Equilibrium Insurance Pricing, Market Value of Liabilities and Optimal Capitalization

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Paper aims

- Review of some previous research on optimal capitalization
  - Taylor JRI (1995) determined an optimal capitalization, related this to insurance prices and calibrated to Australian market data, based on CAPM assumptions
- Consider the model and assumptions in more detail and relate to standard results in Finance Theory
- Derivation of assumptions underlying a capital allocation consistent with a method used in practice based on actuarial value of liabilities

Model assumptions

Single period model

Individuals, productive firms and insurance firms

Individuals hold real assets, consume, invest in productive firms, invest in insurance firms, and purchase insurance over losses on their real assets.

Productive firms only issue shares and are 100% equity financed.

Insurance firms issue insurance policies on real assets, issue shares and purchase shares in productive firms.

Individual h maximizes a utility function

$$U_h\,(\mu_h, v_h, C_h)$$

μ_h is expected terminal wealth

v_h is variance of terminal wealth, and

C_h is current consumption of individual h

The budget constraint

$$W_h = \sum_i v_{hi} V_i + \sum_k a_{hk} A_k + \sum_f s_{hf} S_f + \sum_{fk} n_{hfk} P_{fk} + C_h$$

V_i is the price of one share in productive firm i, A_k is the price of a unit of real asset k, S_f is the price of one share in insurer f, P_{fk} is the premium per unit of insurance for asset k charged by insurer f.

Decision variables for household h are v_{hi}, a_{hk}, s_{hf}, n_{hfk}, and C_h.

Terminal wealth

For household h

$$T_h = \sum_i v_{hi} R_i + \sum_k a_{hk} \left(R_k - X_{hk}\right) + \sum_f s_{hf} Y_f + \sum_{fk} n_{hfk} X_{hk}$$

R_i is the end-of-period value of share i, R_k is the value of real asset k, Y_f is the end-of-period value of a share in insurer f (assumed to ignore limited liability), X_{hk} are the losses for individual h on real asset k.

First order conditions

for shares in productive firms (assuming a risk free asset with return r_f)

$$V_i = \frac{1}{\left(1 + r_f\right)} \left[\overline{R}_i + 2\lambda_h cov\left(R_i, T_h\right)\right] \qquad \text{for } i = 2, \ldots, I$$

$\lambda_h = \frac{U_v'}{U_\mu'}$ is the marginal trade-off between variance (risk) and return for individual h

Similar results for real assets, shares in insurance firms and insurance policies.

Market clearing

When asset markets are in equilibrium, aggregate supply (the fixed endowment) for each of the productive firms shares, $\sum_{hi} \widehat{v}_{hi} + \sum_{fi} \widehat{v}_{fi}$, is equal to aggregate demand, $\sum_{hi} v^*_{hi} + \sum_{fi} v^*_{fi}$, and the aggregate supply of real assets, $\sum_h \widehat{a}_{hk}$, will equal aggregate demand, $\sum_h a^*_{hk}$.

Consumption good markets are also in equilibrium with aggregate supply (the fixed endowment of consumption goods) equal to aggregate demand, $\sum_h \widehat{C}_h = \sum_h C_h$.

Insurance contracts are in net zero supply in this model economy, as is the risk free asset.

Equilibrium

Shares in productive firms (similar results for real assets, shares in insurance firms and insurance policies)

Sum first order conditions over all individuals in the economy

$$-\frac{1}{2}\sum_h \frac{1}{\lambda_h}\left[\overline{R}_i - \left(1 + r_f\right) V_i\right] = cov\left(R_i, \sum_h T_h\right) \qquad \text{for } i = 2, \ldots, I$$

$$V_i = \frac{1}{\left(1 + r_f\right)}\left[\overline{R}_i - \lambda cov\left(R_i, M\right)\right] \qquad \text{for } i = 2, \ldots, I$$

where $\lambda = -2\left[\sum_h \frac{1}{\lambda_h}\right]^{-1}$ and M is the end-of-period economy wide wealth.

Market price of risk

Aggregate first order conditions across all assets and solve for λ in terms of economy wide wealth (expected end of period value $\overline{M}$)

Market price of risk

$$\lambda = \frac{\left[\overline{M} - \left(1 + r_f\right) M_0\right]}{\sigma_M^2}$$

Insurance CAPM

$$P_{fk} = \frac{1}{\left(1 + r_f\right)} \left[\overline{X}_k - \lambda cov\left(X_k, M\right)\right] \qquad \text{for} \left[\begin{array}{c} k = 1, \ldots, K; \\ f = 1, \ldots, F \end{array}\right]$$

Equilibrium capitalization

Taylor assumes the price of insurer f depends on net assets (equity) K_f so that $S_f = S_f\left(K_f\right)$, then differentiates with respect to K_f to determine optimal capitalization

We show in the paper that

$$S_f\left(K_f\right) = K_f$$

and that this approach does not produce an endogenous optimal capitalization

No optimal level of capital for an insurer in this model - compare with Modigliani and Miller result in Finance Theory

Limited liability

If insurer assets A_f are less than the liabilities X_f then policyholders share the loss

$$
\begin{aligned}
X_{hk}^{L} &= \min\left(X_{hk}, \frac{X_{hk}}{X_f} A_f\right) \\
&= X_{hk} - \max\left(X_{hk} - \frac{X_{hk}}{X_f} A_f, 0\right)
\end{aligned}
$$

Fair price of the insurance contract

Assuming bivariate lognormal distribution for asset and liabilities,becomes

$$P_{hk}^{L} = P_{hk} - \left\{ P_{hk} N\left(d_1\right) - P_{hk}\frac{A}{L} N\left(d_2\right) \right\} = P_{hk}\left[1 - \left\{ N\left(d_1\right) - \frac{A}{L} N\left(d_2\right) \right\}\right]$$

where

$$d_1 = \frac{\ln\frac{L}{A} + \frac{1}{2}\widehat{\sigma}^2}{\widehat{\sigma}}$$

$$d_2 = d_1 - \widehat{\sigma}$$

and $\widehat{\sigma}^2$ is the variance of $ln\left(\frac{X_{hk}}{\frac{X_{hk}}{X_f}\sum_i v_{fi}R_i}\right)$ which is the variance of $\ln\left(\frac{X_f}{A_f}\right)$.

Capital allocation

Commonly used method is to allocate capital in proportion to value of liabilities by line of business

Equity can be written as the difference between assets and liabilities ignoring insolvency plus the insolvency option value

On the assumptions made earlier, the fair or market value of equity is

$$A - L + L\left\{N\left(d_1\right) - \frac{A}{L}N\left(d_2\right)\right\}$$

Myers and Read refer to $A - L$ as surplus.

Payoff to line of business hk

$$\max\left(X_{hk}-\frac{X_{hk}}{X_f}\sum_i v_{fi}R_i, 0\right)=X_{hk}\max\left(1-\frac{A\left(R_f\right)}{X_f}, 0\right)$$

with total

$$\sum_{hk}X_{hk}\max\left(1-\frac{A\left(R_f\right)}{X_f}\right)=\max\left(X_f-A\left(R_f\right), 0\right)$$

Market value of total equity is

$$A-L+\sum_{hk}P_{hk}\left\{N\left(d_1\right)-\frac{A}{L}N\left(d_2\right)\right\}$$

Noting that total liabilities ignoring default have value

$$L = \sum_{hk} P_{hk}$$

total equity value is

$$A - \sum_{hk} P_{hk} \left\{ 1 - \left\{ N\left(d_1\right) - \frac{A}{L} N\left(d_2\right) \right\} \right\}$$

Allocate assets on a proportionate basis to policies to obtain an expression for total equity value of

$$\sum_{hk} P_{hk}\left[\frac{A}{\sum_{hk} P_{hk}} - \left\{1 - \left\{N\left(d_1\right) - \frac{A}{L}N\left(d_2\right)\right\}\right\}\right]$$

So if we allocated the total equity of the insurer to the policies according to

$$P_{hk}\left[\frac{A}{L} - \left\{1 - \left\{N\left(d_1\right) - \frac{A}{L}N\left(d_2\right)\right\}\right\}\right]$$

then these would "add up" and will reflect the insolvency risk of the insurer at a company level.

Allocation to lines of business is determined by valuing the losses for each line of business, adjusting for the risk of the liabilities but ignoring the possibility of default by the insurer.

An allowance is made for market risk in the liability value but not default risk of the insurer.

Each liability value is multiplied by the same factor

$$\left[\frac{A}{L} - \left\{1 - \left\{N\left(d_1\right) - \frac{A}{L}N\left(d_2\right)\right\}\right\}\right]$$

to determine the allocation of capital to the individual risk.

This makes clear the assumptions for this simple method of allocating capital to line of business.

This is not being advocated as a basis for allocation of capital.

The “correct” method for allocating capital on an economic basis is not generally agreed (Myers-Read, Merton-Perold etc)

Conclusions

Taylor's optimal capitalization result is shown to not provide an endogenous optimal capitalization - it is really just the exogenous current level in the economy

The model does not provide a basis for differences in insurance companies (no transaction costs or other frictions)

A simple capital allocation result used in practice is derived by assuming a proportional allocation of assets based on policyholder liability ignoring insolvency