

# RISK / RETURN HISTORIES: MEASURING FUND MANAGER PERFORMANCE OVER TIME

BY

L. A. BALZER, PhD (Cantab), BE, BSc (NSW), FIMA(UK), FIEAust, CMath

Manager, Investment Technology  
Lend Lease Corporate Services Limited  
Level 64, MLC Centre  
19-29 Martin Place  
SYDNEY NSW 2000  
AUSTRALIA

Tel: 61 2 225 7030

Fax: 61 2 225 7011

## Summary

Fund manager performance has been shown traditionally by 'league tables' of manager returns and their rankings. In the eighties, the now familiar risk/return scattergrams began to appear and allowed advisors and trustees to make explicit trade-offs between risk and return. Being 'snapshots', which show only the average performance over a fixed period of time, they fail to reveal more than they actually show. By using rolling periods for the analysis and by plotting the path traced out over time in the risk/return plane, a manager's complete history of performance can be revealed. Furthermore, if the median return and median risk for the manager's peer group are subtracted at each point in time, a *Relative Risk/Return History*, or '*Snail Trail*' as they have become known, evolves. By plotting both return and risk relative to their respective medians, the manager's value adding and risk management skills *relative* to its peer group are laid bare for all to see. These Histories are also valuable in showing the effects of mixing managers in split funding arrangements. In some cases, real diversification of risk does occur but in others, qualitatively different management styles produce actual returns which exhibit surprisingly similar patterns over time. Hence, the diversification is illusory rather than real. Correlations of relative returns are also shown to provide a good guide to real diversification. Finally, *Risk/Return Ranking Histories* or '*Snail Trails Mk III*' are introduced. These show a manager's skills relative to the *whole* peer group and not just the *median* risk and return. Furthermore, a non-parametric rank correlation coefficient, which arises naturally from these diagrams, proves to be an excellent indicator of real diversification of risk in multi-manager arrangements.

**Historiques des risques/rendements :  
Mesure des performances des gestionnaires de fonds  
dans le temps**

L.A. Balzer, PhD (Cantab), BE, BSc (NSW), FIMA(UK), FIEAust, CMath

Directeur, Investment Technology  
Lend Lease Corporate Services Limited  
Level 64, MLC Centre  
19-29 Martin Place  
SYDNEY NSW 2000  
AUSTRALIE

Téléphone : 61 2 225 7030

Télécopie : 61 2 225 7011

**Résumé**

Les performances des gestionnaires de fonds étaient évaluées traditionnellement par les 'league tables', tableaux des rendements obtenus par les gestionnaires, en établissant un classement. Dans les années 1980, les diagrammes de dispersion de risques/rendements à présent familiers ont fait leur apparition et ont permis aux conseillers et aux administrateurs de prendre les décisions en faisant explicitement la part du risque et du rendement. Mais ces diagrammes sont en fait des "photographies instantanées", qui n'indiquent que les performances moyennes au cours d'une période fixée et ils cachent beaucoup plus de choses qu'ils n'en révèlent actuellement. Il est possible, en utilisant des périodes glissantes pour l'analyse et en établissant dans le temps une courbe située dans le plan risque/rendement, de faire apparaître un tableau chronologique complet des performances d'un gestionnaire. De plus, si le rendement moyen et le risque moyen du groupe de pairs du gestionnaire sont soustraits à chaque point temporel, on obtient un *historique relatif du risque/rendement, dit "trainée d'escargots"*. En établissant les courbes du rendement et les courbes du risque par rapport à leur moyenne respective, on peut faire apparaître clairement les capacités du gestionnaire à ajouter de la valeur et à gérer le risque, par rapport à celles du groupe de ses pairs. Ces historiques permettent également de montrer les effets obtenus en réunissant plusieurs gestionnaires dans le cadre d'arrangements de financement répartis entre eux. Dans certains cas, il se produit bien une véritable diversification des risques, mais dans d'autres, les styles de gestion qualitativement différents produisent des rendements réels qui présentent des modèles étonnamment semblables à terme. La diversification est donc plus illusoire que réelle. On démontre également que les corrélations des rendements relatifs fournissent une bonne indication de la diversification réelle. Enfin, on présente ici les *historiques de classement des risques/rendements ou "trainées d'escargots Mark III"*. Ces historiques font apparaître les aptitudes du gestionnaire par rapport à *l'ensemble* du groupe de ses pairs et pas seulement par rapport aux risques et aux rendements *moyens*. De plus, le coefficient de corrélation de rang non-paramétrique qui se dégage naturellement de ces diagrammes se révèle être un excellent indicateur de la diversification réelle du risque dans les arrangements à gestionnaires multiples.

## INTRODUCTION

In the 'dark ages', or at least the 'grey ages', all we had were league tables showing fund managers ranked according to their returns over the last 'x' months or years.

Then came the now familiar risk/return scatter diagrams with return on the vertical axis and standard deviation of that return, as a proxy for risk, on the horizontal axis. These diagrams were a great step forward and allowed explicit trade-offs between return and the variability of that return to be made. Their greatest limitation, however, was and still is, that they present a single snapshot in time – one which shows the *average* performance over a set period of time, often three years.

As we shall see later, some fund managers can and do undergo quite significant, even dramatic, changes over such periods. Unfortunately, the traditional risk/return 'snapshot' fails to capture, let alone highlight, such changing performance.

## RISK/RETURN HISTORIES

Rather than present a static snapshot showing many managers on one graph, a dynamic history of a single fund's performance over time can be shown. To do so, the first point on the risk/return graph is plotted as usual. Both the beginning and end points of the time period are then rolled forward by one month and the return and risk for the new period calculated and plotted on the same graph. By repeating this process, a trajectory is traced out dynamically in risk/return space, as shown in Figure 1. (The MLC Balanced Fund is a highly diversified Australian superannuation/pension fund totalling \$A3.2 billion at present.)

This type of *total* risk/return history is useful in portraying how a fund's performance has varied over time, but it fails to reveal how much of that performance is due to the manager's skills and how much is due to fortuitous or unfortunate market movements enjoyed or suffered by all managers.

A good indication of the skill component can be obtained by constructing a *Relative Risk/Return History*, where the median risk and median return (for an appropriate universe of similar funds) are subtracted from the fund's results. This process highlights a manager's value adding and risk management skills relative to its peer group.

Figure 2 presents such a *Relative Risk/Return History* for the same fund as shown in Figure 1. The results are quite enlightening. Using three year rolling periods since August 1989, this fund has consistently delivered above median returns with below median volatility, compared with its Australian peer group.

Showing results relative to the median or average fund manager is clearly not only a reasonable approach, but a very powerful tool for comparing fund managers.

### Relative Return

To be clear on the interpretation of Relative Risk/Return Histories, we must first be clear on exactly what is being plotted and what is not.

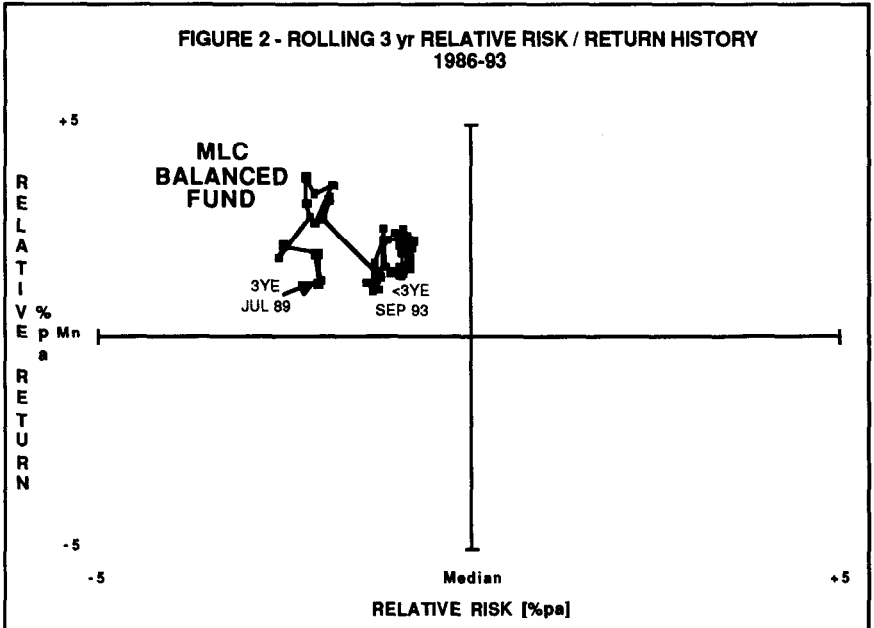
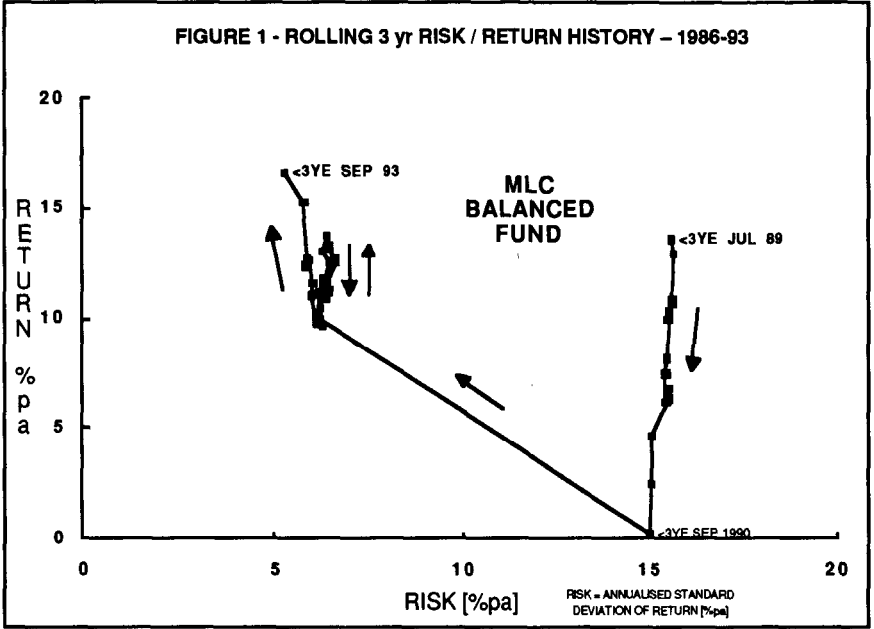
Relative return is plotted on the vertical axis.

In Figure 2, three year periods are used, as in many industry standard analyses. The first three year period starts in August 1986. (This starting point coincides with full implementation of Lend Lease Corporate Services' current disciplined investment management process.)

After the compound average annual return over the three year period is calculated, the median return for the same period for the universe of medium and large pooled superannuation funds is subtracted to give the *relative* return. Figure 2 shows that the relative return for the first period is about 1% pa above the median return for the peer group.

### Relative risk

Relative risk is plotted on the horizontal axis.



Whilst there are a number of alternative proxies for, or measures of risk, the most commonly accepted one, namely standard deviation of return, has been chosen for these analyses. There is no reason why Relative Risk/Return Histories could not be constructed using other measures of risk.

The standard deviation of monthly returns over the three year period is calculated and annualised using the standard  $\sqrt{12}$  factor (which implicitly assumes a random walk stochastic model for the return series). The *relative risk* is then calculated by subtracting the median standard deviation for the peer group and is plotted on the horizontal axis. In Figure 2, the relative risk for the first period is about -2% pa, ie. 2% below median.

It is important to note that the horizontal axis is not the standard deviation of the relative return, but the relative standard deviation of the total return. Use of the former would show how well a particular fund manager tracked the median return. This might or might not be a useful measure depending on one's purpose. A low figure would simply indicate that the manager was nicely tracking the median return, which itself might be undesirably volatile. On the other hand, a low figure for relative standard deviation, as defined in this paper, implies low volatility of returns in the absolute sense, which is clearly desirable.

### Rolling periods

After the first point is plotted, the three year period is rolled forward one month and the calculations repeated. This includes calculating a new median return and a new median standard deviation.

The period of analysis is rolled forward another month, the calculations repeated and so on.

The importance of using rolling periods results from the fact that performance analyses are often very sensitive to end point effects. For example, financial year figures often differ noticeably from calendar year results. One of the major advantages of the

above rolling approach is that, except for intra-month effects, all results are revealed – good or bad.

### Interpreting the relative risk/ return histories

The performance figures used in this paper are both 'after tax' and 'after fees'.

One way of visualising the Relative Risk/Return Histories is to think of fixing the median 'crosshairs' on a conventional risk/return scatter diagram in the centre of the page and then watching how a manager moves *relative* to them as the period of analysis is rolled forward in time.

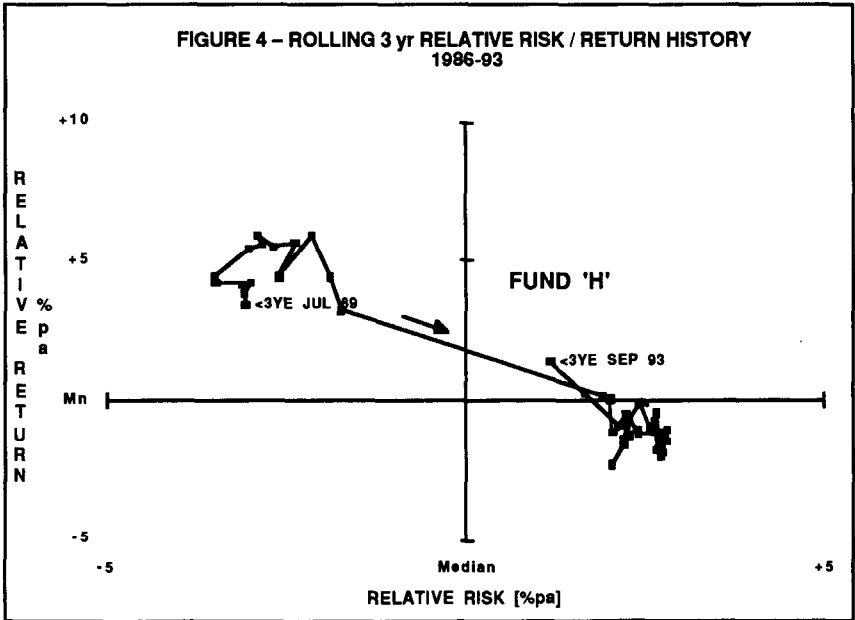
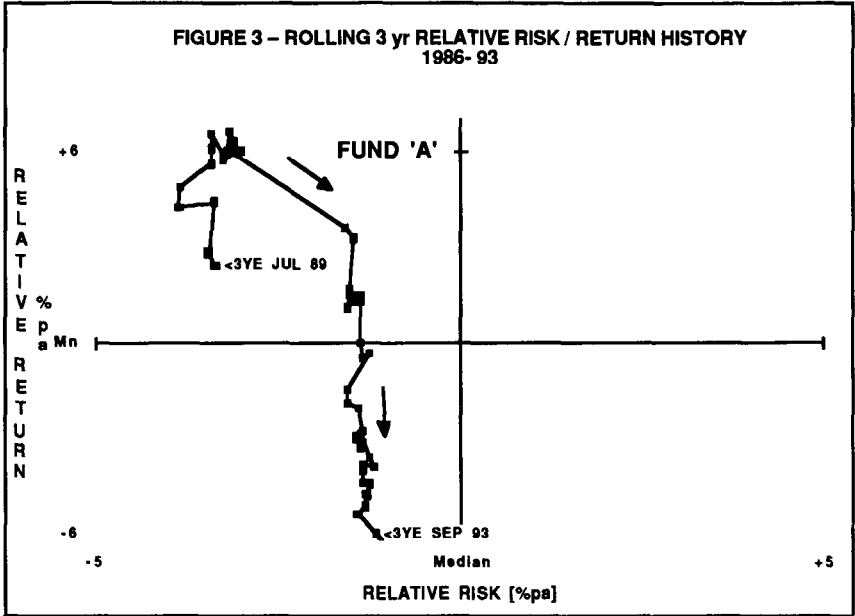
Fairly obviously, desirable above median returns appear above the horizontal median return line and less desirable below median results, below it. Similarly, below median volatility appears on the left hand side of the diagram and above median volatility on the right. The most desirable region is the top left 'high return/low risk' quadrant, while the least desirable is the bottom right 'low return/high risk' quadrant.

Irrespective of the absolute position of a set of points on the graph, the tightness of their grouping is a direct indication of the consistency of a manager's risk/return performance.

### Examples of three year histories

Figure 2 demonstrates that Relative Risk/Return Histories are useful in displaying consistent performance over changing market conditions, but are they also useful in revealing changing performance? Figure 3 shows an example of a major fund, the performance of which has been falling steadily over recent years, while Figure 4 shows another major manager whose performance has changed more rapidly. (The letters 'A', 'H' etc. denote the same funds as shown in Table 1.)

Clearly, Relative Risk/Return Histories can play a valuable role in capturing and displaying the dynamics of changing fund manager performance.



### One year histories

In Figure 4, we saw the performance history of a fund manager who "got the 1987 Stock market Crash right" but whose performance in the post-Crash period has been noticeably poorer. With a rolling three year period, the dramatic change in performance does not show up until October 1990. This is hardly early warning!

Figure 5, however, shows the Relative Risk/Return History for the same manager using a rolling one year analysis. The fall off in performance becomes obvious much earlier than in Figure 4.

Theoretically, the statistical reliability of any single point on the graph is reduced due to the reduction in the number of monthly samples from 36 to 12. From a practical point of view, however, the absence of extreme outliers on the graph, tends to indicate adequate statistical reliability.

Pragmatically, any concern about statistical reliability is more than outweighed by the increased speed of response to changing performance.

A very dramatic example of changing performance is shown in Figure 6. The manager is generally thought to take large bets, some of which appear to pay off and some of which appear not to. This fund has been near or at the top of the league table of returns on several occasions over recent years. Figure 6, however, shows how dangerous it can be to neglect either the risk dimension or the history of a manager's risk/return performance.

The value of displaying the dynamic history of both risk and return is clear. In fact, this example shows that the single snapshot approach can be very misleading.

### How consistently do Australian fund managers add value and reduce volatility?

Figure 7 shows the proportion of time which the medium and large balanced pooled super funds have spent in the top left quadrant of the rolling one year Relative Risk/Return Histories. Only two funds in this group have

been able to stay in this quadrant for more than 50% of the time<sup>1</sup>.

### SPLIT FUNDING & MIXING MANAGERS

Relative risk/return histories clearly provide a very powerful aid to understanding and selecting individual fund managers. An obvious extension is to use them to analyse split funding arrangements with multiple managers.

Figure 8 shows the Relative Risk/Return History for a major balanced fund managed with an aggressive style at both the asset allocation and security selection levels. Higher than median returns are achieved at times but at the expense of higher than median volatility.

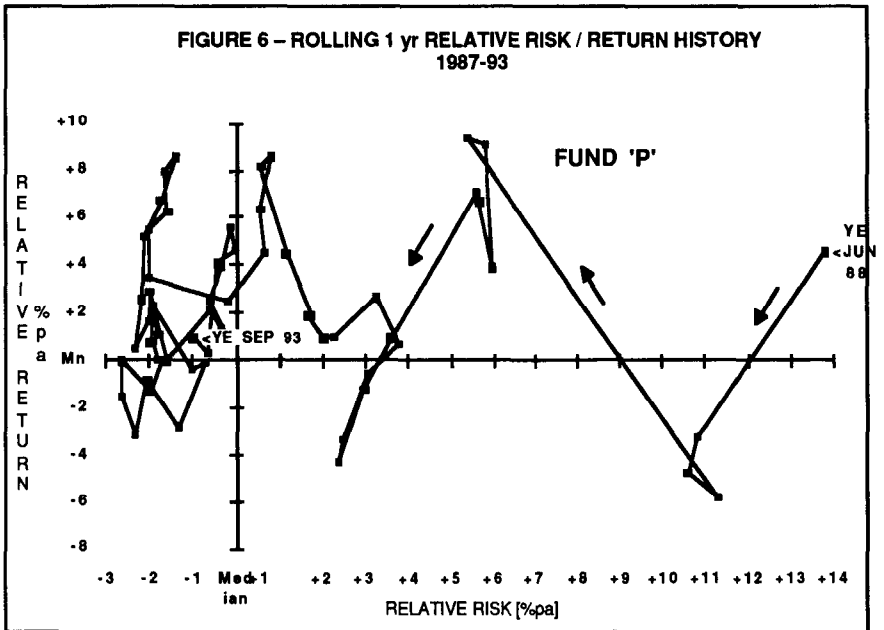
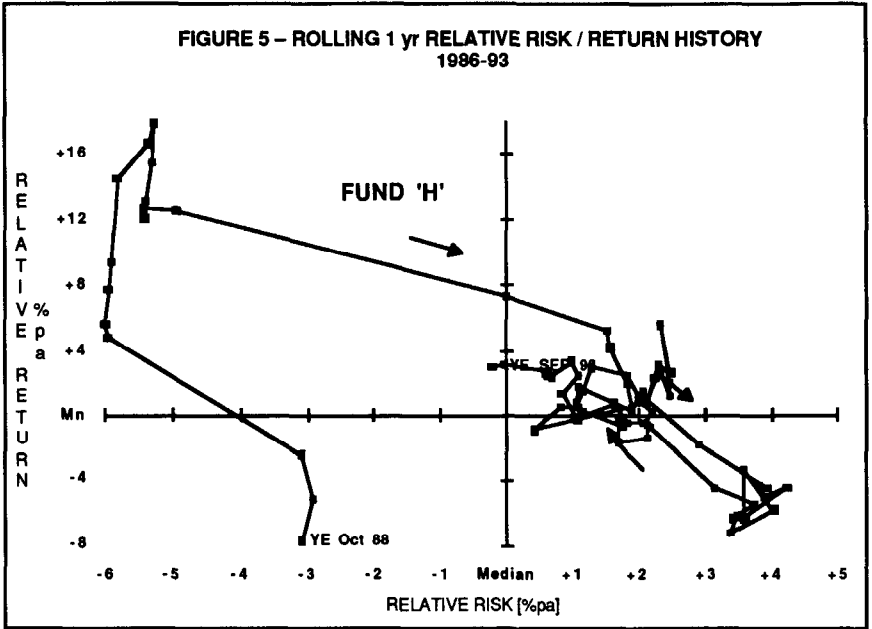
Figure 9 then shows the results when this fund is mixed 50/50 with the quite different fund shown in Figure 2. Volatility is pulled back noticeably and uncertainty of return reduced. Figure 9 provides a clear example of the power of Relative Risk/Return Histories in graphically displaying the benefits of mixing complementary managers.

Figure 9 also shows that peak returns for the combination are reduced. Is this too high a price to pay? To answer this, the History needs to be complemented by some conventional calculations. In this example, the total post-Crash cost is negative – an *increase* in return! In general, this probably means that the benefits of reduced volatility can be achieved without any significant cost to returns, if the managers are selected sufficiently carefully using snail trails.

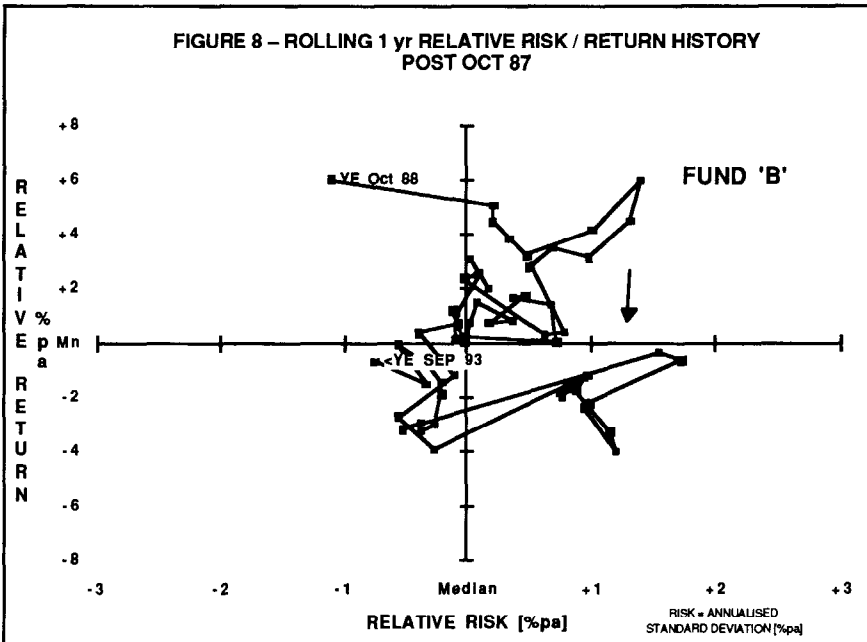
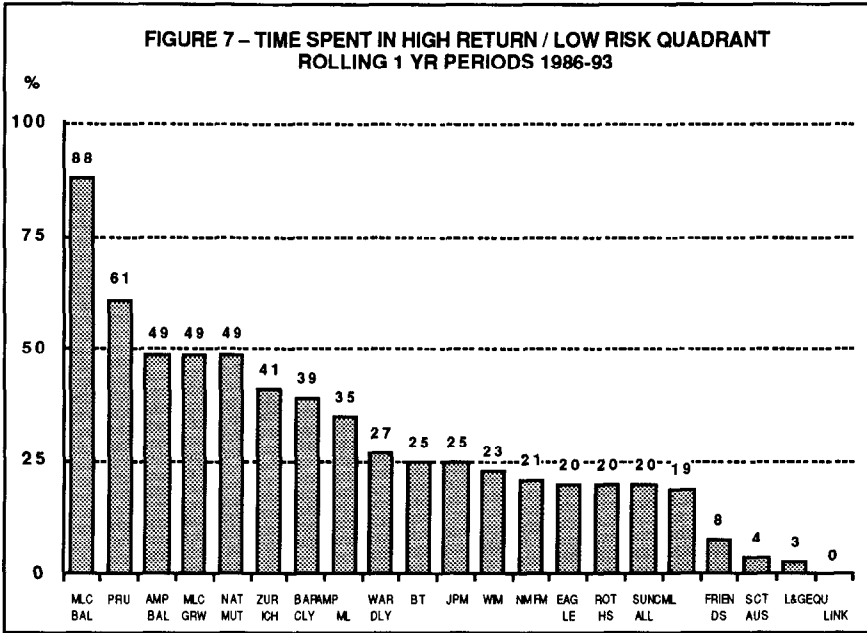
Other than laboriously trying promising combinations by trial and error, how can one narrow the search for a good combination of managers?

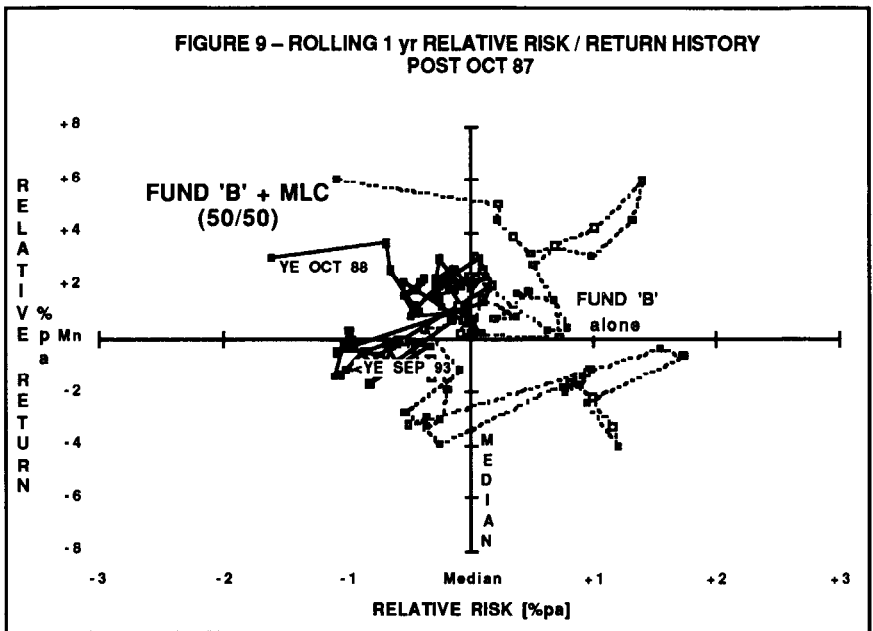
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<sup>1</sup>Based on data from Frank Russell Australia.









### Finding good combinations of managers

Whilst *individual* Risk/Return Histories give a good indication of those managers who should be on a short list, they give only a limited indication of which managers are likely to complement one another in split funding arrangements.

A common approach is to assess managers by style and then to select a complementary mix of styles. As we shall see later, however, style is only a partial indicator of the complementarity of investment returns.

Another approach could be to treat managers as asset classes and to determine the optimal mixes of managers lying on the efficient frontier. This approach is far more complex for *relative* risk and return than it is for total risk and return. Standard asset optimisers typically presuppose a relationship between portfolio variance and constituent asset class variances which does not apply in the case of *relative* risk as defined in this paper.

The simplest approach is to look for fund managers having negative correlations of relative returns with other managers.

### Correlation of relative returns

Table 1 presents the results of a correlation analysis of the relative returns of 10 major funds.

The results shown in Table 1 are surprising in two ways. Firstly, the relative returns of most of the funds are positively correlated with each other even though their investment styles differ markedly in many cases. If one averages the correlation coefficients for each fund with the other nine, then only three have negative averages. Secondly, the negative correlation of one fund (MLC Balanced) with all but two of the others will surprise many.

How important are the results in Table 1?

The theoretical basis for seeking a mix of managers with negative correlations is considered later. While negative correlation of *relative* returns is a good indicator of real

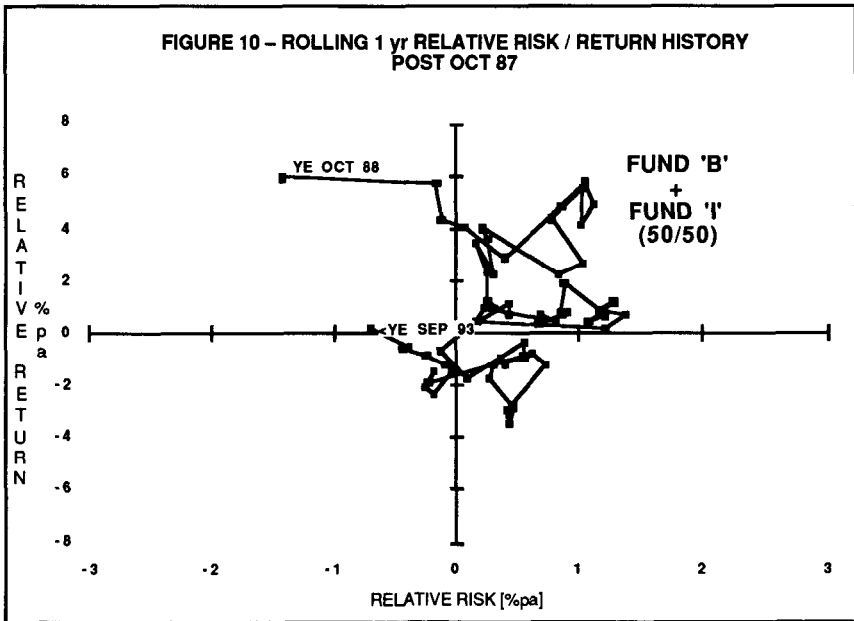
diversification of risk in practice, it is not a sufficient condition from a mathematical point of view. The important point, however, is that *in practice* superior risk reduction is achieved when managers with negative correlations in Table 1 are mixed together.

Figures 8 & 9 showed the reduction in volatility which could have been achieved by mixing the negatively correlated Fund 'B' and the MLC Balanced Fund. On the other

hand, mixing the positively correlated pair, 'B' and 'I', would have achieved only minimal reduction in volatility, as shown in Figure 10. This occurs in spite of the managers for Funds 'B' and 'I' having very different styles, namely aggressive asset allocation and security selection with 'B' versus a large indexed core plus active management around that core with 'I'.

**TABLE 1 – CORRELATION OF FUND MANAGER EXCESS RETURNS**  
(Based on one year rolling returns relative to median; Nov 87 - Sep 93)

FUND	A	B	C	D	MLC	E	F	G	H	I
A	1.0	0.9	0.7	-0.4	-0.1	1.0	-0.4	0.7	-0.1	0.8
B	0.9	1.0	0.6	-0.3	-0.1	0.9	-0.5	0.8	-0.1	0.8
C	0.7	0.6	1.0	-0.4	-0.1	0.6	-0.1	0.4	-0.2	0.7
D	-0.4	-0.3	-0.4	1.0	-0.3	-0.3	0.1	0.0	0.5	-0.2
MLC	-0.1	-0.1	-0.1	-0.3	1.0	0.0	0.0	-0.3	-0.6	-0.3
E	1.0	0.9	0.6	-0.3	0.0	1.0	-0.5	0.7	0.0	0.8
F	-0.4	-0.5	-0.1	0.1	0.0	-0.5	1.0	-0.6	0.2	-0.3
G	0.7	0.8	0.4	0.0	-0.3	0.7	-0.6	1.0	0.2	0.8
H	-0.1	-0.1	-0.2	0.5	-0.6	0.0	0.2	0.2	1.0	0.2
I	0.8	0.8	0.7	-0.2	-0.3	0.8	-0.3	0.8	0.2	1.0



### Relative versus total returns

Should not total rather than relative returns be correlated? They could be, but the results would be less illuminating. Mathematically, negative correlations of relative returns are a necessary but not a sufficient condition for high levels of volatility reduction.

Consider an analogy. A balanced equity portfolio contains a number of stocks to diversify 'specific' risk. Similarly in split funding, the aim is to diversify risk, this time by using multiple managers. But just as there is a residual or undiversifiable 'market' risk in any equity portfolio, so too a residual undiversifiable risk arises when mixing managers. A convenient proxy for the undiversifiable risk is the median or average manager. Removing the median return effectively removes the undiversifiable component and focuses attention on the diversifiable.

Thus correlations of *relative* returns provide greater insight. To torture an innocent metaphor, "Using total returns is like putting the bath water back before looking for the baby".

### Period for correlation analysis

The analysis in Table 1 deals with the post-Oct 87 period. There are (at least) three reasons for restricting attention to this period.

Firstly, a number of managers overtly or covertly changed their style significantly following the events of October 1987. Some adopted more conservative asset allocation strategies, some implemented more disciplined systems for security selection, for example the BARRA system for equities, while some saw significant changes in key staff. It is unrealistic to pretend that such changes have not significantly changed the performance of a number of managers.

Secondly, including returns for the month of October 1987 distorts what are otherwise 'statistically well-behaved' data<sup>2</sup>.

Thirdly, including an event like the Crash in a five year (say) analysis implicitly specifies that a similar event will occur *every* five years. This is unrealistic.

Consequently, the analysis in Table 1 focuses on the post-Crash period.

### Rolling returns

The use of rolling returns in the correlation analysis is a little unusual. The analysis for the Risk/Return Histories, however, is performed in a rolling risk/return context. Hence the use of non-rolling correlations would seem to be inappropriate. In any case, the real test is whether the results are useful or not in practice. Our results show that they are a good indicator of where real risk reduction can and cannot be expected. This overrides any concerns about unequal weighting of monthly results at the extremes of the period of analysis and other technicalities.

A detailed technical analysis is appears in the Appendix.

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<sup>2</sup>For example, consider two fund managers whose excess returns are negatively correlated. The negative correlation will show up as a left to right downwardly sloping pattern on a scatter graph of the returns of one manager versus the other. If however, both managers 'get the Crash right' relative to the median fund manager, then an outlying point will appear way out in the top right quadrant of the scatter plot. If 12 month rolling returns are used, 12 points will appear in that region. The linear regression, implicit in calculating a correlation coefficient, will then swing the result from negative to positive.

### RISK/RETURN RANKING HISTORIES or "SNAIL TRAILS Mark III"

The *Relative Risk/Return Histories* or "Snail Trails" show performance relative to the median level of risk and the median level of return. Whilst these graphs have been of considerable assistance to trustees and consultants in both selecting and monitoring fund managers, there is scope for a "history" which can provide additional comparative information.

How good or bad is any particular level of above- or below-median performance? Is it the best available? Is it the worst available? Is it first or fourth quartile? Where does it lie within that quartile?

An important additional consideration is that performance relative to median has changed over time. Figure 11 shows the boundaries between first and second quartile returns over rolling one year periods from the year ending December 1886. The boundary between third and fourth quartile returns is also shown on Figure 11. Clearly, a return of 3% pa above median five years ago is very different from 3% pa above median now. In technical terms, the axes of the risk/return history need to be normalised.

Additional light can be shed on such matters by looking at the *ranking* of the performance level in question. Is it 1st, 6th or 36th? Positions or rankings could be used directly but if the "histories" are to be truly meaningful, the variation in the number of managers or funds in the peer group must be taken into account. By using percentile rankings such problems are avoided.

For example, a ranking of '1st' becomes a percentile ranking of 100%, while a ranking of 36th out of 36, or 42nd out of 42 becomes a percentile ranking of 0%. Obviously the median always falls at 50% and the upper and lower quartile boundaries are always 75% and 25%, respectively. Furthermore, since the relationship between position or rank and percentile ranking is linear, no distortions are introduced. A percentile ranking of 90% for a fund means that 9 out of 10 funds are below it.

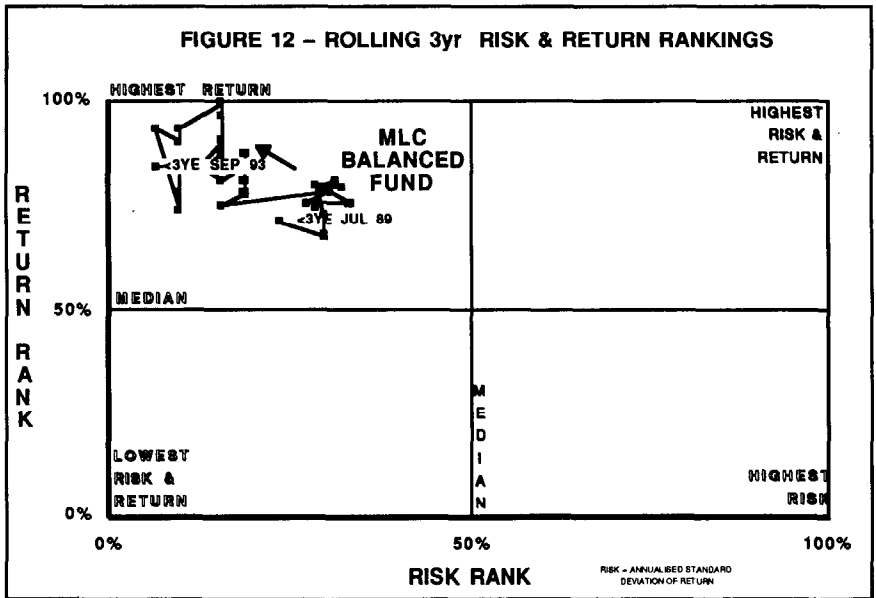
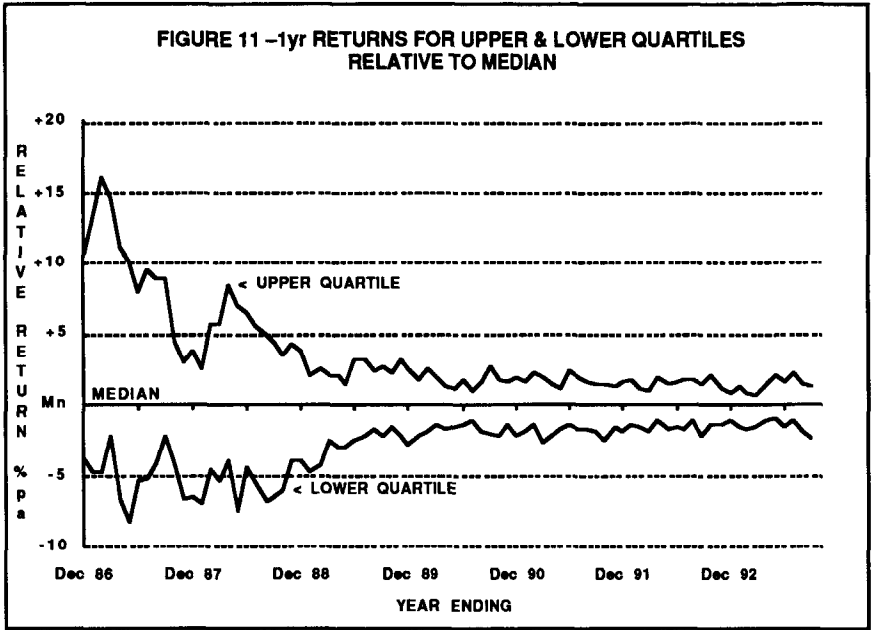
From a value added point of view, a percentile ranking of 100% (the highest) is the most desirable. From a risk point of view, however, a percentile ranking of 0%, the lowest, is the most desirable.

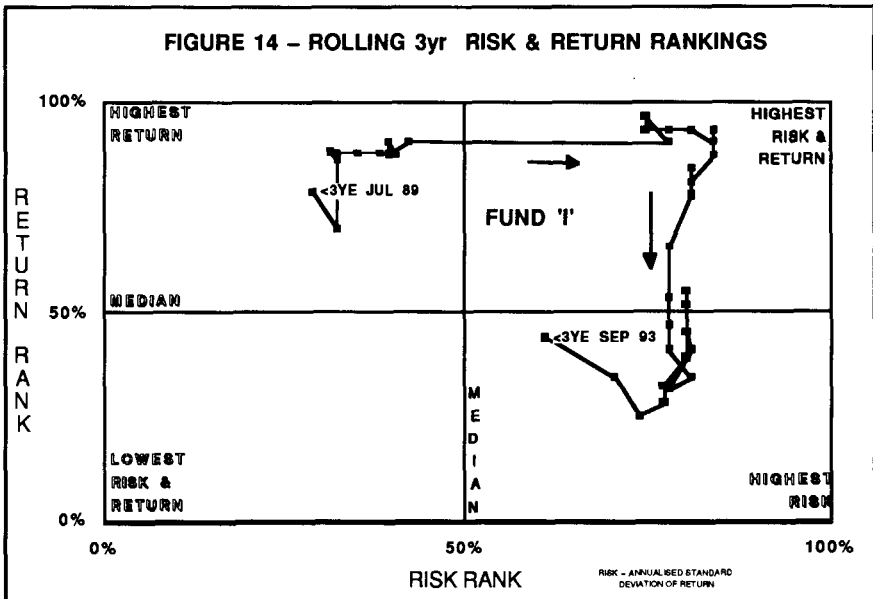
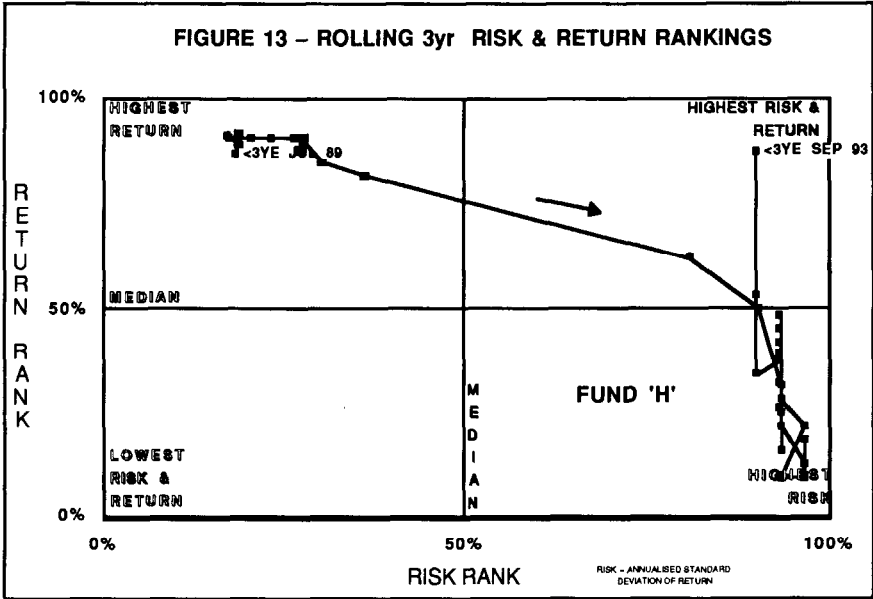
Figure 12 shows the Ranking History of the same fund as shown in Figure 2. From Figure 2, one might wrongly surmise that the fund is becoming less competitive, with lower value added and poorer risk management. The ranking History in Figure 12, however, shows just the reverse. As markets became tougher and as more professionalism became required in the late eighties and early nineties, this fund's rankings for both value added and for risk management relative to its peer group actually improved.

Figure 13 shows the Ranking History for the same fund as shown in Figure 4. In Figure 4, the fall below median following October 87 dropping out of the rolling three year results appears moderate. The Ranking History in Figure 13, however, reveals that the drop was in fact quite severe relative to the peer group's achievements.

Figure 14 shows the way in which the Ranking Histories can capture and reveal dramatic changes in risk management and value adding as they occur.

By showing performance using percentile rankings, an individual manager's skill relative to not only the median but *all* managers is laid bare.

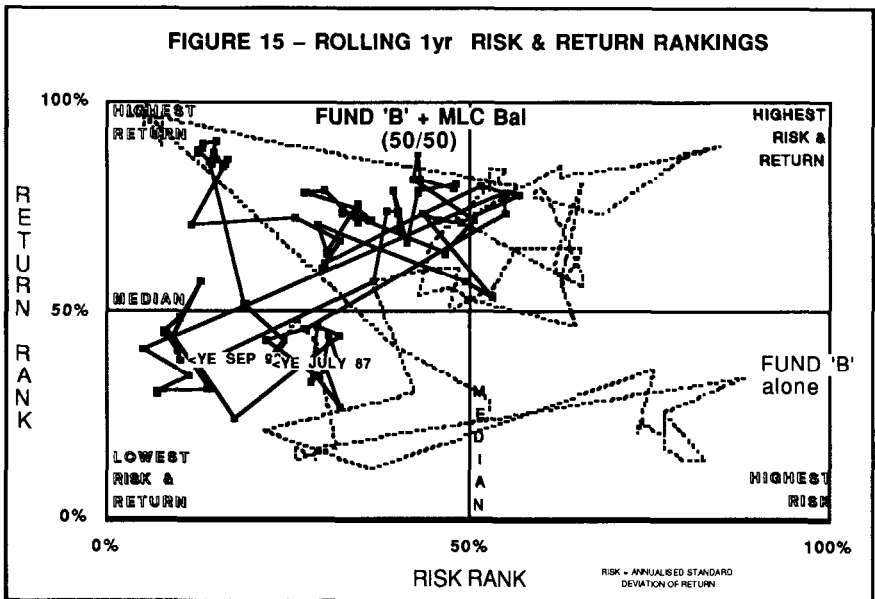




**SPLIT FUNDING REVISITED**

Since the original Relative Risk/Return Histories provided a powerful means of displaying whether the mixing two funds provided real or only illusory diversification, it would be surprising if the Risk/Return Ranking Histories did not do so also. —>

Figure 15 shows the Ranking History for the combination of funds shown in Figure 9. The significant reduction in volatility is clearly visible. Figure 15 also has the added advantage that the reduction is put into greater perspective due to the additional peer group information provided by the rankings.



**NON-PARAMETRIC CORRELATIONS**

From the point of view of diversification of fund manager risk, the natural correlation coefficient to use in conjunction with the Risk/Return Ranking Histories is the correlation of the return rankings.

This approach has additional theoretical appeal. A separate research project confirms that the distributions of fund returns for the medium and large wholesale funds in Australia are far from normal or gaussian. They range from a surprisingly almost uniform distribution in one case, through roughly normal but with outliers, to highly skewed. Even if the returns of all the funds

are aggregated into one data set, the distribution is still not normal. It is distinctly leptokurtic.

Under such circumstances it is both appropriate and prudent to use non-parametric statistical methods. The rank correlation coefficient is one such non-parametric measure of correlation and can be used with the Spearman test of significance to give confidence levels, if required.

Table 2 shows the rank correlation coefficients for the 10 major funds shown in Table 1. The results in Table 2 are surprisingly similar to those in Table 1 and provide additional confirmation of the usefulness of the original approach.



**TABLE 2 – NON-PARAMETRIC CORRELATION OF FUND MANAGER RETURNS**  
(Spearman rank correlation coefficient based on one year rolling returns; Nov 87 - Sep 93)

FUND	A	B	C	D	MLC	E	F	G	H	I
A	1.0	0.8	0.8	-0.6	-0.3	1.0	-0.4	0.4	-0.2	0.8
B	0.8	1.0	0.7	-0.6	-0.2	0.8	-0.5	0.6	-0.4	0.8
C	0.8	0.7	1.0	-0.7	0.0	0.7	-0.3	0.4	-0.4	0.7
D	-0.6	-0.6	-0.7	1.0	-0.2	-0.5	0.4	-0.2	0.6	-0.5
MLC	-0.3	-0.2	0.0	-0.2	1.0	-0.3	-0.2	-0.2	-0.6	-0.3
E	1.0	0.8	0.7	-0.5	-0.3	1.0	-0.4	0.4	-0.1	0.7
F	-0.4	-0.5	-0.3	0.4	-0.2	-0.4	1.0	-0.4	0.5	-0.3
G	0.4	0.6	0.4	-0.2	-0.2	0.4	-0.4	1.0	-0.1	0.5
H	-0.2	-0.4	-0.4	0.6	-0.6	-0.1	0.5	-0.1	1.0	-0.1
I	0.8	0.8	0.7	-0.5	-0.3	0.7	-0.3	0.5	-0.1	1.0

## CONCLUSIONS

The power of the Relative Risk/Return History arises from two sources. The first is its ability to capture and display the dynamics of a fund's performance over time. The second is its ability to remove the 'market' effect and to highlight a manager's value adding and risk reducing skills relative to other fund managers.

The *Risk / Return Ranking History* provides an additional dimension by showing the positioning of the performance relative the whole peer group.

Together, the two forms of History constitute a pair powerful aids to trustees and consultants in the important but complex process of selecting and monitoring fund managers.

## Disclaimer

The views expressed in this paper are those of the author and not necessarily those of any organisation.

Whilst the statements, calculations, data and graphs contained in this paper are believed to be correct, no warranty thereto is given and the author expressly disclaims all liability for any loss or damage which may arise from any person acting partly or wholly on the basis of any such statements, calculations, data and graphs.

## APPENDIX – TECHNICAL DETAILS

Some detailed technical issues need to be addressed for anyone wishing to apply the Relative Risk/Return Histories presented in the first part of this paper.

Care must taken in constructing Relative Risk/Return Histories for combinations of fund managers. Whilst relative returns do add linearly, relative standard deviations do not.

### Relative returns for combinations of managers

Consider a situation where two fund managers, A and B, manage half of a super fund, F, each. Denote the total return in rolling period  $t$  by  $r(t)$ . The return for the fund is then

$$r_F(t) = 0.5r_A(t) + 0.5r_B(t) \quad (1)$$

If relative return is denoted by  $R(t)$  then by definition

$$R(t) = r(t) - r_M(t)$$

where the subscript  $M$  denotes the median return for the relevant universe of fund managers.

The relative return for the fund is then

$$\begin{aligned}
 R_F(t) &= r_F(t) - r_M(t) \\
 &= 0.5r_A(t) + 0.5r_B(t) - r_M(t) \\
 &= 0.5r_A(t) - 0.5r_M(t) \\
 &\quad + 0.5r_B(t) - 0.5r_M(t) \\
 &= 0.5R_A(t) + 0.5R_B(t) \quad (2)
 \end{aligned}$$

Clearly, the linear relationship in Equation (2) applies to mixes other than 50/50 and for more than two managers. Hence relative returns can be added linearly in the same way as normal returns.

**Relative standard deviations for combinations of managers**

If the standard deviation of the total (not relative) return during the 't' th rolling period is denoted by  $\sigma_r(t)$  then the relative standard deviation,  $S_r(t)$ , of total return is defined as

$$S_r(t) = \sigma_r(t) - \sigma_M(t)$$

where  $\sigma_M(t)$  is the median standard deviation for the universe of fund managers, not the standard deviation of the median return,  $\sigma_{r_M}(t)$

Hence the relative standard deviation for the fund, F, is

$$S_{r_F}(t) = \sigma_{r_F}(t) - \sigma_M(t) \quad (3)$$

where  $\sigma_{r_F}(t)$  is the standard deviation of the total fund return found from Equation (1), with the proportions varied as necessary.

The standard deviation of the total return of the fund is a nonlinear function of the standard deviations of the individual managers. Consequently, relative standard deviations cannot be added linearly. The simplest approach is to construct the return series for the fund using Equation (1) and then to calculate its standard deviation directly.

**Correlations**

The case for correlating relative returns was argued at an intuitive level earlier. In this section, some precise mathematical relationships are presented.

It is possible (but tedious) to show that the relative standard deviation of the total

return of the fund is related to the covariances of both relative and total returns. For example,

$$S_{r_F} = \left[ \begin{array}{l} 0.25\sigma_{r_A}^2 + 0.25\sigma_{r_B}^2 \\ + 0.5 \text{cov}(r_A, r_B) \end{array} \right]^{1/2} - \sigma_M \quad (5) \text{ and}$$

$$S_{r_F} = \left[ \begin{array}{l} 0.25\sigma_{R_A}^2 + 0.25\sigma_{R_B}^2 \\ + 0.5 \text{cov}(R_A, R_B) \\ + \text{cov}(R_A, r_M) \\ + \text{cov}(R_B, r_M) + \sigma_{r_M}^2 \end{array} \right]^{1/2} - \sigma_M \quad (6)$$

where  $\text{cov}(\cdot, \cdot)$  denotes covariance.

From these relationships a number of conclusions can be drawn or proven.

Firstly, mixing managers with low total or relative standard deviations results in low relative standard deviation for the fund.

Secondly, the minimum and maximum relative standard deviations for the fund can be shown to be bounded by

$$S_{F_{\max}} = (S_A + S_B) / 2$$

$$S_{F_{\min}} = |S_A - S_B| / 2 - \sigma_M$$

depending on the degree of correlation between A and B. The upper bound confirms that at least an 'averaging' effect occurs when managers are mixed. The minimum bound also clearly shows that the relative standard deviation of the fund can be lower than that of either of the two fund managers under certain circumstances.

Since the  $\sigma^2$  terms in Equations (5) and (6) must be positive, any reduction in total or relative volatility can only occur through low or negative covariances and hence low or negative correlations.

Negative correlation of relative returns is clearly a necessary but not a sufficient condition. Separate analyses indicate that negative correlation with most managers tends to coincide with negative correlation with the median. Hence negative correlation of relative returns tends to be a good pointer to diversification of risk. It should be complemented by constructing the risk/return history for the proposed combination.