Using Canadian Index Linked Bonds To Model Real Interest Rates and to Detect Inflation Risk Premiums

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Summary

An index-linked bond (ILB) is a security whose coupon is linked to the price level of an arbitrary price index. Unlike a standard non-linked bond (NLB) whose coupon is fixed and, thus, promises a constant nominal coupon rate, an ILB's coupon rate will vary in nominal terms but remain constant in real terms. Therefore, the ILB preserves purchasing power over time and is able to serve as a vehicle for inflation-proof investment.

Fischer's (1975) model for pricing ILBs suggests that the difference between the nominal return on an NLB and the real return on an ILB is comprised not only of a compensation for expected inflation but also of a premium for bearing the risk of uncertain inflation. Therefore, models of real interest rates such as Evans, Keef and Okunev (1993), which measure real rates as the difference between nominal interest rates and inflation, are contaminated by the inclusion of this inflation risk premium.

This study models real interest rates by an approach similar to Evans, Keef and Okunev (1993) but uses ILB's and NLB's in order to investigate the stochastic properties of real interest rates on both linked and nominal debt as well as to find evidence of the existence of an inflation risk premium.

Test results indicate strong mean reversion for expected real rates on both ILB's and NLB's although a unit root test could not reject non-stationarity for the ILB. No evidence of serial correlation or heteroskedasticity was found in either case. The mean difference between the expected real interest rates on the NLB and the ILB is significantly positive which is consistent with the existence of an inflation risk premium.

Emploi des obligations canadiennes indexées pour établir le modèle de taux d'intérêt réels et détecter les marges de variation du risque d'inflation

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Résumé

Une ILB (obligation indexée) est un titre de placement dont le coupon est indexé sur le niveau du prix d'un indice arbitraire des prix. Au contraire d'une NLB (obligation non indexée) dont le coupon est fixé et promet donc un taux d'intérêt nominal constant, le taux d'intérêt d'une ILB va varier en termes nominaux mais rester constant en termes réels. Par conséquent, une ILB préserve le pouvoir d'achat au bout d'un certain temps et peut servir de moyen d'investissement non affecté par l'inflation.

Le modèle de Fischer (1975) pour la fixation du prix des ILB suggère que la différence entre le rendement nominal d'une NLB et le rendement réel d'une ILB comprend non seulement une compensation de l'inflation anticipée mais également une marge de variation tenant compte du risque d'une inflation incertaine. Par conséquent, les modèles des taux d'intérêt réels tels que celui de Evans, Keef et Okunev (1993), qui mesurent les taux réels comme étant la différence entre le taux d'intérêt nominal et l'inflation, sont contaminés par l'inclusion de cette marge de variation du risque d'inflation.

Cette étude établit le modèle des taux d'intérêt réels en fonction d'une méthode similaire à celle de Evans, Keef et Okunev (1993), mais a recours à des ILB et à des NLB pour pouvoir étudier les propriétés stochastiques des taux d'intérêt réels sur une dette nominale et sur une dette indexée et pour trouver des preuves de l'existence d'une marge de variation du risque d'inflation.

Les résultats de nos tests indiquent un fort retour à la moyenne pour les taux réels escomptés à la fois sur les ILB et sur les NLB bien qu'un test unitaire de base ait été incapable de rejeter un état non stationnaire pour l'ILB. Aucune évidence de corrélation en série ou hétéroskédacité n'a été trouvée dans les deux cas. La différence moyenne entre les taux d'intérêt réels escomptés sur la NLB et sur l'ILB est nettement positive, preuve de l'existence d'une marge de variation du risque d'inflation.

I. Introduction

The behaviour of short term real interest rates has been an issue of significant debate ever since Fisher (1930) first postulated that real rates could be expressed as the difference between expected nominal rates and expected inflation. Although not necessarily implied by Fisher, the real interest rate in this relationship has traditionally been assumed to be constant. However, the volatile inflation and nominal interest rate environment of the 1970's and 1980's, as well as the empirical work of Nelson and Schwert (1977) and Hess and Bicksler (1987), amongst others, has cast doubt upon this assumption.

Given that real interest rates have been, until recently, unobservable, indirect measures of the short term rate have typically been employed. The most commonly used proxy is the short term nominal interest rate less a measure of expected inflation. Implicit in the use of this technique, however, is the assumption that no significant "inflation risk premium" is included in nominal interest rates. Fischer's (1975) model for pricing index linked bonds (ILB's) suggests that the difference between the nominal return on a nominal bond (NLB) and the real return on an ILB is comprised not only of compensation for expected inflation but also of a premium for bearing the risk of uncertain inflation. As such, models of real interest rates which measure real rates as the difference between nominal interest rates and inflation are contaminated by the inclusion of this inflation risk premium. In essence, what these models actually investigate is the stochastic properties of expected real interest rates on nominal debt which is not necessarily risk free in a real sense.

Nonetheless, Evans, Keef and Okunev (1992) use this proxy to model real interest rates as a Ornstein Uhlenbeck process and find evidence of significant mean reversion and a lack of heteroskedasticity using U.S and U.K. data. They argue that the significant heteroskedasticity found in models of nominal interest rate processes such as those summarized in Chan, Karolyi, Longstaff and Sanders (1992) is a result of the mechanism driving inflation expectations.

The existence of ILB's, however, provide a previously unavailable method to directly measure uncontaminated real interest rates and, subsequently, to model their stochastic properties over time. Furthermore, any differences detected between real interest rates on ILB's and the concurrent nominal interest rates on NLB's less expected inflation constitute evidence for the existence of an inflation risk premium as shown by,

$$IRP = r_n - r_l$$

where r_l and r_n represent the real returns on an ILB and a NLB, respectively.

The purpose of this study is to model real interest rates by an approach similar to Evans, Keef and Okunev (1993) but to use real rates on both the ILB and the NLB in order to investigate each of their stochastic properties as well as to find evidence of the existence of an inflation risk premium.

The outline of this paper will be as follows. Section Two provides a brief description of ILB's and discusses characteristics of the Canadian government issue. The third section demonstrates the theoretical relationship between real and nominal returns based on the ILB pricing model formulated by Fischer (1975) and extended by Landskroner (1981). The fourth and fifth sections develop the empirical approach and describe the data to be used in the study while the final two sections outline the results and provide conclusions.

II. A Description of Index Linked Bonds

An index linked bond (ILB) is a security whose coupon is linked to the price level of an arbitrary price index. Unlike a non-linked bond (NLB) whose coupon is fixed and, thus, promises a constant nominal coupon rate, an ILB's coupon rate will vary in nominal terms but remain constant in real terms. Therefore, the goal of the ILB is to preserve purchasing power over time and to serve as a vehicle for inflation-proof investment.

ILB's were first issued in France and Finland following the second world war in order to stabilize their respective economies while Israel, Argentina, and Brazil issued linked securities in the 1970's in order to market government debt in the face of hyper inflation. By 1980 all Israeli government debt had become inflation linked. Britain, which introduced inflation linked gilts in 1981, became the first country to issue ILB's in an economy without runaway inflation. In that case, the government was motivated by the large volume of high coupon debt outstanding in 1980 which, if inflation were to fall, would dramatically increase the government's real debt burden. As well, the issuance of ILB's served as a signal to the market of the government's commitment to its anti-inflation policy (Rutterford 1983).

Canada issued a 30 year 4.25% semi-annual ILB on December 10 1991, representing the first time that a government issued security of this nature had become available in North America. The Canadian 2021 ILB is linked to the percentage change in the monthly CPI which is lagged by three months for reporting purposes. Accrued interest is calculated on a proportional basis which differs from the calculation of accrued interest on the U.K. index linked gilt which is determined on an absolute basis thereby requiring a lag of six months to determine the coupon and an additional two months for reporting purposes.

III. The Relationship Between Real and Nominal Interest Rates

Fischer (1975) was the first to investigate the pricing and demand for an ILB from a utility optimization perspective and demonstrated that the expected real return on an ILB will be lower than that of an NLB if the market, in general, is not a hedge against inflation. [5]

In a continuous time framework, the relationship between nominal and real returns on an ILB can be represented as,

$$R_l = r_l + \pi \tag{1}$$

and the relationship between the nominal and real returns on an NLB as,

$$R_n = r_n + \pi - \sigma_p^2 \tag{2}$$

where π is expected inflation, σ_p^2 is the variance of inflation and where upper and lower case symbols represent expected nominal and real returns, respectively.

Extending Fischer's (1975) analysis into the Friend, Landskroner, and Losq (1976) framework, and assuming constant relative risk aversion, Landskroner (1981) demonstrated that the equilibrium relationship between the expected nominal returns on an ILB and an NLB can be expressed as,

$$R_l - R_n - \sigma_p^2 = \Theta\left(\bar{\alpha}_N \sigma_{pm} - \sigma_p^2\right) \tag{3}$$

where σ_{pm} is the covariance of inflation with the market portfolio.

The market portfolio consists of the demand for all nominal risky assets aggregated over all households and $\bar{\alpha}_N$ represents the ratio of the market portfolio to total nominal wealth such that,

$$\bar{\alpha}_N = \sum_k \gamma_k(\alpha_s^k + \alpha_l^k)$$

where α_s^k and α_l^k represent household k demand for equity and the ILB, respectively, and where,

$$\gamma_k = \frac{W_k}{\sum_k W_k}$$

The coefficient Θ represents the wealth weighted harmonic mean of the k household's degree of relative risk aversion and can be expressed as follows,[4]

$$\Theta = \left[\sum_k \frac{\gamma_k}{\theta_k}\right]^{-1}$$

[5] Mundell (1963) initially suggested that a negative relationship existed between inflation and the real return on equity and was supported by empirical investigations conducted by Bodie (1975), Fama and Schwert (1977), Solnik (1983) and Chu (1988).

[4] as utilised in the Friend, Landskroner, and Losq (1976) model.

Thus, Landskroner showed that overall risk is reduced by a factor reflecting the degree to which the market serves as a hedge against inflation. The risk of a single asset is reduced by a similar factor reflecting the covariance of the return of that asset and inflation.

In real terms the pricing relationship between the NLB and the ILB can be stated as,
[5]

$$r_n - r_l = \bar{\alpha}_R \Theta \nu_{nm} \tag{4}$$

where ν_{nm} is the covariance between the real return of the NLB and the market portfolio.

The market portfolio in this case consists of all real risky assets aggregated over all households and $\bar{\alpha}_R$ represents the ratio of the market portfolio to total real wealth such that,

$$\bar{\alpha}_R = \sum_k \gamma_k(\alpha_s^k + \alpha_n^k)$$

where α_n^k represents household k demand for the NLB. Therefore, in real terms the sign of the difference between the return on the ILB and the NLB depends only upon the covariance of the market portfolio and inflation. In other words, if the market portfolio is a hedge against inflation (i.e. $\nu_{mp} > 0$), then the NLB will be sold at a premium over the ILB and, hence, $r_l > r_n$. If the market portfolio is not a hedge against inflation, then the ILB will be sold at a premium over the NLB. [6]

The intuition behind these results can be illustrated within a CAPM framework in both nominal and real terms. In nominal terms, the difference between the return on the NLB and the ILB can be expressed as,[7]

$$R_l - R_n - \sigma_p^2 = \beta_{lm}(R_m - R_n - \sigma_{mp}) \tag{5}$$

where,

$$\beta_{lm} = \frac{\bar{\alpha}_N \sigma_{lm} - \sigma_p^2}{\bar{\alpha}_N \sigma_m^2 - \sigma_{mp}}$$

or, in real terms, the difference between the real return on the NLB and the ILB can be expressed as, [8]

$$\begin{aligned} r_n - r_l &= -\beta_{lm}(R_m - R_n - \sigma_{mp}) \\ &= b_{nm}(r_m - r_l) \end{aligned} \tag{6}$$

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[5] which is the real equivalent of the Friend and Blume (1975) model.

[6] as first demonstrated by Fischer (1975).

[7] Landskroner (1981)

[8] Siegel and Warner (1977)

where,

$$b_{nm} = \frac{\nu_{nm}}{\nu_m^2}$$

Intuitively, if uncertain inflation is negatively correlated with the real return on the market portfolio, then the return of the NLB must be positively correlated with the market portfolio and, as a consequence, will be priced unfavourably in the sense that $r_n > r_l$. In this case, the difference between the real returns on the NLB and the ILB represent an inflation risk premium which compensates investors for the bearing of risk associated with uncertain inflation. Substituting the real return on the ILB, equation (1), into equation (6) yields the following relationship,

$$R_n = r_l + \pi - \sigma_p^2 + b_{nm}(r_m - r_l) \tag{7}$$

which differs from the well known Fisher (1930) equation by the inclusion of the inflation risk premium. The existence of an inflation risk premium suggests that investors not only value the compensation for expected inflation but also the elimination of the risk associated with uncertain inflation.

Barro (1986) and Cooper (1987) both describe the existence of an inflation risk premium based on the relationship between expected nominal returns on NLB's and expected real returns on ILB's while Flesaker and Ronn (1987) and Chu (1991), using U.S. CPI futures data, estimate the magnitude of inflation risk premiums in the order of .52% to 1.63%.

IV. Empirical Approach

Similar to the approach used by Evans, Keef and Okunev (1993), the instantaneous change in real interest rates in this study will be modelled by the following mean reverting Ornstein Uhlenbeck process,

$$dr = \beta\left(\frac{\alpha}{\beta}e^{kt} - r\right)dt + \sigma_r dz \tag{8}$$

where, $\frac{\alpha}{\beta}e^{kt}$ represents the long term mean (with growth factor e^{kt}) of the real interest rate and, where β represents the speed of reversion coefficient. Although this process does not incorporate dependent volatility directly, neither the results of Evans, Keef and Okunev (1993), using U.S. and U.K. data, nor the results of this study find evidence of heteroskedasticity.

In their empirical study of the relative fits of competing nominal interest rate models, Chan, Karolyi, Longstaff and Sanders (1992) conclude that dependent volatility is the primary factor that distinguishes between these models. Nonetheless, Evans, Keef and Okunev argue that, if the dependence of volatility inherent in nominal rates and inflation is from the same source, then real interest rates need not necessarily exhibit the same phenomenon.

The discrete time approximation of the above real interest rate process is as follows,

$$\Delta r = \left(\frac{\alpha}{k+\beta}\right)e^{kt}(e^{k\Delta t} - e^{-\beta\Delta t}) + (e^{-\beta\Delta t} - 1)r + e^{-\beta(1+\Delta t)}\int_{s=t}^{t+\Delta t}\sigma_r e^{\beta s}dz \tag{9}$$

where the expected change in real interest rates is,

$$E[\Delta r] = \left(\frac{\alpha}{k+\beta}\right) e^{kt}(e^{k\Delta t} - e^{-\beta\Delta t}) + (e^{-\beta\Delta t} - 1)r \tag{10}$$

and where the variance can be expressed as,

$$\sigma^2_{\Delta r} = \frac{\sigma^2_r}{2\beta}\left(1 - e^{-2\beta\Delta t}\right) \tag{11}$$

After testing for the stationarity of real interest rates using a unit root test, the parameters α, β and k are then estimated using non linear least squares estimation.

V. Data Description

Two sources of weekly data will be used in this study encompassing the period from January 1, 1991 to September 1, 1993. The first source will be a proxy for expected short term real interest rates on nominal debt and the second will be a proxy for expected real interest rates on linked debt.

Following Evans, Keef and Okunev (1992), the expected real interest rate on nominal debt will be estimated by the Canadian twelve month treasury bill yield less expected inflation where expected inflation is assumed to be the percentage change in the CPI over the previous twelve month period. The justification for the use of this particular expectations mechanism is that, if additional inflation lags truly had been significant to the expectations process, then the model for the real interest rate process should exhibit serial correlation. Neither the results of Evans, Keef and Okunev (1993) nor the results of this study exhibit this phenomenon.

Following Pittman (1992), the yield to maturity of the Canadian 2021 4.25% semi annual ILB will be used as a proxy for the expected real interest rate on linked debt. Although the yield on a zero-coupon short term ILB may be a more appropriate measure, index linked treasury bills have, thus far, been unavailable. The justification for the use of this proxy rests on the assumption of a flat term structure of real interest rates and a negligible lag effect of inflation linking. While Barro (1986) and Flesaker and Ronn (1987), using data from the short lived U.S. CPI futures market, found some evidence of a downward sloping real yield curve, Pittman (1992) using a much greater data base of U.K. index linked gilts found no significant difference between quarterly yields of gilts whose maturities differed by greater than 15 years.

Pittman (1992) also demonstrates that the eight month lag causes minimal distortion to the real yields of U.K. index linked gilts if the time to maturity is significantly large. Given the thirty year maturity and the relatively short three month lag period of the Canadian ILB, the lag effect will also be ignored in this study.

VI. Results

Over the period encompassed by the study, the mean expected real interest rate on the NLB was .050 with a standard deviation of .008. Using the Dickey-Fuller (1981) technique to test for a unit root, rejects non-stationarity at a level of 5%. Table 1. provides estimates and asymptotic t-statistics for the variables α, β and k from the non-linear least squares fit of the Ornstein Uhlenbeck model. Strong mean reversion is exhibited with a positive speed of reversion coefficient. The long term mean real interest rate appears to be declining slowly with time and can be described as $.058e^{-.005t}$. No evidence of heteroskedasticity was found in the residuals using Engle's (1982) ARCH test which is consistent with both Evans, Keef and Okunev (1993) and the assumption of constant volatility in the model. The residuals also exhibited no serial correlation up to twenty lags which again was consistent with Evans, Keef and Okunev (1993).

The mean expected interest rate and standard deviation of the ILB over the sample period was .046 and .001, respectively, which was both lower and less volatile than the NLB. Although the unit root test fails to reject non-stationarity, the Ornstein Uhlenbeck model was still fit to the data. Table 1. provides estimates and asymptotic t-statistics for the variables α, β and k from the non-linear least squares fit of the model. Significant mean reversion is demonstrated with a positive speed of reversion coefficient. The long term mean in this case appears to be constant at .046 as the growth factor k is not statistically significant. No evidence of heteroskedasticity or serial correlation was found in the residuals of the expected interest rates on the ILB.

The mean and standard deviation of the inflation risk premium was found to be .005 and .008, respectively. The unit root test rejects non-stationarity at the 5% level. Table 1. provides estimates and asymptotic t-statistics for the variables α, β and k from the non-linear least squares fit of the Ornstein Uhlenbeck model. Strong mean reversion is again exhibited with a positive speed of reversion coefficient. The long term mean real inflation risk premium appears to be declining at a relatively rapid rate and can be described as $.024e^{-.095t}$. Again, no evidence of heteroskedasticity or serial correlation was found in the residuals of the inflation risk premium.

Table 1

Non Linear Least Squares Estimates for the Real Interest Rate Model

Variable Modelled	Parameter Estimates (t statistics)			
	α	β	k	$\frac{\alpha}{\beta}$
Real Interest Rates on NLB				
	.578	9.892	-.005	.058
	(2.16)	(2.19)	(1.67)	
Real Interest Rates on ILB				
	.229	4.974	-.001	.046
	(2.06)	(2.04)	(-0.65)	
Inflation Risk Premium				
	.271	11.118	-.095	.024
	(1.51)	(2.18)	(-1.13)	

The expected real interest rate on nominal debt r_n is estimated by the Canadian twelve month treasury bill yield less expected inflation where expected inflation is assumed to be the percentage change in the CPI over the previous twelve month period. The expected real interest rate on linked debt r_l is estimated as the yield to maturity of the Canadian 2021 4.25% semi annual ILB. The sample covers weekly intervals is from January 1, 1991 to September 1, 1993. The inflation risk premium is calculated as $r_n - r_l$.

VII. Conclusions

Following Evans, Keef and Ornstein (1993), this paper utilizes an Ornstein Uhlenbeck process to model real interest rates. This paper, however, distinguishes between expected real interest rates on nominal debt and expected real interest rates on inflation linked debt in order to investigate differences between their respective stochastic properties and to test for the existence of an inflation risk premium.

Test results indicate strong mean reversion for expected real rates on both ILB's and NLB's although a unit root test could not reject non-stationarity for the ILB. No evidence of serial correlation or heteroskedasticity was found in either. The mean difference between the two measures of real interest rates was significantly positive which is consistent with the existence of an inflation risk premium.

The long term mean real rate on the ILB appears to be constant whereas the long term mean real rate on the NLB may be declining slowly. This result is consistent with the results obtained for the inflation risk premium and may be explained by a 1990's economic environment which has been characterized not only by very low inflation but also by very low price volatility.

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