[e^{-rx_1}]/r \cdots -\ln E[e^{-rx_n}]/r$$
$$= EV_{x_1} + \cdots + EV_{x_n}$$

Thus, the expected utility value of a sum of independent random variables is equal to the sum of their expected utility values.

SECTION II - EXAMPLE OF OPTIMAL PRODUCTION

One example of how the concept of utility can be used is in analyzing marginal profits for a given product or line of business. Consider a simple, one year term product which has the following characteristics:

> The company expects to spend 20% of premium in marginal costs and additionally has $3 million in fixed costs.
>
> The company expects to spend 70% of premium on claims, but the distribution of expected claims is a 10% probability of getting claims equal to 140% of premiums, 20% of getting 100% of premiums, 50% of getting 60% of premiums and 20% of getting 30% of premiums.
>
> The company has an exponential utility function with r = 5.8 using the ratio of gains (losses) to surplus as input to the utility function. The company's surplus is $700 million.

On an expected value basis, the company expects to break even at $30 million of production and make an additional $2 million for every additional $20 million of production. However, if we recognize the company's risk averseness, the volatility in results will cause our outlook to change a great deal. The peak result now occurs at about $100 million of production and the result becomes negative again for $180 million or more of production as shown below:

Amount of Production	Expected Profits	EV of Profits
$20 million	$-1 million	$-1.17 million
40 million	1 million	0.29 million
60 million	3 million	1.38 million
80 million	5 million	2.09 million
100 million	7 million	2.40 million
120 million	9 million	2.29 million
140 million	11 million	1.77 million
160 million	13 million	0.81 million
180 million	15 million	-0.58 million
200 million	17 million	-2.41 million

This analysis shows an alternative view of the value of new business which is counter to the idea that "more is always better" in selling insurance products. Concentration of risk in a particular product (or group of products with similar risks) can reduce the true value of a company to a risk averse owner. In most businesses, there are issues of critical mass to cover fixed costs as illustrated by this example.

The "dis-economies" of scale in the utility function adjustment identify a window of profitable operation. However, if the fixed costs had been $6 million instead of $3 million, the expected profits would simply have been $3 million lower with a need to sell $60 million to have expected profits which break even. In contrast, the EV would never be positive implying that the product is inherently unprofitable at any level of production when management's risk aversion is taken into account. This says that the project's margins are not sufficient to overcome its fixed cost and to provide enough profit to compensate for risk.

If the level of surplus is increased in proportion to the fixed cost, the optimum production level increases by the same ratio as does the EV of profits at that level. Thus, more can be better, but only if we have enough capital to support it. The main point is that capital is needed to support risk as well as to pay the costs of acquiring and administering insurance business and this need may be enough to make some business opportunities inadvisable.

The same kind of analysis can be made using probability distributions of the outcomes.

In fact, for some distributions, our exponential utility function has the added advantage of allowing us to determine EV in closed form. For purposes of illustration, let's assume that our gain, G, is related to premium, P, less benefits and expenses which are expressed as a percentage of premium, L, and fixed costs, C:

$$G = (1-L)P - C$$

Also, assume that L is normally distributed with mean μ and standard deviation σ. This assumption implies that negative benefits and/or expenses are possible. This is not likely to be true for most insurance products and a better assumption would be something which is non-negative and positively skewed (e.g. lognormal). However, using a normal distribution enables us to obtain a simple formula for EV_G so that we can illustrate some of its properties. In addition, as long as the standard deviation is small relative to the mean, adverse effects from this optimistic tail will be minimized. Due to the utility function's relatively lower weighting of favorable results, this is even more true.

Finally, we'll assume that our utility function is exponential in terms of G in proportion to surplus, S:

$$U(G) = 1-e^{-rG/S}$$

From the above

$$EV_G = -\ln E[e^{-rG/S}]/r = -\ln E[e^{-r((1-L)P-C)/S}]/r$$
$$= -\ln[\frac{\mu}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{\frac{(L-\mu)^2}{2\sigma^2}} e^{-r((1-L)P-C)/S} dL]/r$$

Expanding and rearranging the terms in the exponent

$$EV_G = -\ln[\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{-\frac{(L-Q)^2 + \mu^2 - Q^2 + 2r\sigma^2(P-C)/S}{2\sigma^2}} dL]/r$$

Where

$$Q = 1 + rP\sigma^2/S$$

However, the first part of the integrand in this expression is a normal p.d.f., albeit with the mean increased by $rP\sigma^2/S$, so that

$$EV_G = -\ln[e^{-\frac{\mu^2 - Q^2 + 2r\sigma^2(P-C)/S}{2\sigma^2}}]/r$$
$$= -\mu P/S - 1/2R\sigma^2P^2/S^2 + (P-C)/S = -1/2r\sigma^2P^2/S^2 + (1-\mu)P/S - C,$$

This relatively simple expression gives the same type of result as was shown in the above example in that there is an optimum level of production. If we take the first derivative with respect to $p = P/S$, we have:

$$dEV_G/dp = 1-\mu-r\sigma^2P$$

So EV_G has a maximum at $p = (1-\mu)/r\sigma^2p$ and is zero at

$$p = [(1-\mu) \pm\sqrt{(1-\mu)^2 - 2r\sigma^2C/S}]/r\sigma^2$$

as long as the result is not imaginary. If we substitute the values from the example above ($\mu = .7 + .2 = .9$, C = \$3 million, S = \$700 million and r = 5.8) and use the variance of the four ratios of claims to premium ($\sigma^2 = .104$) we have maximum EV_G at P = .17 and "breakeven" points at P = .05 and .28. This is in fairly close agreement to the values shown above.

The effects of volatility are also evident. The smaller the variance, the larger the value at which the maximum occurs (i.e. the higher the tolerance for the business in question). Also, the points at which results are zero tend to diverge (as long as positive values of EV_G are possible).

Of course, this assumes that σ does not vary with the amount of business written. There are situations where this is true (e.g. increasing volumes of risks which cannot be diversified). However, to the extent that the in force volume is comprised of a number of risks which are fully independent, increasing production will reduce σ. Thus if σ is valid for some level of in force premium volume, P_0, under increased volume P our normal distribution will have a reduced variance

$$\sigma' = \sigma \sqrt{P_0/P}$$

If we substitute this into the above we have

$$EV_G = -1/2\ r\sigma^2\ P_0/S^2 + (1-\mu)P/S - CS$$

Again, letting p = P/S and taking the derivative with respect to p:

$$dEV_G/dp = 1-\mu-r\sigma^2 P_o/2S$$

Note that this value does not vary with p, which says thatmore volume is better, as long as dEV_G/dp is positive and the risks are completely independent.

SECTION III - ACCUMULATION OF RISK

Of course, in actual practice, the risks of most insurance business are not as black and white as those in the last section. For example, for a group of individual life policies the mortality risks may be almost completely independent for each insured life. On the other hand, the risk that expenses may increase beyond those assumed in pricing is not independent to a large degree. Similarly, the default disintermediation, and reinvestment risks of these policies appear to be correlated as well.

The above example can be modified to illustrate a potential problem with using utilities on anything less than an enterprise basis if risks are not fully independent. If we have two areas in the company which are isolated from each other and they each follow the strategy given in the example, they will each try to write about $100 million of premium. Unfortunately, if they both succeed, the company will be collectively far past the optimum level of production. In fact, they will have destroyed value in the sense that the utility to the enterprise of their combined business is negative.

To the extent that a material amount of a company's risks are not independent, some recognition of this fact must be made for utility based analysis to be worthwhile. One approach is to monitor the company's entire portfolio and encourage or discourage growth accordingly, even to the point of reducing existing in force. This can be done as an adjunct to any other process of resource allocation which may be in place but requires some means to monitor business plans and to quantify the utility adjusted results. In addition, some process for deciding which lines to support and which lines to cut back needs to be in place.

If the results for a company are projected using a corporate model, the results can be run across a number of economic scenarios which will produce an array of financial results. The impact of writing more or less of each line can then be projected and the impact of doing so evaluated.

The concept can also be applied in pricing for an individual line by looking at the array of results for a particular line, but this will not capture the impact of the correlations among the lines. While it may be difficult to obtain cross line results, failing to consider the impact of correlations is equivalent to assuming independence. Said another way, this ignores increasing concentration of risk for lines whose results are correlated (e.g., a number of accumulation oriented annuity lines) and the spread of risk on independent lines.

It may be that a satisfactory approximation can be found for the inter-line effects which can then be used to increase or decrease the risk charge used in pricing a particular product depending on the way it interacts with the other lines. Alternatively, surplus can be allocated to each line. In addition, each line might be allowed to have its own measure of risk aversion. This "profit center" approach has the advantage of letting each line operate fairly independently once its capital allocation has been established. However, the need for a process to make these allocations on a periodic basis remains.

SECTION IV - IMPACT OF DEPENDENCY ON A COLLECTION OF RISKS

The degree of correlation of a collection of risks has a profound impact on the utility of that total compared to the individual risks. Let's consider a simple example using an exponential utility function with $r = .1$ and the following two independent outcomes:

A1 with probability of 80% and value 3.
A2 with probability of 20% and value -5.
B1 with probability of 50% and value 5.
B2 with probability of 50% and value -2.

The expected utility value (EV) of event A is .8078; and the EV of event B is .8996. If A and B are independent, we get the following compound distribution:

The probability of A1 and B1 is 40% with value 8 = 3 + 5.
The probability of A1 and B2 is 40% with value 1 = 3 - 2.
The probability of A2 and B1 is 10% with value 0 = 5 - 5.
The probability of A2 and B2 is 10% with value -7 = -5 - 2.

The EV of this distribution is 1.7074 = .8078 + .8996 because of the additive property discussed in Section II. However, if we change the probability of B1 and B2 to be dependent upon the outcomes A1 and A2, the result is different. For example, consider the following change in the assumed probabilities:

Let the conditional probability of B1 be 60% if A1 occurs and 10% if A2 occurs. Note that the probability of B1 is still 50% in total. The probability of B2 is the complement of B1.

The probability of A1 and B1 is now 48% with value 8.
The probability of A1 and B2 is now 32% with value 1.
The probability of A2 and B1 is now 2% with value 0.
The probability of A2 and B2 is now 18% with value -7.

$$\begin{aligned} U(EV) &= .48(1-e^{-.8}) + .32(1-e^{-.1}) + .02(1-e^{-0}) + .18(1-e^{.7}) \\ &= .264322 + .030452 + 0 - .182475 \\ &= .112299 \end{aligned}$$

$$EV = -LN\,(1 - .112299)/.1 = 1.1912$$

The EV of compound distribution has decreased from 1.7074 when the risks were independent to 1.1912. This is true even though the expected value of 2.9 and the probabilities of each outcome considered separately are unchanged. The positive correlation of the outcomes has increased the risk and lowered the equivalent value.

To look at a more complex situation, we built a model which simulates the results of multiple identical risks using Monte Carlo techniques. Used with an assumption of independence, it produces an approximation to the binomial distribution. However, the program allows a degree of dependence to be introduced.

Suppose we have a company with 10 distinct, but identical, lines of business. In each line of business, the probability of making money is 75% and the probability of losing money is 25%. When a line is profitable, it will make $10 million. When it loses money, it will lose $30 million. Hence, the expected gain is zero.

If we put this in the context of a company with $700 million of surplus by dividing our results by this surplus and use a risk parameter of $r = 5.8$ (which matches our experimental results), we find that the EV USINGprecise values from the binomial distribution is -$13 million. This means that each line would need to charge an additional $1.3 million as a risk charge for this level of risk.

Now we will compare three different versions of this same basic situation. The first is calculated under the assumption of independence using the binomial distribution. The second version illustrates the impact of a positive correlation of the events. As the program calculates the probability of profit for each line of business, a positive result from the preceding line causes the probability of a given line making money to increase to 80%. A loss from the preceding line results in the probability falling to 60%. This would be similar to having several lines of business which are vulnerable to the same shift in some factor such as interest rates, inflation, the weather, health care trends, or in the economic climate. Note that the total probability of each line of business being profitable remains at 75%.

The third version illustrates the impact of a negative correlation of the events, which would be similar to a situation where the risks of various lines tend to occur at different times such that they tend to offset. An example might be mortality improvements as they impact a life insurance line versus an annuity line. Here, we assume that profit in the preceding line of business now lowers the probability of profit to 70% while the probability rises to 90% after a loss.

EXPECTED OCCURRENCE PER 100,000 TRIALS

Gain(Loss)	Independent Results	Positive Correlation	Negative Correlation
$-300 Million	0	4	0
-260 Million	3	56	0
-220 Million	39	293	1
-180 Million	309	1,107	22
-140 Million	1,622	3,271	360
-100 Million	5,840	7,288	3,464
-60 Million	14,600	13,752	14,679
-20 Million	25,028	20,416	30,695
20 Million	28,157	23,495	31,824
60 Million	18,771	20,132	15,997
100 Million	5,631	10,186	2,958
Expected Gain (000s)	-1	-32	-35
Expected Utility Value (EV)			
r = 5.82	-13,146	-19,734	-8,794

The spread of the results is much greater under the positive correlation. The greater risk entailed by the higher probability of large losses is captured by the utility function calculation. The opposite is true of the negative correlation. The results are clustered nearer to the expected value and the utility function calculation reflects this lower level of risk. The necessary risk charge increases by 50% as a result of the positive correlation, but reduces by 33% as a result of the negative correlation.

SECTION V - REINSURANCE AND RETENTION LIMITS

When insurance companies determine the amount of reinsurance that they will buy on their life insurance portfolio, they are implicitly or explicitly applying utility concepts. In general, they must pay more for the reinsurance than the expected value of the claims or the reinsurers would all go out of business (and some have). Therefore, the ceding company is making a cost/benefit decision about eliminating volatility from their results. To illustrate this situation, we will examine a small, hypothetical portfolio of 120 lives in two groups. The first group has 100 people with 2 units of insurance each and a probability of death of 2% during the year. The second group has 20 people with 10 units of insurance each and a probability of 4% of dying during the year. The following table shows the distribution of losses expected from the portfolio:

Total Losses	Probability	Average Loss
0-10	41.95%	3.635
10-20	37.21	13.464
20-30	15.71	23.298
30-40	4.20	33.136
40-50	0.83	43.159
50-60	0.08	51.371
60-70	0.01	63.506
70-80	0.001	72.529
80-90	0.0001	82.380
90+	0.00001	92.629
Expected Value of Loss		12.000
Expected Utility Value (r = .1)		17.601

This shows that recognition of the volatility of risk can add substantially to price. If the company wishes to examine options for reinsurance, it might consider two options: stop loss on total claims and proportional reinsurance. The ceding company will reduce its expected claims and the EV of claims. The following table shows these reductions under several reinsurance schemes assuming r = .1:

Aggregate Stop Loss Retention	Expected Value of Reinsured Loss	Improvement In Expected Utility Value
15	2.503	6.806
16	2.194	6.323
17	1.952	5.923
18	1.711	5.499
19	1.502	5.112
20	1.294	4.700
Proportional Reinsurance		
40%	4.800	8.602
50	6.000	10.387
60	7.200	12.046

In each case, the expected value is decreased, but the expected utility value is decreased more. This is a result of transferring a portion of the risk to the reinsurer. However, the reinsurance will not be free. If we assume that the reinsurer will charge 200% of the expected value to assume a risk, we get the following result:

	Reinsurer Risk Charge	Total Risk Charge
Aggregate Stop Loss Retention		
15	5.006	15.801
16	4.388	15.666
17	3.904	15.582
18	3.422	15.524
19	3.004	15.493
20	2.588	15.489
Proportional Reinsurance		
40%	9.600	18.599
50	12.000	19.214
60	14.400	19.955

The total risk charge is the sum of the ceding company's EV on its retained business and the reinsurer's risk charge. The best coverage would be the aggregate stop loss with a limit of 20. The proportional reinsurance is a bad deal because the EV of the original risk (17.601) is less than 200% of the expected claims (24.000). A higher constant load on expected value for stop loss coverage would simply raise the optimal stop loss level. The negative utility value of large losses is unbounded and will exceed any constant charge at some stop loss level. However, the amount of the expected claims transferred may be small.

In the above discussion, we assumed that the reinsurer would be willing to accept any of the risks for a fixed percentage of the expected claims transferred. If the reinsurer is using the same type of utility function to evaluate the charge for offering reinsurance, the volatility of the risk that we transfer will increase the price of the coverage. With the reinsurer using the same r = .1 factor, the optimal coverage is 50% proportional reinsurance with a total risk charge of 14.427. If the reinsurer is using a r = .05 factor (or if the reinsurer has twice as much surplus and is normalizing the results), the optimal reinsurance is 67% proportional reinsurance. These results imply that the best reinsurance deal is to allocate the risk proportionately to the ability and willingness to bear risk. Thus, the stop loss coverage tends to be the best deal when the reinsurer is pricing as a percentage of expected claims (and that percentage exceeds the ratio of the ceding company's EV to expected claims), but proportional reinsurance is optimal if both insurers are using the exponential utility function. In practice, pricing would vary by reinsurance company and any preconceived notions about which coverage would be optimal are potentially dangerous.

Footnotes:

1. Easley, Matthew S. and Sedlak, Stephen A. J. Risk Based Pricing of Life Insurance Products. Second AFIR International Colloquium, Brighton, 1991, Volume 3, pp. 199-217.

2. DeGroot, Morris H. Probability and Statistics. Menlo Park, CA: Addison-Wesley, 1975, pp. 164-165.

ATTACHMENT

UTILITY EXPERIMENT SCRIPT

Initial comments - the purpose is to try to quantify various opinions about risk. There is not a correct answer to these questions. We are doing this with all management teams and eventually with Life Cabinet. This is the second group - the first was a group of actuaries at an offsite meeting. The results will be used for real pricing and for a paper.

We will examine seven situations where the company has the opportunity to take a risk for a price. We will tell you the size of the risk and you will put a price on taking that risk. We will tell you the minimum price needed with a zero risk charge - the excess over that will be the additional price for taking the risk.
Another way of looking at it is that you are setting the house odds for Nationwide. Refusing to take the risk is an acceptable answer.

These risks can only be taken once, not several times. They are in the form of a one time reinsurance deal over a year with no residual risk. No assets transfer except for cash at year end.

After the first answers are done, discuss the appropriateness of these types of risks, relate them to a percentage of surplus and to the Hugo and Andrew losses and the investment in a start up business. Talk about the consequences of such a loss to Nationwide Life, and ask them to use the second page to indicate what they would select as their team answers after the discussion.

After the second answers are done, put the second set of answers on the board (no names) and discuss the range of answers. Ask the management team to come up with consensus answers, not necessarily at that meeting.

ANSWER SHEET

	Loss	Probability	"Breakeven" Gain	Desired Gain
Situaton 1	$ 10MM	$20%	$2.5MM	______
Situation 2	$ 50MM	20%	$12.5MM	______
Situation 3	$100MM	20%	$25MM	______
Situation 4	$100MM	10%	$11.1MM	______
Situation 5	$100MM	50%	$100MM	______
Situation 6	$250MM	20%	$62.5MM	______
Situation 7	$500MM	10%	$55.5MM	______

RESULTS OF UTILITY EXPERIMENT WITH THE MANAGEMENT TEAMS

Situation	Team 1	Team 2	Team 3	Team 4	Team 5	Team 6	Team 7
1	4	4.5	3	4.5	3	3	3
2	25	22.5	14	25	18	17.5	18
3	150	50	35	200	45	40	40
4	35	32	20	75	20	17.5	18
5	NO	NO	NO	NO	450	NO	225
6	NO	NO	NO	NO	190	NO	250
7	NO	NO	NO	NO	NO	NOP	NO
Best Fit for r:							
	10.7	7.2	4.8	11.6	3.7	4.9	3.8

Model Answers Using Various Values of r

Situation	3	5	7	9	11	13
1	2.75	2.80	2.85	2.91	2.96	3.02
2	15.40	17.03	18.96	21.29	24.17	27.83
3	35.90	45.30	60.15	88.52	209.03	NO
4	15.32	18.47	22.70	28.64	37.58	52.95
5	191.46	NO	NO	NO	NO	NO
6	163.93	NO	NO	NO	190	NO
7	451.93	NO	NO	NO	NO	NO