The Efficient Market Hypothesis, the Gaussian Assumption, and the Investment Management Industry

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Abstract

The purpose of this paper is to disclose how the Gaussian form of the concept of market efficiency is at the origin of the contemporary professional debate on passive index-linked management which continues on despite the growing popularity of indexing among investment management practitionners in Europe. This particular Gaussian form entered the investment management industry in the 1970's and carries strong assumptions about the behavior of returns and the structure of the information set. We argue that this ill-defined debate on indexing is due to a confusion between efficiency and Gaussian efficiency. The originality of the paper resides in the point of view choosen as the « main thread ». Instead of focusing on informational issues, now better understood since the seminal paper of Grossman and Stiglitz (1980), we concentrate on the probabilistic aspects included in the testable applications of the concept, so as to connect Fama's statement of 1970 to Bachelier's work (Theory of Speculation) of 1900. We establish the link between Bachelier's dissertation and portfolio management applications of market efficiency. We argue that understanding the precise characteristics of the link associating the informational efficiency concept itself with the underlying probabilistic hypothesis leads to a better approach to the problems facing the investment management industry and allows us to understand the professional impact of the Gaussian form of efficiency. The issues of non normality of empirical distributions (fat tails problem) and concentration of performance are linked with the efficiency paradigm so introducing a rationale for the the continued growth in alternative investment management.

Contents

1. The efficient market concept engrained in the industry

1.1. « Efficiency »: everywhere but where?

Some concepts look like old habits: people don't precisely remember their origin, but everybody agrees, at least tacitly, that they are the source of a large number of practical ways of acting in real world. One such concept is the efficient market hypothesis for the practitioners of asset management. For example, in a professional paper devoted to packaged investment products and intended « to serve as a springboard for further dialogue », one can read that « investment managers' differing approaches are often correlated to their acceptance of the concept of market efficiency »[1]. Another example is indexation; in a professional textbook, the rationale for indexation is given in these terms: « the theoretical arguments revolve around the efficient market hypothesis »[2]. In a special report prepared for the 25th anniversary of indexing and quantitative techniques, the following statement is found: « the debate (for and against indexing) turns on different views of the efficiency of capital markets »[3]. It is the reason why « the market-efficiency literature also produced a demand for performance evaluation »[4].

Of what use is the so-called « market efficiency » concept? Viewed simply and schematically, this concept expresses the fact that the quoted market prices faithfully reflect the underlying economic reality, and so enable the investors or asset managers to make a reliable decision (buy or sell), meaning with full knowledge of the relevant facts necessary for this decision. If the relevant information necessary for a transaction is correctly transmitted in the price, and considered as an exchange mechanism, the market is then called « efficient ». As this efficiency concept involves modeling information, the words « informational efficiency » are also used to describe market efficiency (as opposed to other forms of efficiency, like economic Pareto-efficiency for example). It is a concept which is at the basis of modern financial theory, as it has been developed over fifty years. Is it but an intellectual abstraction created *a posteriori* to account for the expanding use of a large number of professional practices and financial instruments, or is this empirical knowledge, are these consensual practices themselves, generated by a concept which precedes their development?

In fact, since the development of financial theory, first with the growth of the asset management industry in the US, and more recently with derivative markets around the world, there has always existed an interaction between empirical knowledge and theoretical models. As noticed by Ross, « the feature that truly distinguishes finance from much of the rest of economics is this constant interplay between theory and empirical analysis »[5]. On the one hand, developments in world financial markets have encouraged the extension of financial theory, on the other hand, this theory has often been at the origin, either of the creation of new financial instruments, or new ways of thinking old topics, as for example portfolio management. For example, « before (the Sharpe-Lintner-Black model) became a standard part of MBA investment courses, market professionals had only a vague understanding of risk and diversification »[6]. In his 1991 review of the academic literature, E. Fama emphasized: « it is well to point out here that the efficient-markets literature is a premier case where academic research has affected real-world practice »[7]. These studies have

[1] PwC (1999), p.3.
[2] Bishop (1990), p. 7.
[3] PwC (1998), p. 9.
[4] Fama (1991), p.1608.
[5] Ross (1989), p.30.
[6] Fama (1991), p. 1593.
[7] Fama (1991), p. 1608.

« changed the *views* and practices of market professionals »[8]. One of these market professionals confirms this assertion: this new approach of the markets « seemed [...] to prescribe radical changes in the way I should carry out my responsibilities »[9]. The change mover was the new understanding of the market returns behavior: « it has changed our *views* about the behavior of returns »[10], and « along with recent empirical work on securities' prices, the profession's *view* regarding the returns available from professionally managed mutual funds has undergone a similar change »[11].

From this fertile interconnection between theory and practice were born the global financial markets of today. They are like risk transfer mechanisms on the scale of nations, in which the agents (retail or institutional) can sell their financial risk to others, who receive for this a premium (called, for this reason, a "risk premium") which is more or less remunerated according to the quantity of units of risk bought. For example, if the defined benefits retirement schemes can provide secure streams of payments, or if individuals can buy financial products offering a guarantee against risk of loss, it is because, as a last resort, somewhere, there existed a buyer of this risk which could not be retained by those who had sold it. Hence, the possible dysfunctions in the financial markets will concern their efficiency: transfer of risk and transfer of information about the exchanged assets.

This allocational efficiency of the capital markets represents the theoretical foundation for their existence. Without good allocation of risk and information, market agents will not be able to work in the financial markets, or not in the same way. To understand how important transparency of information and the right evaluation of the price of the risk is for the smooth functioning of a stock market, it is enough to consider the problems raised by the misdemeanors of insiders or the bankruptcies resulting from the absence of risk control. Without these conditions, the market will not be efficient in allocating resources.. Hence, informational markets efficiency refers to its allocational efficiency. This concept is at the basis of modern financial theory, and paves the way for all financial instruments, professional practices, hedges in trading rooms, techniques of asset management, etc. For example, in a reference textbook for finance students, one can find this statement: « the efficient market concept is the ground on which the whole of modern financial theory rests »[12].

Following Kuhn's statement of « paradigm », it is possible to state that the informational efficiency of markets is a paradigm for the financial practices in the asset management industry, and perhaps for the whole financial industry itself. Discussion about the relevance of the application of Kuhn's concept of paradigm in economics can be found in Coats' 1969 paper[13]. Considering the scope of application of this concept, it could be useful to come back to its beginning, its intellectual roots. All the more so since on account of certain dysfunctions which have appeared in certain markets, efficiency is in a state of turmoil: questions have been raised concerning the validity of the efficiency concept, without any theoretical foundation appearing for the moment to replace it.

1.2. The two-sided aspect of the efficiency concept

It is well recognized that, on its economic side, this paradigm is closely related to the idea of economic equilibrium. For example, one can find in a Mandelbrot paper that « « market efficiency is an aspect of economic equilibrium »[14]. More recently, LeRoy states that « at its most general level,

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[8] Fama (1991), p. 1576.
[9] Bernstein (1992).
[10] Fama (1991), p. 1576.
[11] Malkiel (1995), p.550.
[12] Jacquillat, Solnik (1989), p. 43.
[13] Coats (1969).
[14] Mandelbrot (1971), p. 227.

the theory of efficient capital markets is just the theory of competitive equilibrium applied to the financial asset market »[15] Another example emphasizes this relationship: « in neoclassical equilibrium theory, efficiency refers to Pareto efficiency. [...] The relation between the definitions of efficiency is not obvious, but it is not unreasonable to think of the efficient market definition of finance as being a requirement for a competitive economy to be Pareto efficient »[16].

But it is clear that this paradigm also refers to a probabilistic representation of the behavior of stock market prices: the concept of informational market efficiency is closely associated with a statistical-probabilistic view, which implements a toolkit borrowed from probability theory and physics. This is because the property of efficiency assumed for the market necessarily requires the complementary property of arbitrage. As Mandelbrot said, « Roughly speaking, a competitive market of securities, commodities or bonds may be considered efficient if every price already reflects all the relevant information that is available. The arrival of new information causes imperfection, but *it is assumed that every such imperfection is promptly arbitraged away*. (…) it can be shown that if an arbitraged price is implemented, it must follow a martingale random process »[17]. The classical case mainly used by the practitioners is the Gaussian random walk. The so-called and well documented « anomalies » observed in the market's behavior are mainly departures from this probabilistic framework. In other words, *the market efficiency concept is inextricably linked to the random walk model*. Understanding the precise characteristics of the link associating the informational efficiency concept itself, and the underlying probabilistic hypothesis, leads to a better approach to the problems facing the investment management industry.

This paper examines the emergence of this paradigm in the finance industry from a historical probability perspective. More precisely, byemphasizing the probabilistic preconditions of market efficiency to show how this concept has been influenced by a choice of probability distribution. This paper therefore invites the reader to an archaeological excavation of the probability theory inherent in the design of efficiency and its role in the investment management industry: an investigation into the intellectual roots of quantitative techniques and asset management practices.

Given this objective, it appears fruitful to come back to the historical birth of the concept: to understand its nature, we should consider its origin. The story begins in 1900, a crucial year for the birth of efficiency in the modern sense of the term. It was on the 29th of March 1900 that the French mathematician Louis Bachelier defended his doctoral dissertation in mathematics, before Henri Poincaré, on the subject of stock market fluctuations in the Paris stock exchange. This thesis, entitled « Theory of speculation », can be considered as the beginning of modern finance, and the distant root of the concept of informational efficiency in its probability sense. As Cootner acknowledged, « so outstanding is his work that we can say that the study of speculative prices has its moment of glory at its moment of conception »[18].

2. The birth of the « random walk » model in Bachelier's pioneer work

2.1. The mathematical finance emerging with Louis Bachelier

Louis Bachelier is an atypical figure in the history of science. Little is known of him as most of the archives concerning him were burned. It is therefore difficult to reconstitute his life as a mathematician and, more generally, as an individual. One short biography exists now[19], which gives

[15] LeRoy (1989), p. 1583 (opening of the article).
[16] Ross (1989), p.2.
[17] Mandelbrot (1971), p. 225.
[18] Cootner (1967a), p. 3.
[19] Courtault *et al.* (2000).

some information and new lights about the main facts of Bachelier's life. Born on 11 March 1870 in Le Havre, he had a relatively late university career. It is known that he applied without success in 1926 for a position as professor at the University of Dijon. « It was really a disgrace that the Council of the Faculty of Sciences classified him as the second candidate, a disgrace not because of the result but the pretext : Professor M. Gevrey blamed Bachelier for an error in the paper of 1913 readiliy confirmed by a letter of Paul Lévy, professor of Ecole Polytechnique »[20]. Lévy relates this episode in these terms[21]

> I first heard of him (Bachelier) a few years after the publication of my *Calcul des Probabilités*; that is, in 1928. He was a candidate for a professorship at the University of Dijon. Gevrey, who was teaching there, came to ask my opinion on a work by Bachelier published in 1913. In it, he defined Wiener's function (prior to Wiener) as follows (...). Gevrey was scandalized by this error and asked my opinion. I agreed with him, upon his request, confirmed it in a letter which he read to his colleagues in Dijon. Bachelier was blackballed.

Bachelier exposed the details of this story in a letter[22] :

> Due to a sequence of incredible circumstances…, I have found myself at the age of 56 in a situation worse than I had during the last six years ; this is after twenty-six years with the doctor degree, five years of teaching as free professor at Sorbonne, and six years of official replacement of a full professor. My career happened to be blocked in a deplorable and extremely unfair way without a minor reproach which could be given to me.

One possible deeper reason for Bachelier's difficulties was the subject of his thesis itself. Poincaré had said, in ambiguous terms: "The topic is somewhat remote from those our candidates are in the habit of treating"[23]. This comment indicates a sort of surprise at the intrusion of the stock market into the sphere of mathematics, as well as reservation, even a certain skepticism, on the part of Poincaré faced with this new interest. In his notebook, Lévy himself gave the comment : « Too much on finance »[24]. As Mandelbrot pointed out : « Unfortunately, no organized scientific community of his time was in a position to understand and welcome to him. The gain acceptance for himself would have required political skills that he did not possess, and one wonders when he could have gained acceptance for his thought »[25]. However, this « traditional public view according to which Louis Bachelier was a mathematician prodigy whose remarkable insight had no impact on the development of the theory of stochastic processes (…) perhaps because the title of his thesis was no attractive to mathematicians » is challenged by Courtault *et al.* who assert that « our study of documents shows that this simplified story is far from being correct »[26]. The question is opened.

Finally, after several setbacks, Bachelier was appointed professor at the University of Besançon. He died on 28 April 1946. The precursory and radically new character of some of his ideas was not to be perceived by his contemporaries. Even Paul Lévy later acknowledged that he had not properly appreciated Bachelier's work[27]:

[20] Courtault *et al.* (2000), p. 345.
[21] Letter from P. Lévy to B. Mandelbrot, dated 25 January 1964, cited in Mandelbrot (1977), pp. 251-252.
[22] Courtault *et al.* (2000), p. 345.
[23] Poincaré's report, related in Mandelbrot (1977), p. 253, and in Courtault *et al.* (2000), p. 348.
[24] Personal papers of Paul Lévy, related in Courtault *et al.* (2000), p. 346.
[25] Mandelbrot (1987), p. 86.
[26] Courtault *et al.* (2000), p. 346.
[27] Letter from P. Lévy, *op. cit.* in Mandelbrot (1977), p. 252.

> We became reconciled with him. I had written him that I regretted that an impression, produced by a single initial error, should have kept me from going on with my reading of a work in which there were so many interesting ideas.

Anyway, his thesis is and remains surprisingly modern, and still constitutes very "food for thought" reading for today.

2.2. The Bachelier's dissertation from a financial point of view

2.2.1. « Zero expected profit for the speculator »: giving up the search for predictability

In his doctoral thesis, Louis Bachelier writes[28]:

> The influences which determine fluctuations on the Exchange are innumerable; past, present, and even discounted future events are reflected in market price, but often show no apparent relation to price changes (p. 17).

These are « natural causes » of price changes: economic, financial, or political factors which move the level of prices. Besides these « natural » factors, Bachelier also mentions other causes, which he calls « artificial», which refer to self-generated price changes in the markets.

> Besides the somewhat natural causes of price changes, artificial causes also intervene: the Exchange reacts on itself, and the current fluctuation is a function, not only of previous fluctuations, but also of the orientation of the current state (p. 17).

These two aspects of stock market fluctuations are to be found in most analyses of market behavior. In today's words, one would speak of new available and relevant information arriving on the market and integrated by market agents into their price change expectations, and of the self-reinforcing system due to stock market prediction models, which leads to price-volume analysis. It is remarkable that this twin causality is also reflected in the two ways of approaching the markets, which in general separate market professionals into two groups: the financial analysts and economists in one hand, and the technical analysts in other hand. The first are more "fundamentalist" in the sense that they are seeking the "true" value of the stock or the market, while the second, voluntarily ignorant even of the financial characteristics of the assets examined, concentrate only on the "technical" elements of the market, and mainly on the time series made up of past prices.

But Bachelier adds:

> The determination of these fluctuations depends on an *infinite number of factors* (our italics); it is, therefore, impossible to aspire to mathematical prediction of it. Contradictory opinions concerning these changes diverge so much that at the same instant buyers believe in a price increase and sellers in a price decrease (p. 17).

Against the analysts' point of view, who try to predict the next price change, Bachelier takes the side of the unpredictability of future returns, by pointing out the existence of an "infinite number of factors". One discovers in this argument (an "infinite number of factors") Gauss' question of the sum of small independent random variables, which led to the law of large numbers and to the central limit theorem: if an overall random phenomenon is the sum of a very large number of

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[28] Bachelier (1900). The quotations and the page numbers refer to the English translation of the thesis. The translation is made by A.J. Boness, and edited in Cootner (1967), pp. 17-78.

elementary random phenomena, each having some distribution or other, but all being weak in magnitude, then the distribution of the overall random phenomenon is a Gaussian distribution. Using modern financial language, one would talk about the bits of elementary information arriving on the market, so that their aggregation leads to a limit probability distribution (in the sense of the previous theorem) for the stock market fluctuations.

Hence Bachelier switches the stock market analysis from predictability of returns to their probability distribution. But what type of probability?

> One can consider two kinds of probabilities: 1. The probability which might be called "mathematical" which can be determined a priori, that which is studied in games of chance. 2. The probability dependant on future events and, consequently, impossible to predict in a mathematical manner. The last is this probability that the speculator tries to predict. He analyses causes which could influence a rise or a fall of market values or the amplitude of market fluctuations. His inductions are absolutely personal, since his counterpart in a transaction necessarily has the opposite opinion (p. 25).

Cootner noticed that "this classification of probabilities (subsequent to Bayes) foreshadows the current popular dichotomy of objective and subjective probability". A key assertion, regarding the theoretical as well as the practical consequences which it was to have to this day, is next put forward by Bachelier.

> It seems that the market, the aggregate of speculators, at a given instant can believe in neither a market rise nor a market fall, since for each quoted price there are as many buyers as sellers (p. 26).

Modifying the quoted prices, to take account of bond coupons, in order to consider only the price variations without the impact of this coupon, Bachelier introduces the notion of "true price", meaning by this a price which one would call today a "price index", as opposed to a "total return index". Then with this new concept, he postulates that

> From the consideration of true prices, one could say: at a given instant, the market believes in neither a rise or a fall of true price (p. 26).

That is to say, considering the value of true prices,

> the mathematical expectation of the speculator is zero (p. 28).

This zero mathematical expectation for each player represents the application to stock market variations of the concept of "fair game". This is a remarkable assertion, for the wide consequences that it was to have on the practices in portfolio management and the modern development of operating on the capital markets. Any prediction became not only impossible but useless:

> Clearly the price considered most likely by the market is the current true price; if the market judged otherwise, it would quote not this price, but another price higher or lower (p. 28).

Using probabilistic framework, this means that the stock market fluctuations follow a stochastic process for which the future does not depend on the past except through the present; in such a process, the best prediction of the subsequent price is the value of the current price.

This characterization of the stochastic process governing stock market fluctuations refers to the reflection property of the available and relevant information in the last quoted price. Thus, to operate in a market with efficiency, there is no need to know all the past prices; the value of the last quoted price is enough, and only the future is considered. By way of analogy, let us look at chess and bridge. To operate during a bridge game, it is necessary to know exactly all the previous declarations and moves; the future depends strictly on the past, and the absence of this knowledge prevents playing. But in the case of chess, a game called "perfect information game", it is possible to enter the game at any point and take the place of a player, without needing to know the preceding moves and captures; all the information which is necessary to the action is condensed in the present position of the pieces on the chessboard. The succession of captures in chess is characteristic of a process without memory. But in the case of bridge, a memory of the game exists, a memory which influences future moves.

The debate opened by Bachelier is framed in exactly these terms. Is the process which characterizes the successive price changes in the markets without memory or not? Is there a memory in the market, which could be used to determine systematically winning strategies, and, in this case, where is the good information to be found? Or, if all the information necessary for a successful trade is already contained in the quoted prices, then the only cause of new variations could be elements of information which are not predictable, and thus random. Bachelier's argument can be expressed in the following simple and intuitive manner: if the prices contain the predictable, later fluctuations are exclusively a matter of the unpredictable, that is, of chance. What is evident, with the introduction of this argument, is the importance given to the concept of information. And, in fact, it is from the study of information that the definitions of the efficiency concept will develop.

2.2.2. « Fluctuations of a given amplitude »: from predictability to Gaussian volatility

With the loss of the predictability dimension of stock market variations, another object of research is taken up. Even if one cannot predict the future development of the market,

> it is possible to study mathematically the static state of the market at a given instant, i.e., to establish the law of probability of price changes consistent with the market at that instant. If the market does not predict its fluctuations, it does assess them as being more or less likely, and this likelihood can be evaluated mathematically (p. 17).

For Bachelier, the buyers and the sellers will then exchange expectations of variability. He expresses this as follows:

> If the market believes in neither a rise nor a fall of true prices, it may suppose more or less likely some fluctuations of a given amplitude (p. 26).

The modernity of Bachelier's ideas does not cease to surprise. This amplitude of stock market movements has become an essential parameter in contemporary models: it is what is called the "volatility" of the markets, i.e. standard deviation of their successive price changes. And the question which is posed to practitioners of the quantitative approaches to the markets (calculations of the hedges for positions in the trading rooms, and of artificial replication of portfolios) is not to predict the development of future prices, but only to estimate their potential variability. This instantaneous variability represents a key element in the designing of synthetic portfolios (role of the variance-covariance matrix, which expresses the joint variability of assets which compose the portfolio, or their co-movements in pairs) as well as the hedge of positions on derivative financial instruments (role of the price gradient of the financial asset). For example, an option price basically depends on the volatility of the underlying asset on which the option is written. The derivative markets have become markets of volatility, in which levels of price variability are exchanged between traders, without needing any information related to the market trend.

With the introduction of the volatility criterion, one sees how a shift is produced in the way stock market variations are viewed; rather than seeing the markets from a *directional* perspective, or from the perspective of the search for predictability, one considers them from a *distributional* perspective, meaning the perspective of the analysis of their variability. From that moment, the complete characterization of a market behavior will rely on the probability description of its successive price changes. So Bachelier writes:

> The determination of the law of probability which the market admits at a given instant will be the purpose of this study (p. 26), and particularly, "Research for a formula which expresses the likelihood of the market fluctuations", a formula which "does not appear to have been published to date" (p. 17).

Faced with treating the question of the arrival of information on the market, the « infinite number of factors », Bachelier addressed the problem of the sum of little independent random causes. One knows that, if the « little random causes » are not too scattered[29], their resulting sum is clustered according to the normal distribution of Laplace-Gauss: it is well known that, if the variables have finite variance, the limiting distribution of their sum will be the normal distribution (asymptotic normality of cumulative sums property). But, more generally, one can approach the question of the limiting distribution of the sum of independent random variables, without setting particular conditions for their dispersion[30]. This relatively complex problem was not resolved until 1925 by Paul Lévy, with the introduction of « stable » probability distributions, which determine the form of the basins of attraction of the limit laws for the sums of independently and identically distributed random variables, by generalizing the question of Gauss' little random causes[31]. In the absence of this theory, Bachelier could use the only tool which was available: the normal distribution of Laplace-Gauss (which belongs to the stable family), the only limiting distribution known for the sum of independently and identically distributed random variables, at the time of defending his thesis. This comes down to saying that the random shocks of information bits which cause the price movements of financial assets are of the same nature and « not too dispersed » (that is, with finite variance). This is a very strong hypothesis on the nature of information shocks, but the only one possible in Bachelier's time[32]. Developing an elegant and keen demonstration, he finds a Gaussian density for the sought expression of the probability.

By applying the normal distribution to successive price changes, through this work on random processes, Bachelier had set out the premise of Brownian motion before Einstein, based on research into the formalization of stock market fluctuations. The model of the Gaussian « random walk », or the so-called Gausso-Markovian representation of stock market fluctuations, was born. This was a

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[29] More precisely, using probability words, it is necessary that the mathematical expectation and variance of these random variables exist. The variance being of the order of the square of the mathematical expectation, it is said that the random variables should be square-integrable. The square-integrability of the random variables is the necessary condition for obtaining the central limit theorem in its classic (Gaussian) form.

[30] Note that, in this case, it can raise a problem with the convergence of the empirical averages of stationary time series towards mathematical expectation. This is the so-called rate of convergence in the law of large numbers, addressing the limit behavior of sums of iid random variables.

[31] Paul Lévy showed that each time the mathematical expectation and the variance are finite, the only possible limiting distribution for sums is Gauss' distribution. In all the other cases, one obtains more complex limit distributions, which generalize Gauss' distribution by giving to the look of the distribution a more leptokurtic form, more stretched (somewhat like the ellipse stretches out the circle).

[32] In fact, Cauchy's distribution (1853) was already in Bachelier's time an exception to the central limit theorem in its Gaussian form. But this exception was considered by the probability theorists as so unrealistic (for example Bienaymé, in 1867) that this law was quite rapidly dismissed from the attention of scientists.

real intellectual break with the previous conceptions of stock market fluctuations, and can be called Bachelier's « coup de force ».

Let us now consider the Bachelier heritage in finance.

3. The rise of statistical-probabilistic paradigm in financial theory and industry

3.1. Earlier empirical works validating the random walk thesis before 1965

Bachelier's model was then tested statistically. But not immediately. As Cootner said,[33]

> Despite Bachelier's very early interest in stochastic analysis of speculative prices and Working's renewed interest in the 1920's, stock market research from this point on was very slow to develop. While professional practitioners displayed a strong and continuing interest in the stock market, it enjoyed relatively little academic attention until *after the debacle of 1929* (our italics)[34]. While such lack of attention was not absolute, it stands out very sharply in comparison with the extensive research on commodity prices and on prices of those financial instruments which came under the rubric of "money". This disinterest was compounded of many parts: the smaller role played by organized equity markets in industrial finance, a conviction that stock markets were the product of mass (irrational) psychology akin to gambling, and a shortage, among economists, of the mathematical and statistical skills necessary for effective research in this field.

The tests began in the 1930's, and took two paths. The first consisted of verification of the absence of memory (and thus of predictability) of the market, that is the independence of successive price changes (in probability vocabulary, « independence of increments »). Basically, this is looking at the form of the past-present-future relationship, to be sure that it does not exist. The second path involves the risk of price change, the variability of the markets, and seeks to characterize statistically the form of returns dispersion, the empirical distributions observed, to verify whether they are actually Gaussian. In short, the tests carried out on Bachelier's model tend to demonstrate first of all that stock market variations are indeed the result of random variable; and also demonstrate that this randomness has a determined distributional structure. These two paths represent descriptive approaches to market behavior, and focus on the minimum required statistical properties, without bothering with causal economic explanations for the statistical mechanism which is sought.

Statistically, the question of the predictability of stock market returns is asked in the following way: if the price changes are the result of independent Gaussian random selections, then the measurement of the interdependence of the variations should cause a zero result to appear. Thus the

[33] Cootner (1967b), p. 79.

[34] Interesting fact for thought is to notice the role of crashes in the development of academic researches. The story will repeat itself after the 1987 crash : today's open academic debate on the fat tail problem is a direct consequence of the debacle of 1987. For this point, cf. below.

choice of dependence tests, which were to be systematically undertaken in the markets[35]. The results appeared to be conclusive, and the model was validated first by Cowles (1933), Working (1934), Cowles and Jones (1937), and then by Kendall (1953), who demonstrated the existence of some randomness in the successive price changes.

It was to be confirmed and supported by the article published in 1959 by Osborne, who tested also the form of the limiting distribution. In relation to Bachelier's hypothesis, Osborne introduced a slight but important modification, by considering, not the absolute price changes themselves, but their logarithm[36]. Relying on the known properties of particle movements in statistical mechanics, he demonstrates that this approach to the markets leads to direct inference of a Gaussian density in the first differences in the logarithms of quoted prices, and to consideration of stock market variations in the same way as Brownian motion for particles:

> It is shown that common stock prices (…) can be regarded as an ensemble of decisions in statistical equilibrium, with properties quite analogous to an ensemble of particles in statistical mechanics. If Y is (the log-return), (…) then the distribution function of Y is precisely the probability distribution for a particle in Brownian motion[37].

Strictly speaking, Bachelier was ignorant of Brownian motion, and thus it was Osborne who in complementing Bachelier's model thanks to developments in statistical mechanics, set forth the first hypothesis of Brownian motion in the logarithms of prices. Osborne's article is an important stage in the application of models to stock market variations.

The publication of these results marked the beginning of an increase in interest of this subject. Statistical investigations continued, facilitated by the introduction of computing machinery. One after another, following Roberts (1959)'s suggestions, Larson (1960), Working (1960), Houthakker (1961), Alexander (1961), Cootner (1962), Moore (1962), Granger and Morgenstern (1963) bring strong support to the random walk thesis, and confirmed that successive price changes can be considered comparable in first approximation to a Brownian motion. It must be noted that at this time, temporal (non-interdependence of variations) and distributional perspectives (Gaussian density) were not dissociated. These two aspects, although different in their consequences on the analysis of the markets behavior, were intermingled at that time: one can speak of a "Gausso-Markovian view" of the markets, characterized by stock market fluctuations without brutal jumps (stock market crashes) or long-term correlations (persistence of rising and falling movements).

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[35] The properties of a stochastic process can be partially summarized by the function of auto-covariance, from which are extracted serial auto-correlations. Dependence tests will take four paths : serial correlation tests, momentum or "runs" tests, filter tests, persistence tests. The serial correlation tests use the functions of auto-correlation and examine the appearance of the correlograms. The momentum or runs tests analyze the successive values taken by the random variable. The filter technique is a trading method for operating on the markets : if detectable trends exist, then the performances obtained with this method will be superior to those obtained by a classic procedure of buy and hold. The persistence phenomenon corresponds to a sort of "memory" of the price changes : the big changes are followed by big changes (rising as well as falling), and vice versa, the small changes are followed by other small changes. One finds here the inspiration for the ARCH type models of the 1980's.

[36] Osborne (1959a) and the commentary Osborne (1959b). The choice of a logarithm is justified by considerations concerning the form of the distribution of variations as well as a physical equivalence with the significance of the magnitude of the variations when the price level rises. Although Osborne used Weber-Fechner's law on the responses of physiological sensations to stimuli, from an economic point of view, his argument involves a function of decreasing usefulness on the variations of absolute price levels : the Weber-Fechner'law is a law of proportionality. From a financial point of view, the justification of this choice would be given in 1965 by Fama with the notion of a continuous interest rate.

[37] Osborne (1959a), p. 145.

From now on, the hypothesis of Gaussian random walk acquired consistency and strong academic recognition, and that which is called the Bachelier-Osborne model became a paradigm for the study of market behavior in academic research. Developments in portfolio theory and options theory reinforced this thesis, which was not to be brought into question until the stock market crash of October 1987.

3.2. The passage of random walk thesis in the financial industry after 1965

3.2.1. The portfolio theory entering the investment management industry

A third dominating trend began to penetrate into the world of fund management in the US: in 1952 Harry Markowitz presented a radically new way of managing portfolios on the markets, different from the traditional methods used until then[38]. The theory was designed to cope with the choice between risk and return, and the way of obtaining the best risk-return profile. For example, what would be the best asset allocation between stocks and bonds? What is the part of the portfolio which has to be dedicated to cash? The theoretical results consist of maximizing return under constraints, for a fixed level of risk, or to minimize a level of risk, for a expected return: with his pioneering work, Markowitz introduced to finance the methods of optimization used with success in other scientific fields, and paved the way for the quantitative approach of portfolio management. He justified rigorously the concept of diversification, and he demonstrated that any investor should consider, not only the assets by themselves, searching for their theoretical underevaluation, but the portfolio as a whole, with global risk and global return, i.e. global trade-off between risk and return. To sum up, quantitative approach of investment management is based upon statistical-probabilistic principles, and uses rational analysis to construct portfolios.

As noticed by many professional investment managers, among them A. Rudd, « unfortunately, the computational requirements were too burdensome for the approach to be implemented on a large scale until the mid-70's »[39]. Having remained confidential for several years, the portfolio theory began to emerge at the beginning of the sixties, thanks to the simplification of the necessary calculations implemented by Sharpe in 1963. An understanding of all the co-movements of financial assets taken in pairs was in fact essential to achieve Markowitz' optimal portfolio. But practical use of this method by portfolio managers had been almost impossible with the limited means of calculation available at the time, because of the problem of determining all the pairs of these relationships, the so-called calculation of the variance-covariance matrix. By a clever statistical transformation which resulted from a daring economical hypothesis, Sharpe made the calculation of the optimal portfolios of Markowitz accessible to asset management firms[40]. The mathematical complexity of the calculations was reduced in such a way that it became possible to implement the simple linear formula of the Sharpe' CAPM model.

By introducing the idea of the optimal portfolio, called « mean-variance efficient » (or « MV-efficient ») Markowitz and Sharpe used and thus validated the Bachelier-Osborne conception. As a matter of fact, to solve the problem of optimizing portfolios, it was necessary to hypothesize a stochastic process for the time series of securities'returns on the market. For the calculation of the variance-covariance matrix implies the existence of a probability characterization of all the co-

[38] Markowitz (1952). Then developed in Markowitz (1959). These two contributions founded modern portfolio theory. It was to be complemented and bettered, but not fundamentally changed.
[39] Rudd (1989), p. 20.
[40] Sharpe (1963). This simplification consists of the introduction of a linear relation between a market index and all the securities which fluctuate on this market. The model is called « diagonal », and the number of parameters to estimate, for example in a portfolio of 100 stocks, passes from 5050 to 200. This reduction thus allows more rapid and thus more operational calculations.

movements of these securities, by means of a probability vector concerning the whole market: a joint distribution of returns for all the securities. And of course this was a joint multinormal distribution. This multinormality in the price changes of securities allowed the necessary calculations to be made, and for the construction of Markowitz' MV-efficient portfolios to be achieved. Therefore, the foundations for the quantitative approach for investment management are the researches related to the random character of stock market prices. And from that time forward, investors and portfolio managers began to think not only about expected returns, but to take into consideration, for each investment, the risk-return relationship. The risk is expected, and quantified, by means of the volatility of markets and securities, that is, the Gaussian standard deviation of the successive returns of assets, corresponding to stock market fluctuations. From then on, the normal distribution governed and calibrated the quantitative aspects of the markets, and the hypothesis of random walk became necessary for the construction of these portfolios[41]. In one hand, the random paradigm of stock market prices created the portfolio theory; but, in the other, this theory reinforced the paradigm, because of its use in the investment management firms. The investment management industry was entered by the paradigm; next it solidified this paradigm.

3.2.2. The option pricing theory entering the derivatives industry

An identical process, but of even greater intensity, came about a few years later: this is the birth and development of the option pricing theory. Although the field of option theory is more concerned by the concept of « arbitrage » than the one of efficiency, although the period under consideration extends beyond the emergence of the probability aspect of the efficiency concept, this process is nevertheless worth noting, as this phenomenon will lead to a consolidation, a hardening of the hypothesis of Gaussian random walk thesis in the financial industry. The development of option tools became so important in finance in the 1970's and 1980's, with intensive use of second order diffusion processes, that is, of a marginal law with finite variance, that it would be impossible in practice to question the use of the normal law as a marginal distribution governing the behavior of the markets.

In this case also, the precursory character of Bachelier's work remains remarkable. And the influence which he subsequently exerted is certainly perceptible if one understands the manner in which these results were obtained. In the continuation of his thesis, Bachelier sought to calculate the probability that the price of an financial asset would be reached or exceeded at a later given time. This question corresponds exactly to the search for what is called today the price of a call option. In order to resolve the problem which he had set himself, Bachelier was finally to arrive at Fourier's conduction equation, with an argument as elegant as it was daring. He was to come to this result by inventing a concept which he called « radiation of probability »:

> Each price during an element of time radiates towards its neighboring price , over time, an amount of probability proportional to the difference in their probability. (...) The above law can, by analogy with certain physical theories, be called the law of radiation or *diffusion* (our italics) of probability[42]

[41] The innumerable practical difficulties encountered when these models were put into operation must be pointed out : these difficulties resulted from, among others, the non-validation of their premises: on the one hand, the market indexes are not always efficient in the Sharpe-Markowitz sense; on the other, the multi-normality of stock market variations is not verified, which brings about constant instability in the variance-covariance matrix, and difficulty in appreciating the quality of the supposedly optimal portfolio. Finally, the expected return is delicate to estimate, and a change in estimation leads to a modification of the optimal portfolio. Even today, quantitative management has not resolved all the problems of determination of optimal portfolios.

[42] Bachelier (1900), p. 39.

According to Henri Poincaré:

> The manner in which Louis Bachelier obtained the law of Gauss is most original, and all the more interesting as the same reasoning might, with a few changes, be extended to the theory of errors. He develops this in a chapter which might at first seem strange, for the title is "Radiation of Probability". In effect, the author resorts to a comparison with the *analytical theory of the propagation of heat* (our italics). A little [sic] thought shows the analogy to be real and the comparison legitimate. Fourier's reasoning is almost applicable without change to this problem, which is so different from that for which it had been created. It is regrettable that the author did not develop further this part of his thesis[43].

As it happened, this result was not found on the financial markets until seventy-three years later by Black and Scholes (1973) who in turn used the heat conduction equation to evaluate the price of an option; this model was the basis for options pricing theory.

But one discerns clearly, in this approach to option pricing, the overlapping which exists between Gaussian distribution on returns and the capacity to obtain a price on the optional financial instrument. Distributional properties or returns are of vital importance for this objective: to derive a valuation model of option, it is necessary to assume a stochastic process for the underlying asset. As the equation of heat conduction, taken up by Black and Scholes, presupposes filling certain conditions in the form of a stochastic process, used in the probability representation of stock market returns, conditions which impose Gauss'law[44], one sees that Gauss' law will be called upon for calculating this expectation of income flow. And in practice, in the formula which gives the price of an option, terms appear which describe the calculations of the function of Gaussian density. This is to say that one could not conceive, at the beginning of the 1970's, an options market, without validating at least implicitly the Bachelier-Osborne model. More fundamentally, the notion itself of quantified evaluation of an option has its intellectual roots in the Gaussian diffusion processes. And vice versa, the rejection of this model led to an incapacity to evaluate, and so to hedge, the options whose volume was beginning to rise exponentially on the financial markets. From that time on, it became necessary, even vital for financial activity itself, that the random walk model be recognized as usable in first estimation to create models of stock market variations. Thus the statistical-probability paradigm in financial theory was consolidated. It needed the stock market crash of 1987 and the development of the prudential approach in the derivative markets on an international level, for the probability foundations of the Bachelier-Osborne model to be re-examined.

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[43] Report of the thesis dissertation by Henri Poincaré, in Mandelbrot (1977), pp. 253-254 and in Courtault *et al.* (2000), p. 352.

[44] Because one examines the sensitivity of the option price to the variations of the underlying asset, on which the option is set. In particular, it is necessary to be able to calculate the sensitivity of the option price in function of the price of the underlying asset and of time. This differentiability uses stochastic differential calculus, and the derivation formula for Gaussian random variables, called Itô's lemma. The use of Itô's lemma implies then that the random variables are Gaussian.

4. Interplays between academics and practitioners around the dispute « predictability vs. random walk »

4.1. The professional practices before the random walk model

4.1.1. Asset managers searching for the relevant information: predictability is assumed possible

The financial theory which was being developed in the 1950's was one in which the best prediction of a future price consisted of the present quoted price (martingale property of price process). This comes down to stating that it is not possible, even with active portfolio management, to make a gain which is significantly better than that obtained by the performance of the market itself over a long period. This radically new and relatively provocative intellectual construction ran counter to the practices of professionals asset managers, financial experts and technical analysts for whom it is possible to achieve a performance better than that of the market itself, to « beat the market », thanks to the detection, by appropriate means, of underpriced securities, market momentums, or « trends » (the term « trend » contains however a statistical ambiguity: according to the scale of observation, the trend can be a rising one or a falling one. This conceptual ambiguity would not disappear for a certain time, but this did not stop the technical analysts from discussing it, without questioning the significance of the observations or the reliability of the predictions).

In a 1961 paper, devoted to the trend - random walk alternative, S. Alexander pointed out the importance of this opposition. He writes[45]:

> There is a remarkable contradiction between the concepts of behavior of speculative prices held by professional stock market analysts on the one hand and by academic statisticians and economists on the other. The professional analysts operate in the belief that there exist certain *trend generating facts* (our italics), knowable today, that will guide a speculator to profit if only he can read them correctly.

The partisans of the existence of trends belong as much to the professional community of financial analysts (called "fundamentalists") as to that of the technical analysts. The latter are called *chartists* because they use graphic data predicting the future course of securities and indexes. The cornerstone idea in this analysis consists of finding, in charts which represent past price changes, codified patterns which are supposedly indicative of future changes of which one can know the trend and magnitude. The technical analyses for the detection of market trends are sometimes based on a theory attributed to Dow[46].

These two categories of analysts, who differ in the methods to be applied to determine the trend of stock market movements, agree on the postulate of the existence of detectable market surges, of the predictability of future variations.

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[45] Alexander (1961), p. 7.

[46] And also, at times, to Elliot and Gann. Rather than « theories » in the scientific sense of the term, these are rather empirical constructions which use implicit metaphysical visions as justification for their approaches. For example, reference is often made, in technical analysis guidebooks, to a « structural harmony of the universe », to a holistic vision of nature in which certain figures, such as that of gold number, have certain properties, which can be exploited in market prediction. One also encounters conceptions according to which « that which is below is like that which is above », and thus that stock market fluctuations are to be analyzed as tangles of large waves, medium waves, and small waves, of which the art then consists of determining the position on the rises and the falls. In the conceptual foundations of technical analysis there are a certain number of principles borrowed from numerology and Pythagorean gnosis.

> The two main schools of professional analysts, the fundamentalists and the technicians, agree on basic assumption (predictability capacity). They differ only on the methods used to *gain knowledge before others* (our italics) in the market[47].

The principle of this ability to predict later development in the market rests on knowledge of specific information, which is not entirely reflected by the quoted prices.

> These facts are believed to generate trends rather than instantaneous jumps because most of those trading in speculative markets have *imperfect knowledge* (our italics) of these facts, and the future trend of prices will result from a gradual spread of awareness of these facts throughout the market. Those who gain mastery of *critical information* (our italics) earlier than others will, accordingly, have an opportunity to profit from that early knowledge[48].

The key question will then be: where does one find this information which can provide good predictability? The financial analysts (« fundamentalists ») will look for it in financial statements, complemented by macro-economic and econometric forecasts. The technical analysts (or chartists) will seek to capture the structures of movements, the invariances of fluctuations, only by examining the past time series of prices; assuming a sort of determination of market movements (« history repeats itself »), based on technical factors (for example, the volume of transactions, past moving averages, indicators of over- or under-reaction of the markets, etc.). For this community, chart patterns reflect certain pictures revealing psychology of the market. It is assumed that the human psychology tends not to change, and that these pictures will continue to work in the future as well as in the past. The economists think that it is possible to exhibit slow cycles and to predict the future with that. For some of them, supporters of the existence of determinist cycles, the analysis of these cycles should lead to a good predictability of later movements. The cycles are, in this sense, conceived in a mechanical way, and their study can be undertaken in a scientific manner with mathematical tools such as Fourier's transformation; it analyzed time series by making characteristic oscillations appear on a diagram of frequencies (or spectrum) to extricate a periodicity. Spectral analysis of stock market fluctuations should, according to this approach, enable the capture in frequency spectra of the information sought for predictability.

4.1.2. The performance measurement issues: predictability in a state of turmoil

The simplest rationale for active portfolio management is precisely the possibility of capturing the relevant information before the others, i.e. the possibility of detecting underpriced securities or global trends in the markets: stock picking ability or timing ability. The classical approach for this detection was given in the famous Williams' guidebook: « The Theory of Investment Value »[49], upon which generations of financial analysts were educated. Fundamentally, the idea is to evaluate the discounted cash flows of a given security, and to forecast the course of individual stock price, or to predict the course of the stock market as a whole. « Fundamental analysis » consists in forming a projection of future cash flow, and « fundamental prediction » consists in forming scenarii of future events. The financial information is the raw material used to perform this objective. The information process is the procedure for taking information (mainly public information) and processing it in a way to construct an appropriate portfolio. This involves several types of information, as for

[47] Alexander (1961), p. 7.
[48] *id*.
[49] Williams (1938).

example, accounting data, macroeconomic data, brokerage research, company management information, sector data, demographic information, forecast earnings or dividends[50].

Given this « superior information », or « superior judgement », it is theoretically possible for an asset manager to outperform an index, which is, namely, an uninformed benchmark. Yet, the problem with fundamental analysis and the ability to find « superior information », was that it appeared not to produce a better performance than the market performance over long time periods. In 1933, Cowles demonstrated that the recommendations of major brokerage firms failed to exhibit skills in forecasting, and that « the investment policies of major insurance companies, based on the accumulated knowledge of successive boards of directors whose judgement might be presumed (…) fail to exhibit definitely [sic] the existence of any skill in investment »[51]. This study was completed by a second sight in 1944, which affirmed the same conclusions: no ability to predict successfully the future course of the stock market, i.e. no ability to capture relevant information[52]. The implication of these results was that investors who paid to have funds managed by a professional manager were wasting their money. It was the beginning of the performance measurement issue: Cowles' pioneering work paved the way for further academic researches and explicitly launched the conflicting idea that the professional managers are unable to outperform the indexes, an idea which progressively clashed with most professional managers, before becoming a paradigm in the investment management industry[53].

4.2. The clashes due to the introduction of random walk model

4.2.1. The technical analysts : « we totally reject this idea » !

In this perspective, it clearly appears that Bachelier's hypothesis, in which models can be made of stock market variations based on stochastic processes of « random walk » type, is in complete contradiction to this research. It clashes frontally with the whole professional community. The first and most violent shock took place with the social group made up of the practitioners of technical analysis: as one of them said, « the idea that the markets are random is *totally rejected* (our italics) by the technical community »[54]. Indeed, « technical analysis is the study of market action, primarily through the use of charts, for the purpose of forecasting future price trends »[55]. Technical analysis raises a particular problem, in that in its cornerstone, there is the assertion that all relevant and useful information is actually reflected in the quoted price: « market action discounts everything ». For the technical analysts, this is the reason which makes it possible to extract information from the historical series of past prices which enables them to construct an efficient extrapolation for the prediction of future prices. But this hypothesis concerning the true reflection of relevant and useful information is also the origin of the formalization of stock market variations by means of random walk. In other words, based on the same postulate of the representativeness of information in the prices, the academics and the technical analysts put forward opposite conclusions on the ways to use this information. This is not the least of paradoxes in modern finance, and increases disagreement between academics and technical analysts.

In fact, it involves the economic question of the rationality of expectations, and the significance given to the market traders' expectations. In both cases, there is agreement in acknowledging that the market, as a price mechanism, provides information, but the difference

[50] For more details, see for example Rudd (1989).
[51] Cowles (1933).
[52] Cowles (1944).
[53] Walter (1999).
[54] Murphy (1986), p. 21.
[55] Murphy (1986) p. 1.

concerns the nature of the information and its interpretation. According to classic financial theory, the information contained in the prices concerns facts outside the market, i.e. economic or financial facts: Bachelier's « natural causes ». This past information has no use because it refers to a future equilibrium which is already anticipated. The only sources of market variability are then exogenous; this is the arrival of new information, which will in turn be integrated into the price. The price will then reflect a new future equilibrium. The economic agents are said to have rational expectations; this rationality enables them to correctly anticipate the price of future equilibrium. The academics' position is derived from this.

The chartist technical analysts do not hold this conception of market equilibrium. For them, even if the price reflects all the information, market fluctuations are not exclusively exogenous; there may also be endogenous (for example, psychological) factors, irrational phenomena of enthusiasm in the market, panic movements, or speculative crisis, in which the price fluctuations can take the market beyond all supposedly « fundamental » reference level. And these endogenous sources of important fluctuations may be found in the market's past; if one hypothesizes as they do, that the psychological attitudes of the traders tend not to change, one can find in coded and catalogued patterns of market fluctuations (charts) the traces of this behavior. In one sense, the same causes can produce the same effects, and incidentally, small causes can produce large effects. In other words, for the technical analysts, the information contained in the market's past not only refers to facts *outside* the market, but also to determinants of repetitive group behavior, i.e. *inside* the market. For them, the expectations are not rational, in the classical academic sense.

It is this differing comprehension of the nature of expectations, and thus of the causes of market fluctuations, which is one of the reasons for this confrontation between academics and technical analysts, and for the paradox mentioned above. In fact, for the technical analysts, the expectations would be rather more self-fulfilling than rational. Not until the 1990's did a possibility emerge for resolving this paradox, with the appearance in financial theory of non-rational expectation models in the classic economic sense, which display possibilities of herding behavior leading to the emergence of endogenous fluctuations. But the conflict is still not completely reduced today, as: these models have only recently appeared in the academic literature; the technical analysts are not familiar with the corpus of financial theory; mimetic models with endogenous fluctuations have not yet been integrated into the quantitative techniques of option pricing or portfolio management, which still rest on the hypotheses of rational expectations equilibrium.

4.2.2. The active portfolio managers : « don't tell me about indexation » !

The quarrel between the trends supporters and those of random walk in financial theory is thus as old as the theory itself. The vehemence of the arguments is proportional to the stakes involved. If Bachelier's hypothesis is right, it would be useless to seek to « beat the market », and to keep (and to pay) teams of analysts and active portfolio managers in asset management companies, as they are expensive in terms of personnel and costs: huge sums of money are spent every year on the detection of underpriced securities or arbitrage opportunities. The following excerpt of a recent paper illustrates this topic, emphasizing the stock-picking ability, i.e. selectivity information:

> Currently, over one trillion dollars are invested in actively managed equity mutual funds. Assuming that the fees and expenses of these funds average about one percent of assets – a conservative estimate that ignores the expenses that funds generate from buying and selling stocks – the total costs generated by this industry exceed US 10 billion per year. Although mutual funds provide a number of services, such as check-writing and bookkeeping

> services, more than half of the expenses of mutual funds arise because of their *stock-selection efforts* (our italics)[56].

With a Gausso-Markovian approach to the markets, the expected deviation between the performance of an actively managed portfolio and an index which is representative of the market can be modelized with a random variable with zero expectation, i.e. a « fair game ». This does not imply that this deviation will be constantly near zero, but that, over a long period, if an investor stays long enough with the market portfolio, the empirical performance which he will obtain will converge towards the expected performance of the market portfolio, due to the central limit theorem. Therefore, the best market strategy to follow would be a simple « buy and hold » strategy, as opposed to any active attempt to « beat the market », and this activity (security analysis and active management) becomes entirely unproductive. The value added by active portfolio management becomes questionable, and even third-party management itself. Any investor has to choose between two alternatives: invest directly and manage the securities himself, taking the related risks to achieve the expected return; or call upon an investment manager to manage the portfolio, that is to say, recognizing that somebody else can add some value. In its radical form, Bachelier's statement leads to the abandon of third party management because of the absence of value added by the investment managers.

The force of this affirmation, coming straight from the Bachelier-Osborne model and the MV-efficiency of market indexes and supported by the Cowles' 1933 and 1944 pioneering works, led asset managers in the US to undertake, with an academic consultancy, a very large number of performance tests in the 1960's on actively managed portfolios, compared to « market performances » performed by the stock market indexes. This intellectual movement was the basis for the double emergence of the stock market index industry (there was very little before this assertion), and that of performance measurement of managed portfolios, sold by independent consulting firms. The results of the early studies were disconcerting, confirming in first estimation the consequences of Bachelier's hypothesis and Cowles's works on the impossibility for portfolio managers to obtain a performance better than that of the market, over a long enough period[57]. In short, the conclusions of the performance studies were that stock picking doesn't pay, so investors should simply buy a broadly diversified portfolio.

Thus, at the beginning of the 1970's, one asset management firm, named Wells Fargo Bank, decided with pragmatism that, given these results, the best thing to do was simply to acquire the computers and data bases necessary for the construction of MV-efficient broadly diversified portfolios, and see what happened. In an internal memorandum to staff dated 19th November 1970, the head of the Wells Fargo Financial Analyst Department, Vertin, wrote[58]:

> While only time and experience will tell, it may well be that we are very much on the ground floor of a development which can mean big things (…) for Wells Fargo Bank.

In practical terms, Wells Fargo initiated the era of quantitative investing with the creation of the first indexed funds: the first in July 1971, (6 MUSD contribution from the Samsonite pension fund), indexed on the equally-weighted NYSE, and the second in 1973, named Wells Fargo S&P 500 Index Fund, with the objective of tracking the performance of the capitalization-weighted S&P 500 index. This creation of the Wells Fargo funds marked the birth of indexing techniques in the

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56 Daniel *et al.* (1997).
57 Treynor (1965), Sharpe (1966), Jensen (1968, 1969).
58 Jahnke, Skelton (1990).

investment management industry in the US, which today accounts for an estimated 23% of institutional equity mandates (in the UK, indexing only took off in the mid-1980's)[59].

Since then, indexing has become a popular technique in the US and the MV-efficiency of the market indexes represents the conceptual basis for the « index-linked management » of portfolios. Indexation means that a market index is duplicated by a technique which allows a portfolio to follow a market index, i.e. to track the index performance. Different categories of indexation exist: full replication, stratified sampling (or partial replication), and optimized sampling. The choice between indexing techniques depends on six main considerations: size of fund, type of index being tracked, turnover and transaction costs, performance objective of the fund, average holding period of investments by the fund, likely size of cashflows into and out of the fund[60]. « Passive index linked » is the extreme form of index linked management, and means managed in the absence of a manager or analyst, without using predictions of any sort (and certainly not technical analyses).

The debate for and against indexing is nourished by the performance studies. The main questions can be summarized as follows: can the average active manager beat the market? can an individual active manager beat the market through selectivity information? can an individual active manager beat the market through timing information? can outperformance persist?[61] Empirical evidences of the performance studies suggests that active managers often fail to exploit relevant information over long time periods. But, even today, one can read that « the problem of accurately measuring the performance of managed portfolios remains largely unsolved after more than 30 years of work by academics and practitioners »[62]. This is due to technical flaws in the traditional measures of performance, and the question of measuring the performance remains an open puzzle.

Thus, parallel to the professional practice of technical analysis, another profession practice emerged which applied the principles of financial theory, a practice called « quantitative asset management », as it used quantitative techniques for constructing and managing portfolios. But it is interesting to observe that, in the same financial institutions, one could find traders who used technical analysis and quantitative managers who dismissed it. This apparent paradox was due to a misconception of the efficiency concept, as we will see below.

4.3. The mid-term solution to bring peace

At the beginning of the 1960's, the confrontation between the supporters of one or the other hypothesis was such that the communities rapidly began to mutually ignore each other. Academics dismissed technical analysts, considering technical analysis to be a sort of neo-gnostic numerology, while the technical analysts regarded the academics as lost in their intellectual cathedrals with no relation to the real market. In a 1962 article, devoted to this debate, Paul Cootner does not hesitate to write[63]:

> The subject matter of this paper is bound to be considered a *heresy* (our italics). I can say that without equivocation, because whatever views anyone expresses on this subject are sure to conflict with someone else's deeply-held beliefs.

[59] PwC (1998), p. 2.
[60] PwC (1998), p. 4.
[61] PwC (1998), p. 8.
[62] Ferson, Schadt (1996).
[63] Cootner (1962).

To try to reconcile these two contradictory points of view, Cootner suggested a third approach in which imperfections existed in the market mechanism, impeding the straight reflection of information in the quoted prices and creating opportunities for arbitrage on the markets.

In fact, the issue is whether market movements which seem to exhibit the recognized patterns the technical analysts call « trends » or « heads and shoulders », or other similar terms, are compatible or not with the postulate of random walk. In other words this is, on the financial analysts' side, the possibility of reconciling a predictability of the future dividend, and thus future returns, with an unpredictability characterizing an increase in the price due to Gaussian random toss. On the asset management' side, it is the issue of capturing the relevant information to « beat the market ». To sum up, are these market behavior situations compatible with the hypothesis of random walk, in spite of the presence of apparent trends in the market?

As we have seen, economic thought was unable to address this question in the absence of theoretical developments related to the nature of expectations, informational asymmetries, and the generalization of standard rationality into models of mimetic behavior which could exhibit endogenous fluctuations in the market. A compromise was put forward by Fama, in an article published in 1965[64] In a general view of stock market behavior, most of the work carried out on this question is reviewed, with tests carried out by the author himself on securities composing the Dow-Jones index. Fama states that in general, on the data observed, the tests do not point significantly to interdependence. He concludes:

> Now in fact, we can probably never hope to find a time series that is characterized by *perfect* independence. Thus, strictly speaking, the random walk theory cannot be a completely accurate description of reality. For practical purposes, however, we may be willing to accept the independence assumption of the model as long as the dependence in the series of successive price changes is not above some "minimum acceptable" level.[65]

The question is then to define this « minimum acceptable level » of dependency, which does not contravene the hypotheses of the Bachelier-Osborne model.

> What constitutes a "minimum acceptable" level of dependence depends, of course, on the particular problem that one is trying to solve. For example, someone who is doing statistical work in the stock market may wish to decide whether dependence in the series of *successive* price changes is sufficient to account for some particular property of the distribution of price changes. If the actual dependence in the series is not sufficient to account for the property in question, the statistician may be justified in accepting the independence hypothesis as an adequate description of reality[66].

In short, in 1965 it was considered that a weak interdependence of price increments could exist, a sort of « short memory » of the market, but that this type of autocorrelation does not put the hypothesis of random walk into question. The real problems were to appear later, with the development of the econometrics of time series and probability tools coming out of fractal geometry. These factors, thanks to new and more powerful analyses of data, added to a larger number of available data, together with an increase in the computer capacities for treating high frequency data (stock market fluctuations analyzed minute by minute), led to exhibit new market

[64] Fama (1965).
[65] Fama (1965), p. 35.
[66] *Id.*

phenomena as « long memory », or « infinite memory ». But these phenomena were not observable in the 1960's as this technical apparatus was not available. Thus for this period the Bachelier-Osborne model is in this sense (short memory) a good approximation of the observed reality.

Finally, around the middle of the 1960's, it was henceforth acknowledged that even if the market did not exhibit all the technical aspects necessary for validating the Bachelier-Osborne model, this model could be considered, in first view, to be a good representation of the stock market fluctuations behavioral reality. As Fama stated:

> The independence assumption is an adequate description of reality as long as the actual degree of dependence in the series of price changes is not sufficient to allow the past history of the series to be used to predict the future in a way which makes expected profits *greater than they would be under a naïve buy-and-hold model* (our italics) [67].

The issue of predictability seemed closed, leaving behind two more or less opposing and unreconcilable concepts.

5. The ex post-emergence of the market efficiency concept

5.1. Economic rationale founding the random walk model

Following this accumulation of statistical results which tended to validate the hypothesis of random walk, it was then necessary to find the economic causes of these results. In other words, to pass from an analytical-descriptive view of the stock markets to an economic explanation of their observed behavior. This was achieved with the introduction of the informational market efficiency concept, which establishes a connection between the *allocational role* of capital markets, and the *informational role* of prices.

It is widely accepted that the first statement of the informational efficiency concept, in its modern form, is given by E. Fama, in his 1970 paper[68]. The initial idea appears in some previous articles, but there was no precise definition until Fama's contribution. Fama himself mentions some authors in which earlier statements on the efficiency concept could be found. This first definition was refined by Fama in 1976, and a sequel of this seminal article was also written by Fama in 1991, but these two additions do not significantly alter the basic statement of 1970. Fama's initial approach was as follows :

Basic principle:

> The primary role of the capital market is allocation of ownership of the economy's capital stock[69].

It emphasizes the "quality" of the market, considered as a specific tool, or mechanism, for allocating resources. The efficiency of the tool is assessed by its ability to allocate rare resources. Note that this definition does not include any risk dimension. A more modern form, including risk allocation, is given by Grossman:

> How does a decentralized economy allocate risk and investment resources when information is dispersed?[70]

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[67] *Id*.
[68] Fama (1970).
[69] Fama (1970), p 383.
[70] Grossman (1995).

This question of the allocative efficiency of markets can be found in an article of Working:

> Have the markets served well, keeping prices usually close to the level warranted by existing *economic information*, and recording price changes chiefly in response to *changes in pertinent economic information*? Or have the speculative markets generated a large amount of economically unwarranted and undesirable fluctuation in prices? (our italics)[71]

It leads to the question of the *informational role of prices*. This subject is tackled by Hayek in the well-known passage of his 1945 article on "The use of knowledge in society":

> We must look at the price system as such a mechanism for communicating information if we want to understand its real function. [...] In abbreviated form, by a kind of symbol, only the most essential information is passed on, and passed on only to those concerned. It is more than a metaphor to describe the price system as a kind of machinery for registering change [...].[72]

In other words, it is not possible to think about market efficiency without considering the nature of information conveyed by prices.

We have seen that information conveyed in the price is a notion which is implicitly contained in the Bachelier-Osborne model of random movement. The first attempts at economic justification, *a posteriori* to what appeared to be a validated experimental statistical result, used the concept of random arrival of information onto the market, which immediately transmitted this new bit of information into the quoted price. This idea was to be taken up and developed. For example, in a 1962 article by Cootner:

> The stock exchange is a well organized, highly-competitive market. Let us suppose, for practical purposes, that it is a *perfect market* (then) "the only price changes that would occur are those which result from new information. Since there is no reason to expect that information to be non-random in its appearance, the period-to-period price changes of a stock should be random movements, statistically independent from one another[73].

The essentials are set forth in this statement, which was to be taken up and more strictly formalized. In a more radical form, this argument is stated as follows:

> If successive bits of new information arise independently across time, (…) then successive price changes in common stock will be independent[74].

The smooth transit of information was to be associated with the concept of the « perfect market » of the 19th century, a concept which was to be updated with the introduction of the economics of information. Thus the efficiency of a market will reside in its capacity for smooth transmission of information through prices, in other words, informational efficiency implies that the market will be efficient as a mechanism for conveying information.

The efficient market theory addresses these questions within a neoclassical economic case. Fama begins his 1970 paper by recalling that

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71 Working (1956), p.1436.
72 Hayek (1945), pp. 526-527.
73 P. A. Cootner, art. cited, p. 25.
74 Fama (1965), p. 37.

> The ideal is a market in which prices provide accurate signals for resource allocation: that is, a market in which firms can make production-investment decisions, and investors can choose among the securities that represent ownership of firm's activities[75].

He adds after the following definition, generally considered to be the basic statement of informational market efficiency,

> … under the assumption that security prices at any time «fully reflect» all available information. A market in which prices always «fully reflect» available information is called «efficient». [76]

Although the preceding definitions must be complemented by what is understood through « available » and « relevant » information, Fama's formulation completely defines the efficiency of a market.

It fell to Samuelson and Mandelbrot to completely formalize this hypothesis, by giving it rigorous probability foundations and a defined economic justification. In their articles of 1965 and 1966[77], using martingales models which are generalizations on random walk, they clearly showed that if the information arrives in an unpredictable manner on the market, then it is possible to make models of the variations of the discounted prices of financial assets (the financiers' "present value") as successive results of independent random tosses; the discounted prices of the assets follow a martingale process[78].

This is an important stage in the conceptual evolution of financial theory. The martingale models are introduced into finance, and market efficiency becomes closely associated with the martingale models[79]. Henceforth, an important component in creating models of stock market variations will be the stochastic component contained in the equations of the models, their probability core. The equations used for representing the fluctuations of returns are stochastic differential equations, following the example widely used in physics.

5.2. Building the efficient market framework for the industry uses

5.2.1. Aim: to make the concept testable

It is particularly interesting to note that this formal development of an explanatory economics theory did not occur until after the statistical and experimental evidence was produced: the impetus for the development of a financial theory came from the accumulation of empirical evidence observed before. The role of these results in the formulations which were to be constructed may thus be noticed. For, if new statistical results contradicted the previous ones, it would then be necessary to reconsider some of the foundations of the theory itself. In other terms, this means that because of its intellectual origin, the theory of market efficiency cannot be separated from its probability base. With this probability underpinning, the theory acquires a capacity for experimental verification and thus for refutation (which renders it "scientific" in Popper's sense).

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[75] Fama (1970), p 383.
[76] Fama (1970), p 383.
[77] Samuelson (1965), Mandelbrot (1966).
[78] If the expectations of the investors are rational, then the expected future price of the asset is equal to the mathematical expectation of the future price, conditionally to information passed to the present period. In writing the present price as the expected discounted future price, one obtains a martingale on the discounted future price.
[79] See for example Le Roy (1989).

The postulate of informational representativeness of quoted prices is not, as such, experimentally falsifiable. To know whether the quoted price is a true representation of the theoretical equilibrium price, which the traders are supposed to anticipate rationally, and thus whether the market is in fact informationally efficient, it is necessary to attach to this postulate a complementary piece of information concerning the way in which this representativeness is conceived. It must be defined and measured. Definition must involve the nature of the information, that of the measurement, and of the appropriate instrument. To test this hypothesis, the "conditional mathematical expectation" operator is introduced between the return of a real portfolio and the return of the market itself, defined as being at equilibrium. The "mathematical expectation" operator allows measurement, while the "conditional" aspect refers to the object measured, and allows testing of the efficiency hypothesis; the conditionality means that the mathematical expectation of the profitability gap between the portfolio and the market is conceived conditionally in reference to a set of past information.

5.2.2. Specification of the information set

It follows that the next intellectual stage is naturally the explanation of the content of the information set. In Fama's statement, the information which should be reflected in the price is presented as being « available » and « relevant ». How can this availability and this relevance be characterized? Three types, or levels, of information, corresponding to three forms of informational efficiency: weak, semi-strong, strong, are stated: « The categorization of the tests into weak, semi-strong, and strong form, will serve the useful purpose of allowing us to pinpoint the level of information at which the hypothesis breaks down »[80]. The information set corresponding to the weak form of efficiency is composed by all past quoted prices of the market and only these. The weak form of efficiency rules out the use of technical analysis, which appears as non-efficient for obtaining profits higher than those of the market itself. Here again we encounter the source of conflict with the technical analysts. The information set corresponding to the semi-strong form of efficiency consists of the preceding set of past prices, augmented by financial data of the firms. The semi-strong form of efficiency rules out classic financial analysis, to obtain profits higher than those of the market. This is the source of the conflict with the financial analysts and the economics research departments of banks. Finally, the strong form of efficiency, which includes the two preceding sets of information, is concerned with the existence of private information, i.e. not necessarily public, for example the forecasts to which the professional pension funds managers have access, unavailable to the general public. With a very strong form of efficient market, no professional managers, even provided with a high level of skill, can obtain profits higher than those of the market on a long-term basis. This is the source of conflict with the active managers of portfolios, who spend a large part of their time looking for and choosing stocks which they think will be profitable, and conjecturing as to the future development of the market. This split of the information set into three distinct categories, leads to definition of three fields of investigation into the concept of efficiency, and then to testing the efficiency. In the three cases, the gap between the return on the portfolio and that of the market must be a random variable of zero expectation. The concept of efficiency thus becomes defined by the nature of the chosen information as « available and relevant ».

On the other hand, the introduction of the "mathematical expectation" operator gives an key role to the concept of "average value", as the refutability of the efficiency will depend on the response given to the existence or absence of an "average profit", or average gap between the portfolio and the market. It is known that the simple use of the average implies the existence of the

[80] Fama (1970), p. 388.

variance. The existence of the variance allows the application of a law of large numbers, which ensures the convergence of real returns on the portfolio towards the theoretical return of the market, a convergence which will then guarantee the efficiency. There must be a compensation between the gains (over-performance compared to the market) and the losses (counter-performance compared to the market), a compensation which leads to a reversion of the portfolio returns towards that of the market.

5.2.3. Underlying probabilistic assumptions

But all depends, in this verification, on the convergence speed of successive empirical averages (the successive returns of the portfolio) towards the mathematical expectation, the so-called *rate of convergence* issue in the law of large numbers (the limit behavior of sums of iid random variables). With a slow convergence, real gains can be observed over a period of average duration (about a year), and efficiency rejected if a shorter duration of the compensation is presupposed. A condition with the convergence rate is thus necessary to define a duration of return to market performance, to render efficiency "falsifiable" in the Popper's sense, that is, « scientific ». There is here a second step of reduction of the efficiency concept, which to be scientific, pays the price of losing its general nature. This is because the theorems of convergence which ensure « good » statistical properties for estimators of the average are also very restrictive on the form of probability laws of random variables. In other words, the efficiency is restricted a second time by the specific restraining characteristics of probability laws. Efficiency is, and will always be, an *agented concept*; one cannot speak of efficiency by itself, except through a model of price functioning, an equilibrium model that defines « normal returns », with a representative probability system of stock exchange fluctuations. If efficiency is rejected, this could be due to a strong distributional assumption, as well as the inefficiency of market.

Thus, the restriction of efficiency corresponds to an increase in the accurate description of the statistical framework used for describing the market environmental conditions. This probability projection of efficiency appears in ordinary terminology, which expresses, behind the concept, the framework in which it will be thought about. With a classic Bachelier-Osborne type of approach, or more generally the two-parameters model for financial industry, which uses mathematical expectation and variance as essential parameters of efficiency, one speaks of efficiency in mean-variance (MV-efficiency), for this expresses a property of efficiency, perceived through a Gaussian probability prism. MV-variance, or efficiency from a Bachelier-Osborne model, is in the end a Gaussian efficiency[81].

This projection of efficiency according to a favored way of observation results in an incapacity to test the efficiency itself in isolation; the efficiency tests are always joint tests, on the model itself and on the law of probability. This is *joint hypothesis problem*, meaning that *market efficiency as a theoretical concept can never be rejected*. Empirical tests of market efficiency are in fact tests which *include stochastic process specification*. Hence, all the stakes contained in the statistical tests of analytical characterization of the properties of markets behavior come into play: in the sense that these tests are tests of the stochastic process used for the stochastic component of the models. And the formalizations of randomness which will be adopted for the stochastic component will lead simultaneously to fix a probability sense of informational efficiency. *This overlapping between efficiency and probability law is often the cause of misinterpretation and errors, leading to*

[81] A precise analysis, which more accurately describes the articulation between efficiency, Gaussian efficiency, and marginal probability law, is proposed in Lévy Véhel and Walter (2002).

rejection of efficiency whereas there is only a misspecification of the stochastic process[82] But efficiency is not identically associated with only one type of stochastic process. On the contrary, once the postulate of randomness is accepted, the form of randomness is open to choice; this form is the subject of the second great controversy which displays in our story of efficiency.

6. The Gaussian form of efficiency and its professional impact

6.1. The Bachelier's initial choice of Gaussian distribution and its consequences in the financial industry

6.1.1. Mild randomness for a smooth universe

With regard to stock market fluctuations, Bachelier wrote that « the determination of these fluctuations depends on an *infinite number of factors* ». This is the question of the sum of little random causes, which leads, for Bachelier, to the normal law being a limit probability law. Using a contemporary framework with models of information and assuming that the only source for market variability is exogenous and comes from random shocks of unpredictable information, Bachelier's infinite number of factors are precisely these shocks of exogenous information which prices must appropriately reflect. As the magnitude of price variations result only from exogenous shocks of information, this amplitude is calibrated by the chosen law of probability. Therefore, the description of the stochastic component by a probability law is one of the key aspects of efficiency: besides the fact that it expresses a particular vision of the nature of the reality (the shocks), it is the density of this law that produces and measures the variability (and thus the risk) of the financial assets. The Gaussian choice means that small risks are not too scattered[83]**.** This condition provides a strong hypothesis for the nature of information shocks. Gauss' law is a probability law for which there exist average homogeneous variations, although none contribute more than the others to the final result. This amounts to considering that the random shocks of information are homogeneous and not organized into a hierarchy, which means that no element is more significant than any other, no information has more effect, on average, than any other. There is no remarkable event, only average events.

Following this view, stock market variations can be completely characterized by the two parameters (the two moments) of normal distribution: the first moment, or « mathematical expectation » (which provides the expected return of the asset) and the second moment, or « variance » (which expresses the risk of obtaining a different return). In practice, practitioners use the standard deviation called, in market jargon, « volatility »: when one speaks of the "volatility" of a market, it always refers to the standard deviation from a normal distribution. In using a quantity such as volatility, which relates to the normal law, one supposes that this observable quantity (the calculated variance) corresponds to a theoretical real value, which is not observable (the theoretical variance), but which can be estimated by using relevant statistical methodology. The assumption is

[82] One then speaks of the « non-efficiency » of the markets, bringing up a certain number of anomalies (speculation bubbles, etc.), without seeing that these anomalies are anomalies of measurement in relation to a initially Gaussian gauge. The question of the acceptance or non-acceptance of market efficiency is more complex than that of the experimental setting up of measurement anomalies.

[83] The stochastic components must be square-integrable.

thus explicitly made that this theoretical value, called marginal variance, exists [84]. The probability theory teaches us that, in this case, with a normal law, the distribution of successive empirical averages of a series of repeated events (stationary process) is Gaussian**.** Bachelier, followed by Cootner, applied the central limit theorem and used the asymptotic normality of cumulative sums of iid random variables:

> If the series were a single random walk, the distribution of price changes over successively longer intervals should become more and more normal as the central limit theorem becomes more and more applicable[85].

The formalizations of randomness are carried out with Gaussian quadratic models, called « square-integrable martingales » (existence of marginal variances). Square-integrable martingales today represent the dominant model in the stochastic modeling of financial instruments. These techniques are relatively sophisticated but very much used in market practices, for example in the modern approaches to the option pricing problem[86].

Use of the Laplace-Gauss distribution seriously limits the number of large variations in the markets, in that the magnitude of these variations and their instantaneous variability is calibrated and restrained by the variance of Gaussian density. Gauss' probability universe is a smooth universe, without fractures or jumps, in which « randomness », the occurrence of unexpected events, is domesticated, is « mild ». According to Mandelbrot[87], the prototype of mild randomness is provided by thermal noise, which consists in small fluctuations around an equilibrium value. In this probability universe rare events, such as a stock market crash, do not exist. Their magnitude would be too strong in comparison with the other variations, in relation to the values which could be taken with a Gaussian density. Accordingly, with the choice of this probability law, it is the instantaneous variability of the markets which becomes constrained.

Therefore, because of the probability distribution choices, the random walk model restricted the stock market variability potentialities from the outset. And the informational efficiency concept is thus associated, at its birth, with Gaussian distribution. The problem of this Gaussian restriction of price variation is that, unlike statisticians, investors cannot deal abstractly with large fluctuations, which can mean large losses (or large profits).

6.1.2. Misinterpretations of statistical tests

We have seen how the probability core of hypothesis was enshrined in the efficiency concept. Fama, in his review article devoted to market efficiency[88] had noticed this double probability aspect of efficiency: the first restriction has to do with the introduction of the "mathematical expectation" operator as a means of making the concept empirically testable; the second limitation has to do with

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[84] The fluctuations in the estimates of volatility, which can vary with several orders of size (for example from 5% to 50% on the same market according to the periods of estimation), have raised a lot of technical issues for both academics and practitioners. It would have to wait for the formalizations of the 1980's, which introduced specifications on the residuals terms, with ARCH parameterizations (Auto-Regressive Conditionally Heteroscedastic) to make the confusion disappear which existed among practitioners between marginal-historical variance and conditional-instantaneous variance. Even if the theoretical values which are not time-dependant (historical or marginal values), are for example infinite, the real values observed at a given moment (instantaneous or conditional values), can exist. The ARCH parameterizations have enabled evaluation of the bias introduced into financial models when having replaced a marginal variable by its conditional equivalent, and renovated the arguments related to the fluctuations of Gaussian volatility.

[85] Cootner (1962).

[86] For example, Harrrison, Kreps (1979). The Harrison Kreps formalizations, followed by those of Harrison-Pliska, mark the beginning of the reconstruction of option pricing theory in an quite new probability perspective.

[87] Mandelbrot (1997), p. 127.

[88] Fama (1970).

the restriction of efficiency with the second moment, the variance, which conditioned the deviation between portfolio return and market return. He stressed the fact that efficiency as a concept is not related in an identical way to a particular law of probability. This association between efficiency and probability law only corresponds to the necessity of making the concept refutable, of transforming it into a scientific statement. But the choice of the law is left to individual initiative.

Fama had distinctly separated the definition of efficiency from the choice of the probability law, and in particular from its Gaussian restriction. This Gaussian restriction was however useful for defining a theoretical deviation of the real portfolio return from the market return. But he showed that, as the property of efficiency is not related to a particular law of probability, it would be formally possible to define other forms of informational efficiency, not reduced by quadratic Gaussian models. This would lead to reinterpretation of the results of the efficiency tests in that the lengthening of the time periods for return could lead to an acceptance of efficiency whereas one of Gauss' laws, concerning a short time period of return, signified its rejection.

6.1.3. Implications for the investment management industry

This theoretical question had important practical consequences: if the difference between the performance of an actively managed portfolio and performance of the market index is a random variable with zero expectation, but without variance, then situations could appear with significant over-performance of the portfolio in relation to the market index, through large rising or falling movements, big variations whose magnitude is incompatible with the existence of a constant finite marginal variance. The absence of variance, or an infinite variance, would not impede a revert of the portfolio return towards the market return, and so would not infringe upon the market efficiency over a long period, but it is the very concept of « long period » which would change, as the convergence would no longer occur at sufficient speed. In this case, an active management of the portfolio could again be theoretically justified.

Conceptual alternatives thus had important consequences on portfolio management styles, and on the very organization of management companies and investment process philosophy. Active management, as a way of managing portfolios, as opposed to index-linked management, can be split into two types of :approach: those so-called « top down » investment processes which give more importance to the strategic asset allocations (stocks, bonds, currencies, etc.) and focus on particular sectors expected to outperform the market. The other type of approach, processes called « bottom up », concentrate on tactical changes and stock picking. When it is assumed that the initial structure of the portfolio, called « strategic allocation », acquires a key role and that the ulterior changes of its composition, called « tactical asset allocations », will not have a durable impact on the final performance, the underlying assumption implicitly conveyed, but unfortunately not well discerned, is that there is a sort of stationary increments stochastic process with finite variance governing the deviation between portfolio return and benchmark return, i.e. without time horizon considerations regarding the investor's risk profile. That is because the normal standard deviation of the gap between portfolio return and benchmark return allows any investor to hope for a prompt revertion to the benchmark return (cumulative sums of independently and identically distributed random variables). As one can find in a practical guidebook for professional managers, « patience reduces risk: the obvious [sic] conclusion is that the longer the time period, the less risky stocks and bonds become »[89]. To sum up, this assumption is strongly dependant on a specific law of large numbers. In practice, the processes called « bottom up » are not very much used by investment managers, precisely because of this law of large numbers. However, with the non-Gaussian form of efficiency, non-negligible sources of performance could exist, resulting in tactical changes in the composition

[89] Hammer (1991), p .48.

of the strategic portfolio. The organizational weight given to strategic and tactical stages of the composition of portfolios, through investment committees, was then directly dependent upon an intellectual engrained conception of market efficiency. *The top down approach of investment process is closely related to the central limit theorem and the Gaussian law*, using the Gaussian form of efficiency. Gaussian efficiency is the hallmark of the top down approach to the investment process.

But, in cases where the price change process is abnormal, and to paraphrase the citation, patience doesn't reduce risk: what does the central benchmark contribute to the final performance of a managed portfolio? In my 1999 paper, I suggest that in the context of non Gaussian efficiency, stock selection and tactics lead to considerable added value, possibly more than strategic asset allocation[90]. This issue is addressed differently, but leads to the same conclusion, by some authors who analyse the information-diffusion process, and develop models of asymmetrical information. In the two cases, the limiting form of the efficiency led to a limiting form of the investment processes. But, as a professional noticed, « indexed products are difficult to differentiate; at the end of the day, they all provide the same things. There is little differentiation other than, perhaps, the degree of skill with people service and market it »[91]. Hard conclusion, coming from the unchecked use of the Gaussian law.

It is a fact that, among the investment managers, these two types of investment processes coexist. The coexistence of these two approaches illustrates the absence of consensus on the market efficiency vision, and the instinctive reticence of some investment managers with regard to the Gaussian hypothesis.

6.2. The disputes over normality and the Mandelbrot model

Normality tests are at the core of this question. As attractive as the normality or Gaussian random walk model is, it is unfortunately not consistent with the properties of historical returns. As noticed in a recent article, « empirical investigation almost invariably finds that actual returns are too fat-tailed to be lognormal »[92]. Actually, the observation of a large number of so-called « anomalies » in testing the Gaussian form of market efficiency caused a huge debate on the choice of marginal probability distribution governing price changes, a debate which was almost as emotional as that which opposed the supporters of trends to those of random walk. This was the issue of the nonnormality of empirical distributions of returns, also called « fat tails problem », and of the possible choice of nonnormal probability distributions to define the behavior of stock market price changes.

From the beginning of market studies, Cootner, followed by others, had noted that the empirical distribution of returns which were actually observed on the markets deviated from the normal distribution. This departure from normality was produced by the presence of too many large price changes and a too-small number of average changes in comparison to the theoretical number predicted by Gaussian density: the empirical distributions contain too many probability masses in the tail area than the normal. This effect of ill-fitting of the empirical shapes to the theoretical shapes, which exhibits bell shaped distribution more peaked than the Gaussian shape, with thicker and longer tails, was called the leptokurtic phenomenon (from the Greek word « kurtosis », curvature). The too-large leptokurtosis characterizing the empirical distributions of stock market returns represents a serious obstacle to Gaussian adjustments. Does this non-normality signify

[90] Walter (1999).
[91] PwC (1999b), p. 42.
[92] Green, Figlewski (1999), p. 1465.

rejection of random walk? For Cootner, the leptokurtic effect seen in stock market fluctuations implied the rejection of random walk, but it was a reduced form random walk, i.e. a Gaussian random walk. However for Fama it was possible to consider an efficiency without Gaussian restriction. It is clear that the question is related to the choice of the probability law.

Then in 1962 Benoît Mandelbrot proposed replacing Gauss'distribution with Paul Lévy's stable distributions with infinite marginal variance[93]. This involved abandoning an important hypothesis of financial models: the square-integrability of the ups and downs stochastic components causing stock market fluctuations. This meant that the shocks were not homogeneous or of the same nature, but, on the contrary, heterogeneous and strongly hierarchical. Some bits of information could have very important effects on market fluctuations, while others could have none. The importance of the effects could furthermore be disassociated from that of the causes: little events could be the cause of large variations. This hierarchical quality of the shocks corresponded to a hierarchy of singularities (« rare events » or « tail events »): when violent jumps occurred in quoting, the market performances were not distributed in a uniform and homogeneous way over the entirety of stock market days, but rather were found concentrated in a very small number of days[94].

This highly concentrated market performance underscores a strongly structured organization of price changes, themselves highly hierarchical, in which the extent of the hierarchical quality is measured by a determining quantity for the form of market fluctuations: the characteristic exponent of Lévy laws, or tail index (or index of the irregularity of stock market moves). This index quantified the *non-smooth* character, fractured, called fractal, of the stock market charts, and the outcoming performance concentration and therefore risk. Mandelbrot had revived Pareto's law, well known in economics (« very few people have many goods and many people have very few goods ») by discovering a relationship between a Paretian static and a market dynamic. He showed that these concentration phenomena are observed whatever the frequency of market analysis: daily, weekly, monthly, etc. These concentrations weakened the relevance of the average calculations, by making them less significant for market behavior analysis. Using Lévy's stable laws, the goal was to obtain a universality prospect able to describe the market fluctuations with a multiscale resolution approach, meaning without any characteristic time scale: a statistical invariance which was expressed exactly by the new fractal geometry.

Four years later, Mandelbrot wrote,

> The stock market fluctuations are such that there is no hope of applying the simplest, and therefore the most tempting model to them, the model of fluctuation ruled by randomness, that is Brownian motion, which postulates that successive price changes are independent Gaussian random variables. As a stock market prediction which does not involve probability is generally known to be impossible, it is *necessary to modify Brownian motion*[95] (our italics).

In other words, for Mandelbrot, if one wanted to save the random walk hypothesis, it was necessary to abandon Gaussian randomness, which did not stand for a good candidate in respect to probability distribution. So he introduced the concept of « Pareto's randomness », and of « strongly erratic fluctuations », renamed « wild randomness » in his 1997 book[96], new concept for

[93] Mandelbrot (1962).
[94] For example, the yearly performance of the CAC 40 index of the French stock market over 8 years is 9.41%. If the ten best days are removed from the some 2000 stock market days, this performance drops to 3.38%.
[95] Mandelbrot (1966).
[96] Mandelbrot (1997), pp. 125-130.

randomness which he opposed to « Gaussian randomness » equivalent to « benign or Laplacian fluctuations », renamed « mild randomness » in 1997; this was the beginning of the controversy between Gaussian randomness and Paretian randomness[97].

The disputes were fierce, exacerbated by issues raised by developments in portfolio and option theories. Although popular in the 1960's and early 1970's, the stable laws suffer from the absence of a closed form expression of their density, except for three members of the family[98]. One reason is that, in 1963, the statistical techniques and the probability tools existing in Lévy's world had become insufficient, unable to efficiently address financial problems[99]. Hence, they fell out of favor, partly due to the difficulty of creating models, partly due to the impossibility of addressing the portfolio problem and option pricing issue in a financial framework based on the existence of a finite second moment of returns. In his 1965 paper, addressing the portfolio problem in a multistable universe, Fama concluded: « it is our hope that papers like this will arouse the interest of statisticians in exploring more fully the properties of these distributions »[100].

Actually, early studies of stock market returns attempted to capture the fat-tail phenomenon by adequately modeling the empirical distribution of returns, with a Lévy-stable distribution[101]. But, without theoretical tools appropriate to financial problems, Mandelbrot's propositions were not followed; instead of stable laws, financial research has tended to capture the fat tailed empirical distributions by using distributions with finite higher moments (for example, the Student distribution), mixtures of distributions, or conditional approaches of the returns. But these attempts achieved parsimoniously their goal, and therefore, the leptokurtic quality of empirical distributions of stock market price changes was not really taken into account in the probability models used in the concept of informational efficiency.

It is with the provisional closure of this controversy, which concluded with a *status quo* in favor of Gaussian quadratic models, and thus of Gaussian-efficiency, that the genesis period of the first informational efficiency ends.

7. The concept of efficiency crisis since the 1987 stock market crash

7.1. Going back to basics: rethinking Bachelier's statement and related efficiency

7.1.1. Towards generalizations of Bachelier's model

This first form of the efficiency concept was to last twenty-five years. These twenty-five years saw dramatic development in the financial markets, followed by the creation of more and more sophisticated instruments using all the available probability techniques derived from the field of diffusion processes in physics. During this period, the transaction volumes on the various market segments, cash futures, and forwards, increased exponentially, as did the assets under management. Until the stock market crash of October 1987, consolidated by developments in the investment

[97] Conceptually, there exist very strong relationships between a power law of Paretian type, very erratic fluctuations, absence of characteristic scale (scale invariance), hierarchical organization of catastrophes, non-convergence of empirical averages and generalization of the central limit theorem. All these phenomena deal with search for universality classes by using tools from fractal geometry, which unifies comprehension of the different properties of very different physical entities (avalanches, earthquakes, speed turbulence of atmospheric wind, etc.). One can find an introduction in, for example, Mandelbrot (1977).
[98] These three special cases are : the normal, the Cauchy, and inverse Chi-Square.
[99] In particular, the portfolio problem required the replacement of the variance-covariance matrix by an equivalent in Lévy's distributions. This equivalent would not be invented until 1992, and the corresponding estimators, in 1993.
[100] Fama (1965b).
[101] A review can be found in Walter (2002).

management and derivatives industry, the concept of Gaussian efficiency of the markets was not questioned. In spite of a larger and larger number of measurement anomalies, which put into question the probability premises of efficiency, Bachelier's intellectual "coup de force" held for almost ninety years. The professional practices in use on the markets conscientiously and simultaneously applied a financial theory whose fundamental hypotheses rendered useless any attempt at prediction, and a set of empirical rules which were meant to detect trends in price movements. That the separation of the populations concerned partly explains this contradiction does not resolve the internal problem of the unification of financial theory itself, thrown into this paradox by Bachelier's hypothesis of random walk.

It was the stock market crash of 1987, followed by the quakes of 1989, which precipitated a vast intellectual movement to reconsider the fundamental hypotheses at work in market efficiency. The title of an article which appeared in the economic press in 1988 illustrates this awareness: « The Efficient Market was a Good Idea — and then Came the Crash »[102]**.** The subtitle was eloquent: « It (the theory) launched a revolution, but the theory can't explain why investors panicked on October 19 ». In practice, numerous bankruptcies resulting from risk management and hedging techniques based on a Gaussian conception of stock market fluctuations caused a new surge in scientific research into finance, in order to better understand the nature of randomness at work in market fluctuations. This movement was reinforced by a growing preoccupation on the part of supervising authorities and international groups[103]**,** which wished, after these accidents, to establish prudential rules of operation, by imposing minimum levels of solvency ratios on the financial institutions active on these markets, the so-called « value-at-risk » approach. As these ratios calculate the capital equivalent corresponding to confidence intervals on the probability densities of market returns, it became important to better quantify these risks[104].

All these considerations lead to a questioning of the validity of the efficiency concept. It was henceforth acknowledged and now well recognized that the financial markets did not possess the basic statistical behavior assumed by Gaussian density. For example, among many others, this emblematical excerpt of the recent article quoted above [105]:

> Many subsequent derivatives models have generalized the returns process but continue to assume that the stochastic component remains locally Gaussian. However, empirical investigation almost invariably finds that actual returns are too fat-tailed to be lognormal. There are more realizations in the extreme tails (and the extreme values themselves are more extreme) than a lognormal distribution allows for. In other words, the standard valuation models are based on *assumptions about the returns process that are not empirically supported for actual financial markets* (our italics).

Also, if any efficiency existed, it is not Gaussian efficiency. It is still too early today to determine by which route financial theory will rethink the efficiency concept because several competitive currents of thought coexist. However, one can conjecture that this paradigm crisis will probably lead to better unify the theory, in the sense of a reconciliation between probability models

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[102]« The Efficient Market was a Good Idea--and then Came the Crash », *Business Week,* 22 February 1988, pp. 38-39;

[103] For example the « Basle Committee », which brought together governors of central banks in ten countries. In April 1993, this committee decided to focus on surveillance of market risks. The Basle documents acknowledge the non-normality of stock market fluctuations. This comes down to rejecting Gaussian form of efficiency.

[104] For example, the European directive 93/6CEE, or Cad directive (Capital Adequacy Directive), which, taken over by the Commission Bancaire in France, requires each bank to calculate a level of capital corresponding to a viability of the situation of the bank to a probability threshold of 99%, calculated on the position of the markets. One perceives in this recommendation both the choice of the threshold level and the importance given to the law of probability.

[105] Green, Figlewski (1999), p. 1466.

and professional practices of technical analysts. Therefore, the investment management industry will be able to quantify risk even with the bottom up form of the investment processes, as we conclude below.

Whatever the approaches chosen, all present research is attempting to take into account non-Gaussian fluctuations and phenomena of long memory of movements. And parallel to this minimal descriptive approach, the researches on the behavior of agents (« behavioral finance ») are seeking to account for these phenomena, by extending the classic rationality to include the self-fulfilling expectations of agents. More generally, the literature on cognitive psychology provides a promising avenue to explain the so-called « anomalies » of the Gaussian efficiency. This means leaving the domain of Gaussian efficiency with its rational expectations and exogenously-caused fluctuations behind, to enter a world of endogenously-caused fluctuations, corresponding to behavior whose group dimension appears through models of herding. All this is part of a new vision of the markets, in which non-linear phenomena (crises, crashes, abrupt reversal of trends, etc.) are henceforth thought of not as exceptions, as accidents in relation to the so-called « normal » behavior of the market, but rather as elements which are themselves components of the markets.

From the moment that financial theory includes theoretical herding behavior and making it a potential source of non-Gaussian fluctuations, it acknowledges the hypotheses on which the technical analysts have been working for more than a century. It is not an accident if Mandelbrot's fractal geometry corresponds exactly to Elliott's intuition on the entanglement of stock market waves, and if the new sophisticated techniques of wavelets are now partly used to detect trends in the markets: fractal geometry allows technical analysis to pass from Elliott's waves to wavelets, i.e. from alchemy to chemistry. Unlike chartist analysis, for which fractals remain a qualitative intuition which has not been scientifically implemented, the contributions of recent models are leading to quantitative validation of this intuition, by means of an adapted scientific treatment: it is in this sense that these new methods cause technical analysis to pass, as it were, from alchemy to chemistry. Ninety years after Bachelier, most powerful probability tools[106] will lead perhaps to a reconciliation between technical analysts and academics. More generally, the relaxation of the strong restrictions on return distributions required for the Gaussian random walk model could reconcile the practitioners with the academics. This reconciliation will probably lead to the creation of new negotiable financial instruments, as the history of modern financial markets has shown that each risk quantification is always accompanied by the purchase and the sale of units of this risk. If this happens, then once again an innovation in the markets will have originated in a development of probability theory.

7.1.2. A proposed new hypothesis : the shadow of Quételet behind the Bachelier's choice

Let us end by placing the crisis of the Gaussian efficiency concept in a larger perspective. One suggests the hypothesis that Bachelier's initial choice, setting aside the technical restraints, resulted also from an intellectual preconditioning in regard to normality, coming from the « average man » concept put forward by Quételet. The shadow of Quételet would thus appear behind the silhouette of Bachelier, and a conceptual relationship could then be established between certain problems in contemporary finance and the great apostle of the « average man ». But this remains to be examined.

7.2. Conclusion: tomorrow's practitioners beyond Gaussian efficiency

Finally, the concept of informational efficiency in its probability sense brought with it a massive transfer of elements from physics and mathematics to financial theory, with the tools and

[106] For example the fractal generalizations of Brownian motion.

techniques used, accompanied by all the corresponding calculating apparatus. The creation of models in contemporary finance widely uses existing probability tools. Option pricing is a good example: to price options, and hedge and manage optional positions in the markets, it is necessary to use a certain number of fundamental results from the theory of probabilities: the derivatives industry depends heavily on theoretical probability models[107]. Likewise in investment management industry, and even in the evaluations of asset-liabilities studies for pension funds, which are conducted by using stochastic simulations on all the variables of the assets (the financial markets) and the liabilities (the demographic data of the population concerned) of the funds.

In so far as the potential growth of the investment management industry depends in a large way on the amounts of assets coming from retirement schemes, a few comments would be relevant on the influence of efficiency over the management of pension funds. It is remarkable to note how probability models have become essential components of the assets-liabilities models of pension funds. The assets-liabilities models which allow optimal management of assets representing retirement funds presuppose, behind their conceptual approach, a probability-influenced informational efficiency. The analytical or numerical solutions to the optimization problems posed to managers of these pension funds rest on the extensions to liabilities (quantification of constraints stemming from retirement payments) of assets optimization models of Markowitz type[108]. The overlapping between probability theory, demography and financial theory appears perhaps nowhere better than in the field covered today by financial actuaries, who incorporate all the probability hypotheses to combine them in a smooth Gaussian vision of fluctuations of random variables, validating once again Gaussian-efficiency, whose limits of applicability are now beginning to be better understood.

Perhaps the most striking conclusion for the investment management industry could be found in the 1965 Fama paper concerning the cornerstone of the quantitative management techniques, the concept of diversification. Using non-Gaussian stable distributions with infinite variance, Fama showed that in some cases, although increased diversification is believed to reduce the dispersion of return on a portfolio, it is the contrary which occurs: increasing diversification actually causes the dispersion of the return to increase[109]. It emphasizes that, in erratic markets, diversification becomes irrelevant, and top down processes are ruled out by the excess of volatility. The reason comes from the properties of laws of large numbers. When there is a Gaussian law of large numbers at work, it makes the return of a portfolio more certain as the number of securities is increased. But, as the law becomes weaker and weaker, as in the non Gaussian case, the return of a portfolio becomes more and more uncertain. And, when the law of large numbers work in reverse, the return of a portfolio becomes less certain as the number of securities is increased (see Fama 1965 for details). Relationships with investment processes are quite clear: in non Gaussian markets, or markets governed by non Gaussian efficiency, the investment manager has to take account of a highly concentrated performance and risk, and has to manage these concentrations. One argues that it is precisely the reason why the investment managers, having to face the fat-tails problem, are obliged to « underpin their active management with computer-modeling techniques »[110].

Thus, probability theory has become an indispensable component of the derivatives business and investment management industry today. The problems posed by financial modeling are

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[107] In particular stochastic integro-differential calculus. For example, Itô's lemma, which defines the differentiability of Gaussian random variables, without which no market maker today could manage and hedge its positions on the options market.
[108] For example Wise (1984), Wilkie (1985), Leibowitz *et al.* (1992), Keel, Müller (1995).
[109] Fama (1965b), p.412.
[110] PwC (1999), p. 41.

problems of probability. Financial theory itself has become probabilistic: some of the risk management issues are probability problems (for example, risk adjusted performance measurement, capital allocation). The stock market crash of October 1987 led to a reexamination of the specifications of models and, in particular, of the choice of their stochastic component: this is new since the consensus of the 1960's, and introduces the emergence of a new type of risk, called *model risk*: how much inaccuracy is conveyed in the firm strategy by assuming Gaussian efficiency in managing assets or building a valuation model?

But this questioning, if it follows the model's development logic, could appear natural (inevitable?) in the end, given the genesis of the informational efficiency concept. Once the way in which Gaussian distribution restricts market variability, and thus efficiency itself, was understood, it became possible to reconsider efficiency, but without strong Gaussian restriction. In one sense, it is the probability origins themselves of the informational efficiency concept which will allow the concept to be saved and again used in the industry, in spite of the violation by the markets of the initial probability framework in which it was conceived. To sum up in a word, tomorrow's praticioners will work in a non-Gaussian efficiency framework. On the centenary of *Théorie de la spéculation*, the Bachelier héritage is still fertilizing our thoughts.

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