Quality Option

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Abstract

We calculate in this paper the value of the quality option for the french future contract notional. We construct our model in a very intuitive way under the general non arbitrage framework. We show that in certain circumstances the quality option can have very a significant value that must imperatively be taken into account.

Option de qualité

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Résumé

Nous calculons dans le présent exposé la valeur de l'option de qualité pour les contrats à terme français. Nous élaborons notre modèle de façon très intuitive dans un cadre de non arbitrage général. Nous montrons que dans certaines circonstances l'option de qualité peut avoir une valeur significative dont il faut impérativement tenir compte.

Introduction

A future contract is written on a basket of deliverable securities. At maturity the holder of a short position on the future contract choose the asset he delivers. The money he receives depends on the delivery factor of the asset and on the quotation of the future contract on the last trading day. So, at maturity in general there is one asset that is cheaper to deliver than the other. The opportunity that the holder of a short position on the future contract has to choose which asset to deliver is called the quality option. In the US, the holder of a short position on the future can also choose, within a fixed period, the day at which he delivers the asset, this is the timing option. We will study, in this paper, the case of the french notional contract Matif for which only the quality option exists. There has been already many articles on this subject : P.Boyle 1989 [1] modelizes the price of the deliverable assets with a multivariate lognormal distribution, for which he must estimate the correlation matrix, and develops an algorithm for computing the expectation of the mean (P.Boyle and Y.K.Tsee [2]). Other more recent articles such as Ritchken & Sankarabrumanian [8] or Cherubini and Esposito [3] use more advanced modelisations of the term structure of interest rates, as the one designed by Heath Jarrow Morton [5] or Jamishidian [6], to derive in a general non arbitrage framework the price of the quality option. Koenigsberg [7] extends the study to the case where there are callable assets in the basket. In our paper, we show that with very few calculations and very few assumptions, we can derive very simply, the price of the quality option.

Hypothesis and Notations

In the economy there is an instaneous riskless interest rate r_s, a future contract f, and several deliverable assets.

f_t is the price of the future at time t.

P_t^i is the price at time t of bond i.

F_t^i is the value at time t of a forward for date T on bond i.

Fc_i is the delivery factor for bond i and for delivery date T.

$r_t^i(T)$ is the forward rate for date T calculated at time t for bond i.

r_s is the instantaneous riskless interest rate.

$\beta_t = \exp\left(-\int_0^t r_s ds\right)$ is the actualisation factor between date 0 and date t.

$B_s(T)$ is the price of a zero coupon bond of nominal 1, maturing at T.

P is the risk neutral probability. This means that the discounted price of any asset is a martingale under P.

We have to keep in mind that :

i) on delivery day for all bonds : $Fc_i \times f_T \le P_T^i$

and equality is obtained for the cheapest to deliver.

ii) the price of the future is a martingale under P.

iii) for the forward we have $E\left[\frac{\beta_T}{E[\beta_T]} F_T^i\right] = F_0^i$.

proof:

i) obvious

ii) Consider the strategy which consists in buying one future contract at time t and selling it immediately at time $t+dt$. As there is no free lunch, the expected return of this strategy, which requires no initial fund, must be zero.

So we must have, $E[df_t / \Im_t] = 0$.

This prooves that the future is a martingale under P.

iii) Consider the strategy which consists in buying at date 0 one future contract at price F_0^i and selling it at time T, at price F_T^i. There is no initial investment.

So we must have, $E\left[\beta_T\left(F_T^i - F_0^i\right)\right] = 0$

This prooves *iii)*.

Determination of the quality option

Usually traders price the future by a cash and carry argument.

For most of them the theoretical price of the future equals $Min \frac{F_0^i}{Fc_i}$.

In fact this formula takes into account neither the possibility of a change of the cheapest to deliver nor the effect of the margin accounts.

We showed previously that the price of the future contract is a martingale. This means that the theoretical price of the future is $E\left[Min \frac{F_T^i}{Fc_i}\right]$.

The quality option is by definition the difference between these two quantities, that is

$$Min \frac{F_0^i}{Fc_i} - E\left[Min \frac{F_T^i}{Fc_i}\right]$$

to calculate this quantity we can write in the form

$$Min\frac{F_T^i}{Fc_i}=\sum_{i=1}^{i=n}\frac{F_T^i}{Fc_i}1_{D_i}\quad with\quad D_i=\bigcap_{j\neq i}\left\{\frac{F_T^i}{Fc_i}<\frac{F_T^j}{Fc_j}\right\}$$

we're going to use a single factor deformation model for the yield curve to calculate this quantity.

The forward rates $r_0^1(T),r_0^2(T)\ldots\ldots r_0^n(T)$ between date 0 and date T change with respect to a certain factor Δx (the movement is not exactly a translation).
D_i can be written as a union of subsets $\{\alpha_j^i<\Delta x<\gamma_j^i\}$ (see next section for the determination of this subsets).
so we have to calculate quantities of the type,

$$E\left[\frac{F_T^i}{Fc_i}1_{\alpha_j^i<\Delta x<\gamma_j^i}\right]=E\left[\frac{F_T^i}{Fc_i}1_{\Delta x<\gamma_j^i}\right]-E\left[\frac{F_T^i}{Fc_i}1_{\Delta\gamma<\alpha_j^i}\right]$$

but we can write,

$$E\left[\frac{F_T^i}{Fc_i}1_{\Delta x<\gamma_j^i}\right]=\frac{1}{Fc_i}\times E\left[\left(F_T^i-K^i\left(\gamma_j^i\right)\right)1_{F_T^i>K^i\left(\gamma_j^i\right)}\right]+\frac{K^i\left(\gamma_j^i\right)}{Fc_i}P\left(\Delta x<\gamma_j^i\right)$$

where $K^i\left(\gamma_j^i\right)$ is the price at time T of asset i for a variation of Δx equal to γ_j^i.

Note that the first term is the value of a call of strike $K^i\left(\gamma_j^i\right)$ on asset i.

The model

Our philosophy in this party is to say that you neither need an atomic weepon to kill a fly, nor a whole term structure model to calculate a quality option.

We would like for the simplicity of calculus, forwards to be martingales under the risk neutral probability, and we're going to look at what we need for this.
Note that from remark 1, the martingale property is achieved for example if the discounting factor β_T is independant of the forward rate $r_T^i(T)$.

In general the difference between the forward price at time t and its expectation at maturity is as follows (cf Cox Ingersoll Ross [4])

$$E\left[F_T^i / \Im_t\right] - F_t^i = -\frac{1}{B_0(T)} E\left[\int_t^T \frac{\beta_T}{B_s(T)} d < B_s(T), F_s > / \Im_t\right]$$

(< , > is the quadratic variation operator).

This shows that the biais, due to the covariance of the volatilities, is not very important, for reasonable volatilities and time horizons, and is anyway very difficult to estimate precisely. So when you don't know anything about something, only that it is small, why not considering it to be equal to zero ?

So, theorically our hypothesis of martingale for forward price is not incompatible with a non arbitrage framework, and practically this hypothesis can hardly be improoved as the biais of volatility is very difficult to estimate.

For forward prices to be martingales we need forward rates to verify an equation of the form :

$$dr_t^i(T) = -\frac{1}{2}\frac{\left(F_T^i\right)''\left(r_t^i(T)\right)}{\left(F_T^i\right)'\left(r_t^i(T)\right)}\sigma_i^2 dt + \sigma_i dW_t$$

and we obtain

$$dF_t^i = \left(F_t^i\right)' \sigma_i dW_t$$

We're going to take the same σ for all assets and this seems quite natural as all the rates are for assets of about the same duration. We also take σ constant. For computation of the quality option all derivatives for F are going to be calculated with the forward rates at time zero.

This means that for the volatility of asset i we take $S_T^i\sigma$ where S_T^i is the sensibility of asset i at time 0 and for the forward rate $r_0^i(T)$.

So, forward prices are lognormal in our calculations and Δx is the value of σW_T.

Application

We calculate for every possible value of Δx (quotation figures are discrete), the cheapest to deliver and the intervals $\left[\alpha^i_j, \beta^i_j\right]$.

The 14/10/1993

the price of the french notional future contract is : 124.42

the delivery day for the future contract is : 31/12/1993

and the caracteristics of the deliverable bonds are as follows

	coupon	price	repo	forward	forward rate	factor
25 01 2001	9.5	121.12	6.97%	121.33	5.74 %	97.5163
25 11 2002	8.5	117.48	6.94%	117.49	5.91 %	91.3844
25 04 2003	8.5	117.68	6.97%	117.70	5.96 %	91.0813
25 10 2003*	6.75	105.46	6.94%	105.58	5.98 %	80.1982

* This bond is on the primary market.

The volatility on the future is 4%. (this gives σ=0.69%)

On the delivery day :

25/01/2001 will be the cheapest if Δx < 3.07 %

25/04/2003 will be the cheapest if 3.07 % < Δx < 3.795 %

25/10/2003 will be the cheapest if Δx > 3.795 %

So, in this case, there is "no doubt" on the cheapest to deliver at maturity and we find 0 for the quality option.

Now we study a fictive case where the volatility on the future is 5.8% and where there is a big uncertainty about the cheapest to deliver at maturity.

So, the figure are as follows,

The 14/10/1993

	coupon	price	repo	forward	forward rate	factor
25 01 2001	9.5	97.41	6.97%	96.96	10.12 %	97.5163
25 11 2002	8.5	91.25	6.94%	90.87	10.01 %	91.3844
25 04 2003	8.5	90.95	6.97%	90.58	10.01 %	91.0813
25 10 2003*	6.75	79.99	6.94%	79.78	10.09 %	80.1982

On the delivery day (31/12/1993)

25/01/2001 will be the cheapest if $\Delta x < 0$

25/11/2002 will be the cheapest if $0 < \Delta x < 0.01\ \%$

25/04/2003 will be the cheapest if $0.01\ \% < \Delta x < 0.075\ \%$

25/10/2003 will be the cheapest if $\Delta x > 0.075\%$

So there is a great uncertainty about the cheapest to deliver and we expect the quality option to be significant. It is what the computation gives as we find 0.3 for the price of the delivery option...

Conclusion

What we find in our paper is that the quality option can have a substantial value for people trading the future against the cash. When the price of the future is near 100 and when there is a significant probability of change of the cheapest to deliver, the quality option must be taken into account to price the future correctly. Note that in a practical way the quality option results from a duration spread amongst the deliverable bonds.

Biliography

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