

## **Market risks of insurance companies**

Descriptions and measurement approaches  
from the perspective of solvency requirements

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#### **ABSTRACT:**

*Due to globalisation, high pressure to increase transparency and comparability of results and developments in banking supervision (Basel 2), there is currently much discussion in the world on the way solvency requirements for insurance companies can be harmonised by prescribing a 'standardised model' for them. Market risks are one of the major risk types that affect the insurance business. This paper, firstly, provides a description of the character of the market risks that insurers are subject to. Secondly, it proposes a reasonably simple approach for constructing a standardised approach for a corresponding solvency requirement.*

**Keywords:** *Market risks, solvency requirements, replicating asset portfolio, mismatches, correlations, free assets*

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## 0. Introduction

Several international organisations are currently working very hard to develop new solvency requirements for insurance (and reinsurance) companies. As early as 2000, the *International Actuarial Association* (IAA) established the so-called Solvency Working Party for this purpose at the request of the Solvency & Actuarial Issues Subcommittee of the *International Association of Insurance Supervisors* (IAIS). Its final report was released in February 2002. This Working Party was followed by the so-called IAA Insurer Solvency Assessment Working Party, formerly known as the IAA Risk-Based Capital Solvency Structure Working Party. Its main purpose is to develop a standardised framework for determining (required) solvency capital. This Working Party produced a first draft report in November 2002; its final report is due in July 2003.

In addition, the *European Commission* recently started the second phase of the so-called Solvency 2 project. This project should lead to a better risk-based capital approach to solvency requirements applicable to insurers within the European Union than the current framework. The Insurance Committee of the *Groupe Consultatif Actuariel Europeen* is also providing support for this project. It will probably take several more years before the new approach can be introduced.

Finally, several national insurance supervisors are making a considerable effort in relation to this topic. For instance, in 2002 the *Australian Prudential Regulation Authority* (APRA) issued new requirements for both reserving and solvency with regard to general insurance liabilities (APRA, 2002). The *Pensions & Insurance Supervisory Authority of the Netherlands* (PVK) recently issued a white paper on the so-called 'solvency test' that it wishes to introduce as of the 2005 financial year as part of its new 'Financial Assessment Framework' (PVK, 2003).

Much research and discussion are therefore currently taking place on the various types of risks which are relevant to insurance companies, how they can be measured and how adequate economic capital can be defined for them. This is also being stimulated by the current world-wide discussions on new solvency requirements for banks, that is, the so-called Basel 2 Accord. Partly due to the emergence of financial conglomerates, supervisors naturally strive to apply a similar structure and approaches to banks and insurance companies. For instance, in the case of banks, market risks, credit risks and operating risks are distinguished, while in the case of insurance companies, the same risk types plus so-called underwriting risks are taken into account. Furthermore, the so-called three-pillar approach proposed for banks will probably also be adopted for insurance companies, with the first pillar being the quantitative capital requirements, the second pillar being the more qualitative 'supervisory review process' and the third pillar comprising requirements on disclosure.

This report is about the types of market risks that affect insurance companies and the way a corresponding solvency requirement, which is part of pillar one could be calculated. Alternatively, of course, the approach described here could also be applied in calculating internal solvency requirements.

The discussion below starts by proposing a definition of market risks. Two different types of market risk are then defined in section 2, namely risks relating to uncertainty with regard to the composition and actual market value of the asset portfolio that would replicate the liabilities ('type I'), and risks due to mismatches between this replicating asset portfolio and the asset portfolio actually held by the insurer ('type II'). Section 3 discusses the time horizons that should be considered for both these types of market risks.

Type I risks are discussed further in section 4, with separate attention to the definition of the replicating asset portfolio, embedded options, current incompleteness of the capital market and discount rates for discounting liability cash flows. Section 5 deals with type II risks, that is mismatch risks within a proposed time horizon of one year. This section includes a discussion of different asset types and corresponding mismatches, proposals for methods of measuring these mismatches, a discussion of the way correlations between yields of different asset types can be taken into account, and, finally, a discussion of how free assets can be defined and ignored. Section 6 provides a summary and conclusions.

## 1. Definition of market risk

Market risk relates to the volatility of the market price of assets. It involves exposure to movements in the level of financial variables, such as stock prices, interest rates, exchange rates or commodity prices. It also includes the exposure of options to movements in the underlying asset price. Market risk also involves exposure to other unanticipated movements in financial variables or to movements in the actual or implied volatility of asset prices and options.

It is obvious that this volatility affects the actual market value of the company's assets, including those needed to cover the liabilities, and therefore also affects the company's actual surplus. However, the volatility of the market price of assets will also affect the liabilities. This will always happen in at least one and possibly two ways. Firstly, a change in asset yields will affect the market value of the liabilities through their effect on the discount rate(s) of the liability cash flows. These effects on the liability value should always be taken into account. In particular, market risks should always be considered from the perspective that both assets *and* liabilities are valued at their market values.

Secondly, a change in asset returns/yields will affect future liability cash flows, if the policyholders are entitled to some form of profit sharing which is related, for instance, to actual and/or historical returns on assets. In this respect, the different types of 'interest' profit sharing within the global insurance market can be categorised into the following three groups:

- A. profit sharing that is fully based on objective indicators of the performance of the capital market, for instance, an indicator of the actual interest rate level that is calculated and published periodically by a government agency, or a stock market index;
- B. profit sharing that is somehow related to the actual performance of the company ('performance-linked'), particularly with respect to the company's investments (Note: This type includes systems where the management is entitled to 'declare the bonus rate'); and
- C. profit sharing that is related to the actual performance of the assets that are 'locked-in' at the policyholders discretion, that is policyholders themselves are at least partially responsible for the way their premiums are invested (Note: The typical example of this type of profit sharing in life insurance is profit sharing that is implicitly offered with unit-linked/universal life (UL) products).

All three types of profit sharing may also include certain types of guarantees offered by the insurer, for instance a bonus rate that will never be negative or a minimum maturity benefit.

We therefore propose the following definition of market risk:

*Market risk relates to the volatility of the difference between the market values of assets and liabilities within a certain time frame due to future changes in asset prices, yields or returns. In this respect changes in liability cash flows, due to effects on (expected) future profit sharing, should also be taken into account, while free assets may be ignored.*

## 2. Specific types of market risk

In the literature many different types of market risk are distinguished, for example:

- Interest Rate Risk: the risk of exposure to losses resulting from fluctuations in interest rates;
- Equity and Property Risk: the risk of exposure to losses resulting from fluctuations in the market value of equities and other assets;
- Currency Risk: the risk that relative changes in currency values will result in the depreciation of assets denominated in foreign currencies;
- Basis Risk: the risk that yields on instruments of varying credit quality, liquidity, and maturity do not move together, thus exposing the company to market value variance that is independent of liability values;
- Reinvestment Risk: the risk that the returns on funds to be reinvested will fall below anticipated levels;

- Concentration Risk: the risk of increased exposure to losses due to the concentration of investments in a geographical area or another economic sector.
- ALM Risk: the risk that fluctuations in interest and inflation rates will have different impacts on the values of assets and liabilities;
- Off-Balance-Sheet Risk: the risk of changes in the values of contingent assets and liabilities, such as swaps, which are not otherwise reflected on the balance sheet.

We believe, however, that this categorisation is incomplete or, at least, not explicit enough. Minimum investment return guarantees, for instance, are not taken into account. On the other hand, some of these risk categories, namely Interest Rate Risk, Reinvestment Risk and ALM Risk, are basically the same. More precisely, we believe that most of these risks can be regarded as 'ALM Risks' (in a broad sense) or 'mismatch risks', that is risks relating to differences in the sensitivity of assets and liabilities to changes in the return on assets and therefore, in general terms, risk relating to a residual sensitivity of the surplus to such changes.

Mismatch risks can only be measured appropriately if both the market value of assets and the market value of the liabilities are measured adequately. The market value of assets can generally be derived from the listings in the various securities markets. Of course, due to the lack of a real market for insurance liabilities, the market value of insurance liabilities can only be approximated. The concept of the 'replicating (asset) portfolio', defined in section 4.1, is a useful concept in measuring the market value of insurance liabilities. Nevertheless, the possibly long-term duration and/or complex characteristics of the liabilities (guarantees) may cause problems in determining the replicating asset portfolio. We therefore believe that market risks comprise the following two types:

- I. risks due to uncertainty regarding the composition of the replicating asset portfolio, resulting in uncertainty of its market value (and therefore also of its sensitivity to changes in asset yields); and
- II. given the (assumed) replicating asset portfolio, the volatility of the difference between the market value of the actual asset portfolio and the asset portfolio that replicates the liabilities, due to changes of asset yields.

### 3. Time horizon

The distinction made between the two types of market risk (I and II in section 2) can also be justified from the perspective of the time horizon. To a certain extent, risks I should be considered as systematic (undiversifiable), since they are due to the limited availability of (parts of) the replicating asset portfolio or, at least, uncertainty about its composition; the replicating asset portfolio may even be non-existent. These risks must always be assessed for the full remaining term of the contracts, since all these risks increase the market value of the liabilities.

On the other hand, risks II are basically diversifiable. Assuming that it is possible to purchase the replicating asset portfolio on the capital market, the actual mismatch risks due to deviations between the market value of this asset portfolio and the market value of the asset portfolio which is actually available are deliberately chosen by the management. Most of these risks may disappear at relatively short notice by changing the composition of the available asset portfolio in accordance with the composition of the replicating asset portfolio. Such transactions would only imply some minor transaction costs. As a very conservative estimate, we can assume that it may take up to one year to transform the actual asset portfolio into the replicating asset portfolio. Therefore, the time horizon to be used for considering and managing mismatch risks II can be limited to just one year. In addition, however, we believe that 'factor-based' solvency requirements in relation to risks II can be calculated by assuming certain *instantaneous* shocks in asset prices/yields.

Of course, even if the replicating asset portfolio is available in the capital market, it may still be desirable to keep a buffer for mismatch risks (volatility of the surplus due to changing asset yields) beyond the limited time horizon of one year. This is certainly advisable if the management deliberately prefers *not* to invest in the replicating asset portfolio within, say, the first next  $X > 1$  years. However, in that case any buffer that is calculated for such a longer period should sooner be

considered to be a buffer required to support the actual investment policy of the company in the longer term than a buffer required to cover the mismatch risks 'inherent' to the liabilities.

Compared to the situation in which it is (implicitly) assumed that the company will invest in the replicating asset portfolio from the next year onwards, such an *additional* buffer should therefore always be positive. It seems reasonable to take this additional buffer into consideration only in relation to the *second* pillar of supervision, since it is basically only in relation to this that the company's long-term policy, including its actual investment policy, is assessed. In that case, there is no need to assume a time horizon for market risks II within the *first* pillar that is longer than one year.

Market risk types I and II clearly require very different approaches in calculating solvency requirements. The fact that a much longer time horizon is needed for market risks I also implies that the approach needed for risks I is (generally) much more complex than the approach needed for risks II. Generally, it will therefore also be much more difficult to construct a reasonable factor-based approach for determining a solvency requirement in relation to risks I. It may, however, be possible to achieve the latter. The recent report by the Canadian Institute of Actuaries (CIA) on so-called segregated fund investment guarantees clearly illustrates this (CIA, 2002). Nevertheless, due to the wide variety of (financial) embedded options offered in the global life insurance industry, describing a 'generic' factor-based approach for market risks of type I seems to be impossible. In this regard, we also wish to repeat that these risks will hopefully diminish in the future as more advanced financial instruments are introduced on the capital market.

#### **4. Type I risks: uncertainty on the replicating asset portfolio**

##### **4.1 The replicating asset portfolio**

In principle, the replicating asset portfolio generates annual cash flows that 'replicate', that is, coincide with the annual liability cash flows in each individual future year. The replicating asset portfolio therefore provides a perfect 'hedge' against the liability risks.

This is clearly a theoretical concept. Liability cash flows are subject to several types of risks, for instance mortality risks, which cannot be hedged by financial instruments. Therefore, we propose using the following definition of the replicating asset portfolio:

*The replicating asset portfolio (only) replicates the liability cash flows that are adjusted for the systematic non-financial risks, while volatility due to diversifiable non-financial risks (for instance, volatility risk as a consequence of mortality) is entirely ignored.*

Consequently, we believe that the replicating asset portfolio should provide a *full* hedge against the *financial* risks that may affect *all* future insurance liability cash flows.

##### **4.2 Embedded options**

Following the definition above, we believe that the replicating asset portfolio, that is the asset portfolio used to represent the liabilities, should include specific financial instruments that provide a full hedge against (financial) 'embedded options', like minimum investment return guarantees related to profit sharing (if offered by the insurer). In practice, such guarantees are often ignored, particularly if they are considered to be 'out-of-the-money'.

Ignoring embedded options is definitely not correct. Guarantees *always* offer additional value to policyholders, since they imply, implicitly or explicitly, that certain risks are transferred to the insurer. Consequently they always increase the market value of the liabilities. Theoretically, the market value of these guarantees is equal to the market value of the financial instruments that are necessary to hedge these guarantees. As these instruments are generally specific types of options or, if the guarantees also apply to future premiums, swaptions, their market value can generally be approximated by applying calibrated Black-Scholes types of option-price formulas; see for instance,

Bouwknegt and Pelsser (2002) regarding annual minimum investment return guarantees for traditional Dutch regular premium business with profits, and Nonnenmacher and Russ (1997) for rather complex minimum investment return guarantees in German UL business.<sup>ii</sup> If so, it will also be possible to measure the sensitivity of these market values to changes in the asset yields. Including these instruments in the replicating asset portfolio therefore allows for confronting the sensitivity of the total market value of the replicating asset portfolio with the sensitivity of the total available asset portfolio, and therefore for measuring the type II market risk, that is the mismatch risk.

#### 4.3 Incompleteness of the capital market

Unfortunately investment return guarantees in life insurance products are often complex. As a consequence, financial instruments used to hedge the corresponding risks are generally not readily available. These instruments may even be non-existent in practice. Nevertheless it may still be possible to approximate their (fictitious) market values by applying option-pricing theory. Alternatively, their market values can generally be approximated by a reasonably conservative buffer needed to cover these risks. Such a buffer can be calculated by running a large number of stochastic simulations and calculating the corresponding VaR or TailVaR for a certain confidence level.

The lack of adequate hedge instruments is also clear if insurers have only expressed the intention, and not issued a guarantee, to cover certain risks or to provide a certain minimum level of profit sharing. In general, in such cases some conditions, possibly only vaguely defined, have to be satisfied first before granting the (additional) benefits. This is typically so in the case of life insurance benefits that are 'conditionally' indexed for price or wage inflation, or in the case of performance-linked insurance with profits, where the management only offers positive bonus rates if the financial condition of the company, at the discretion of the *management*, allows for the extra payouts. Such embedded options also have a positive value for policyholders (but less than guarantees), particularly if the conditions are not well communicated to the policyholders and historically policyholders have come to hold expectations that may be upheld in court, even if the conditions are not satisfied<sup>iii</sup>.

Furthermore life insurance liabilities resulting from the in-force portfolio, in particular, may extend to more than 30 years, possibly even more than 80 years, into the future. This is much longer than the longest life to maturity of fixed-interest securities obtainable on the capital market (generally somewhere between 20 and 30 years). In such cases the insurer therefore incurs non-avoidable (systematic) reinvestment risks in the long term. We strongly doubt whether it is possible to determine buffers for these types of risks that have a sound probabilistic basis. However, if preferred, (expected) liability cash flows far into the future can always be discounted at rates that may be considered conservative; see also the next section. We also believe that requiring solvency capital for this type of market risk (type I) is superfluous, if a solvency requirement is calculated for the mismatch risk (type II) that explicitly relates to these long-term liabilities; see the method proposed in section 5.3.

More generally, the problems/risks/uncertainties described in this section are due entirely to the limited availability or even non-existence of adequate hedge instruments on the capital market, that is to 'incompleteness' of the capital market. Consequently these basically constitute 'practical' and not necessarily also 'theoretical' problems (for example, it may still be possible to obtain reasonably acceptable estimates of their market values by applying option-pricing theory). Adequate financial instruments to hedge these risks may be introduced in the future. Hopefully these problems will therefore prove to be of a temporary nature.

#### 4.4 Discount rates

Assuming full knowledge of the composition of the replicating asset portfolio, its actual market value (excluding the actual market value of the embedded options that should be valued explicitly and separately) still has to be determined (approximated). This requires discounting the cash flows resulting from the replicating asset portfolio. Assuming these cash flows correspond to the liability cash flows adjusted for the systematic non-financial risks, and ignoring the diversifiable non-financial

risks as advocated above in section 3.1, the discount rates can be set at the actual risk-free spot yields. These spot yields therefore have to be determined.

We strongly believe that the use of different spot yields by different insurance companies should be avoided, as this would not make any sense. We therefore believe that national insurance supervisors should prescribe the levels of the risk-free spot yields to be used for discounting the replicating asset cash flows as part of the process of determining solvency requirements. Of course, this requires an adequate procedure for periodically estimating the actual risk-free spot-yield curve. For this, several methods are available. We cite the specifications proposed by Nelson and Siegel (1987) and Svensson (1994, 1995). For instance, the Nelson-Siegel approach implies estimating the following (non-linear) specification:

$$r_t^{\text{spot}} = \beta_0 + (\beta_1 + \beta_2) * \frac{1 - \exp(-\frac{t}{\tau})}{\frac{t}{\tau}} - \beta_2 * \exp(-\frac{t}{\tau})$$

The parameters to be estimated are  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$  and  $\tau$ . Attractive characteristics of this specification are:

- the specification is reasonably parsimonious;
- the spot yield for the very short duration is equal to  $\beta_0 + \beta_1$ ; and
- the estimated spot yields for the long term converge to  $\beta_0$ .

The drawbacks of this specification are its limited flexibility for fitting the short-term yields and the fact that it cannot describe a decreasing pattern of long-term yields. However, the latter seems inconsistent with economic theory and therefore only occurs temporarily in a limited number of countries.

The outcomes, of course, depend on the estimation procedure itself, including the issue of whether the observations are weighted somehow or not. Generally, observations (prices) are weighted by either one divided by the modified duration or by the level of market capitalisation.

Alternatively, the so-called splines methodology is broadly applied in practice, particularly by central banks and asset management departments of banks and (larger) insurance companies. See Anderson & Sleath (2001) for a recent comparison and assessment of the Nelson-Siegel-type and splines-type methods.

Whatever method is used in practice, it (generally) also provides estimates of spot yields for very long durations, even beyond the terms of (risk-free) fixed-interest securities that are (actively) traded on the capital market. As mentioned above, some conservatism can be built in by lowering these estimates to a certain extent.

## 5. Type II risks: mismatch risks

### 5.1 Methodology

In the following we assume full knowledge of the composition of the replicating asset portfolio, the way its actual market value can be calculated/approximated and, consequently, the way the sensitivity of this market value to changing assets yields can be measured. In that case we can calculate the asset yield sensitivity of the difference between the market value of the available assets, on the one hand, and the market value of the replicating asset portfolio, on the other hand, within a certain time frame (one year). In this section we therefore focus on market risks of type II, that is the 'mismatch risks'.

Solvency requirements for market risks should be as independent as possible of any specific asset allocation, since dependencies on this will make the methodology less transparent. Moreover, it may create opportunities to 'manipulate' the resulting solvency requirement. However, it looks reasonable to always allocate fixed-interest securities to the liabilities and derivatives to options embedded in the liabilities. Additionally, we believe it is fair to ignore the so-called 'free assets', that is a portfolio of assets with a market value equal to the difference between the total surplus and the 'locked-in' surplus required to cover any risks other than market risks. Sections 5.2 and 5.3 are directed to calculating a solvency requirement for all market risks, while section 5.4 deals with calculating a discount for free assets.

Particularly in relation to life insurance, market risks of parts of the total asset portfolio may be transferred to policyholders. This is generally the case in UL business (life insurance), where policyholders (generally) have considerable freedom to decide how their premiums are invested. Clearly such assets and the corresponding liabilities are closely matched (ignoring the non-financial diversifiable risks that may affect these liabilities; see section 3.1) and can be ignored in calculating a solvency requirement for market risks. In summary, the calculation of a solvency requirement for mismatch risks (type II) should (generally) concentrate on:

- all assets excluding those linked to insurance liabilities for which the policyholders carry all the investment risks (UL business); and
- the (fictitious) asset portfolio that replicates the insurance liabilities resulting, if in-force, from:
  - A. traditional Life insurance business;
  - B. health and non-life insurance business; and
  - C. options that are embedded in the liabilities.

Of course, when assuming (instantaneous) shocks in asset prices/yields, the (expected) corresponding effects on the liability cash flows related to with-profits business (category A) should also be taken into account. These effects will generally extend to benefits beyond the first year.

Most other types of liabilities, for instance tax liabilities, can generally be 'netted out' by subtracting the corresponding (expected) cash flows from the (expected) cash flows resulting from the available fixed-interest securities. For those liabilities that are not netted out, the corresponding replicating asset portfolio should be added to the asset portfolio that replicates the insurance liabilities.

The asset portfolio that replicates the liabilities of types A and B above will consist of fixed-interest securities. Consequently, there will generally be substantial offsetting effects resulting from the replicating asset portfolio, if the sensitivity of the difference between the market value of the actually available assets and the market value of the replicating assets for changing bond yields is considered. We believe that a more or less advanced duration analysis is adequate for measuring this sensitivity and calculating a solvency requirement for the corresponding mismatch risks. On the other hand, market risks that are related to any other types of available assets, excluding those required to cover embedded options (C) could be considered to be 'asset-only' risks.

In the following sections, the individual types of assets are considered separately, starting with the main category of fixed-interest securities. Section 6 deals with correlations between the yields of the different types of assets and the way these correlations can be taken into account.

## 5.2 Individual types of assets and liabilities

### 5.2.1 Fixed-interest securities and liabilities

The classical way of calculating a 'mismatch buffer' is a more or less advanced type of duration analysis. A simple calculation method works as follows.

1. Determine the actual market values of the fixed-interest securities and the insurance liabilities of categories A and B, say  $MV^{A(\text{fix})}$  and  $MV^{L(\text{fix})}$  respectively. The market value of the fixed-interest securities can generally be determined (or approximated) by their market prices (or those of similar assets) as listed on the stock exchange. The 'market value' of the liabilities A and B can

be calculated by discounting the risk-adjusted liability cash flows (only considering the systematic non-financial risks) by the actual risk-free spot yields.

2. Determine the 'internal rates of return' of the assets and liabilities respectively, that is the single discount rate that is necessary for equalising the discounted present value of the asset and liability cash flows respectively to their market value as determined in step 1 (say  $r^{A(\text{fix})}$  and  $r^{L(\text{fix})}$  respectively).
3. Determine also the modified durations of the assets and liabilities (say  $\text{dur}^{A(\text{fix})}$  and  $\text{dur}^{L(\text{fix})}$  respectively), assuming 'average' actual yields of  $r^{A(\text{fix})}$  and  $r^{L(\text{fix})}$  respectively.
4. Analyse historical annual changes in the spot yield that corresponds with a duration that is equal to the average of the actual durations of the assets and the liabilities  $((\text{dur}^{A(\text{fix})} + \text{dur}^{L(\text{fix})})/2)$ . From this, estimate a confidence interval for the possible changes (+/-) of this yield within one year, assuming a certain level of confidence (say  $(-\Delta r, +\Delta r)$ ).
5. Define the mismatch buffer ( $\text{Solv}^{(\text{fix})}$ ) as  $\text{ABS} \{S^{(\text{fix})} * \text{dur}^{S(\text{fix})} * \Delta r\}$ , with  $\text{dur}^{S(\text{fix})} = \text{dur}^{L(\text{fix})} + (\text{dur}^{A(\text{fix})} - \text{dur}^{L(\text{fix})}) * \text{MV}^{A(\text{fix})} / S^{(\text{fix})}$  and  $S^{(\text{fix})} = \text{MV}^{A(\text{fix})} - \text{MV}^{L(\text{fix})}$

Note:  $\text{dur}^{S(\text{fix})}$  and  $S^{(\text{fix})}$  may be positive or negative, while  $\text{dur}^{A(\text{fix})}$ ,  $\text{dur}^{L(\text{fix})}$  and  $\text{MV}^{A(\text{fix})}$  are always positive.

This approach has a number of drawbacks. We mention two of these.

- a. The duration approach as described is based on a first-order Taylor approximation of the interest sensitivity of the present value. This approximation is not very good for larger interest changes. A better approximation is possible by including the second-order term, that is the convexity term.
- b. More importantly, the duration approach assumes a parallel shift of the spot yield curve, while non-parallel shifts are equally possible, and possibly even more 'dangerous' for the company. Non-parallel shifts can be taken into account by applying the approach for some duration bands individually and, for example, adding up the resulting buffers. Such an alternative approach can also be regarded as an approach that allows for correlations between the changes of the 'average' spot yields per duration band that are less than one; see more about this in section 5.3.

Changes in bond yields may, of course, be caused by changes in the underlying risk-free rates, changes in the spreads that reflect the liquidity risk and the credit risk of the asset, or changes in both components simultaneously. In section 3.4 we suggested that the replicating asset portfolio (that is, the liability cash flows) should be valued by discounting its cash flows on the basis of the risk-free spot yields. Consequently, changes in spreads only affect the market value of the actual assets available, while changes in risk-free rates affect both the market value of the assets available and the market value of the replicating asset portfolio (liabilities). We believe that it is more logical to consider changes in spreads as typical forms of credit risk. We will not discuss this issue further.

To summarise, the simple approach described by steps 1 to 5 above can be extended and improved in several ways. In any event, all these approaches can certainly be considered to be 'factor-based'.

## 5.2.2 Equity and property

As the procedure described in the section above with regard to fixed-interest securities takes into account all the effects on the market value of the liabilities A and B, the volatility of changing prices/yields/returns of other types of assets, excluding those to cover embedded options (category C; see the next section), can be considered to be a pure 'asset-only' risk. The corresponding market risks can be covered by a solvency requirement that is defined simply as the maximum possible loss of the market value of these assets within the time horizon (one year), for a commonly agreed level of confidence.

This approach is also 'factor-based'. The actual market values of these asset portfolios are the actual risk exposure measures; the factor(s) will probably be somewhere between 20% and 30%.

### 5.2.3 Derivatives and embedded options

In sections 3.1 and 3.2 we stressed the need to value embedded options explicitly. In particular, their value should be set at a level equal to the actual market value of the assets needed to hedge these options. However, these assets may not actually be available. In that case, special attention must therefore be given to possible mismatches between the options embedded in liabilities (category C in section 5.1) and the derivatives, including those that are intended as cover for them. The solvency requirement defined for this should be equal to a conservative estimate of the possible change in the difference between their market values. While these market values should always take the full remaining terms of the contracts into account, the mismatch buffer only needs to cover the possible variation of their difference within the limited time period under consideration (one year).

Calculating a mismatch buffer for embedded options will generally not be an easy task. If it is possible to obtain a reasonable approximation of their actual market value, that is the market value of the replicating asset portfolio, by applying a (calibrated) Black-Scholes type formula, it will generally also be possible to get a reasonably conservative estimate of its possible change. Such formulas generally have two types of parameters, namely the risk-free rate(s) and the implied volatility. Preferably, with respect to the risk-free rate(s), the approach to calculating the mismatch buffer should be consistent with the way the mismatch buffer for fixed-interest securities is calculated (section 5.2.1).<sup>iv</sup> It will generally be more difficult, however, to estimate the potential change of other types of parameters. If, on the other hand, the actual market value of the embedded options can only be estimated by running stochastic simulations, then the mismatch buffer should be calculated in a way that is consistent with this.

### 5.2.4 Other types of assets

Asset portfolios may contain many other types of assets. Some of these may even be off-balance-sheet items. Typical examples are investments in private equity, commodities and all kind of derivatives that are not intended as ways of hedging options embedded in the liabilities. As with equity and property investments, the related market risks are (generally) 'asset-only' risks. The corresponding mismatch buffer can therefore be calculated in a way similar to the way it is calculated for equity and property investments (see section 5.2.2).

Some of these assets may be very much 'over-the-counter', that is illiquid. In that case, of course, both their actual market values and the possible change of these values within the limited time period (one-year) have to be estimated with even greater conservatism.

### 5.2.5 Other types of asset risks

Two more types of asset risks may be mentioned, namely currency risk and concentration risk. Currency risk is only imminent if not all assets and liabilities are denominated in the same currency. In that case, an additional mismatch buffer will be necessary.

Currency risk can be considered as a stand-alone asset risk. Ignoring assets in foreign currency in the calculations suggested in the foregoing sections, a solvency requirement for currency risk can easily be defined in a similar way as a solvency requirement for equity and property risks can be defined, i.e. by setting it equal to the actual market value of the assets denominated in foreign currency times a conservative estimate of the potential change of value within the first next year. The 'potential change' factor can include the effects of both the potential change of the yields/prices and the potential change of the currency.

Concentration risk is more related to credit risk than market risk. Therefore, we will not discuss concentration risk further.

### 5.3 Correlations

Correlations between asset market prices/yields of different asset types, particularly fixed-interest, equity and property, but *excluding* derivatives, are generally low. Correlations between prices/yields/returns of assets in local currency and those of assets that are denominated in foreign currencies may be anything between  $-1$  and  $+1$ , depending on the global and local economic conditions, the type of asset and the specific characteristics of the assets (industry). We nevertheless believe that it is reasonable to assume zero correlation between all these asset types in a factor-based approach. Consequently, the total solvency requirement for market risks can be set at a level equal to the square root of the sum of squared requirements for these individual asset classes.

The market prices of derivatives, of course, including those that hedge options embedded in the liabilities, are closely linked to the market prices of the underlying assets. As was mentioned in section 5.2.3, it is therefore very important to have consistency between the approaches for the 'leading' assets and the derivatives. In particular, if the approach for leading assets is based on an assumed change of the price/yield, the same change should be assumed in determining the change in the value of the derivatives. The resulting solvency requirement can be aggregated to the total requirement by simply adding it to the total defined in the previous paragraph.

Correlations within individual categories are generally high. This is taken into account implicitly by defining and adding up the different solvency requirements of different asset categories, instead of by defining and adding them up for individual assets. Any 'further' correlations due to possible concentration within categories—for instance many investments in shares of companies in the IT-industry—may be 'penalised' by adding solvency requirements for concentration risks.

Within the category of fixed-interest securities, however, special attention must be paid to correlations between spot yields for different durations/maturities, if the factor-based approach for fixed-interest securities is applied independently to individual duration bands (see also section 5.2.1). In that case, a choice has to be made with regard to the way in which the corresponding solvency requirements are combined into a single requirement for all fixed-interest securities (minus liability cash flows). This issue is closely linked to the issue of correlations. Spot yields for various durations generally show high, but not perfect, correlations. The actual spot yield curve may therefore also show non-parallel shifts. The following approach per duration band allows for such shifts.

1. Select a number of (modified) duration bands, for instance 0-2 years, 2-5 years, 5-8 years, 8-12 years, 12-16 years, 16-24 years, and more than 24 years, with corresponding 'median' durations  $dur^{(i)}$  ( $dur^{(1)} = 1$ ,  $dur^{(2)} = 3.5$ ,  $dur^{(3)} = 6.5$ , ...,  $dur^{(7)} = \text{say } 28$ ) and corresponding actual (risk-free) spot yields ( $r^{(1)}$ ,  $r^{(2)}$ , ...) according to the actual risk-free spot-yield curve.
2. Define the 'maximum' potential absolute changes in the spot yields that may occur within the next year, for each of the individual spot yields individually ( $\Delta r^{(1)}$ ,  $\Delta r^{(2)}$ , ...). Preferably, these are based on an analysis of historical changes for each of the spot yields individually.
3. Allocate the cash flows of the available fixed-interest securities and liabilities respectively to the different duration bands, calculate the actual market values as well as their difference per duration band ( $S^{(fix)(1)}$ ,  $S^{(fix)(2)}$ , ...) and define the solvency requirement for each duration band  $i$  as

$$Solv^{(fix)(i)} = ABS \{ S^{(fix)(i)} * dur^{(i)} * \Delta r^{(i)} \}.$$

4. Finally, define the total solvency requirement for fixed interest securities (balanced with the liabilities) as the sum of the requirements for the individual duration bands:

$$Solv^{(fix)} = \sum_i Solv^{(fix)(i)}$$

This implicitly assumes that each of the individual spot yields may either rise or fall within the next year. In this respect zero correlation between individual spot yields is assumed. The final outcome of this approach may therefore be higher than the outcome based on a rise or fall of all spot yields at the same time (by  $(\Delta r^{(1)}, \Delta r^{(2)}, \dots)$  or  $(-\Delta r^{(1)}, -\Delta r^{(2)}, \dots)$  respectively), as it allows for non-parallel shifts. By simply adding up the resulting individual solvency requirements, however, we implicitly assume that the correlations are equal to one.

In practice, we usually see that the duration of fixed-interest assets held by a (life) insurance company is lower than the duration of its insurance liabilities. Therefore, let us assume that  $S^{(fix)(i)}$  is

positive for duration bands  $i=1, \dots, n1$  and negative for duration bands  $i > n1$ . In that case, the procedure described above will automatically lead to assumed *increases* in spot yields  $r^{(1)}, \dots, r^{(n1)}$  and assumed *decreases* of spot yields  $r^{(n1+1)}, r^{(n1+2)}, \dots$ , that is a small *clockwise* rotation of the spot yield curve (approximately) around spot yield  $(r^{(n1)} + r^{(n1+1)})/2$ . It is clear that such a non-parallel shift can indeed be considered to be the most 'dangerous' shift for such a company!

This procedure also automatically results in solvency capital for liabilities with a duration that is too long to be hedgeable by assets that are actually available on the capital market. This solvency capital will serve as cover for a potential decrease in the actual spot yields for the very long term. This is the equivalent of saying that it will cover a certain decrease of the *currently expected* future reinvestment rate (/forward rate). Applying this procedure to type II market risks would consequently obviate the need to calculate an additional solvency requirement because of the non-existence of an asset portfolio that can replicate these long-term liabilities (as part of type I market risks). Indeed, such an additional solvency requirement should be considered as 'double-counting' of the same risk. (See also section 4.3.)

Finally, we note that this approach can be considered as a mix of duration matching and cash-flow matching. The more different duration bands are distinguished, the more insurers will be encouraged to do cash-flow matching.

#### 5.4 Free assets

So far we have suggested the following formula for calculating the solvency requirement for market risks of type II (mismatch risks):

$$\text{Solv}^{\text{MR}} = \sqrt{(\text{Solv}^{(\text{equity})2} + \text{Solv}^{(\text{prop})2} + \text{Solv}^{(\text{fix})2} + \text{Solv}^{(\text{forex})2}) + \text{Solv}^{(\text{deriv})}}$$

where  $\text{Solv}^{(\text{equity})}$  = solvency requirement regarding equity investments  
 $\text{Solv}^{(\text{prop})}$  = solvency requirement regarding property investments  
 $\text{Solv}^{(\text{forex})}$  = solvency requirement regarding assets in foreign currency  
 $\text{Solv}^{(\text{fix})} = \sum_i \text{Solv}^{(\text{fix})(i)}$  = solvency requirement regarding fixed-interest securities  
 $\text{Solv}^{(\text{fix})(i)}$  = solvency requirement for the  $i^{\text{th}}$  duration band  
 $\text{Solv}^{(\text{deriv})}$  = solvency requirement for the difference between the market value of the derivatives and the market value of the options embedded in the liabilities

A solvency requirement such as this, however, would include a solvency requirement for market risks which bear a relation to the 'free assets'. There is no need for this. It therefore seems fair to apply some discount on the outcome of this formula.

We provide two different approaches to calculating a discount. Both of them require full clarity about the size of the 'locked-in' surplus, excluding the solvency requirement for market risks. Let us call this amount  $\text{Solv}^{\text{other}}$  and express it as a fraction  $p$  of the total surplus  $MV^A - MV^L$ :

$$\text{Solv}^{\text{other}} = p * (MV^A - MV^L)$$

1. As the total  $\text{Solv}^{\text{MR}}$  relates to the mismatch risks affecting the total surplus  $MV^A - MV^L$ , the first approach assumes that the same reduction factor  $p$  can be applied to  $\text{Solv}^{\text{MR}}$ . The adjusted solvency requirement for market risks ( $\text{Solv}^{\text{MR}(\text{adj})}$ ) would therefore be:

$$\text{Solv}^{\text{MR}(\text{adj})} = p * \text{Solv}^{\text{MR}} = \text{Solv}^{\text{other}} / (MV^A - MV^L) * \text{Solv}^{\text{MR}}$$

This approach implicitly assumes that the locked-in surplus and the free surplus are invested in assets of equal risk.

2. As an alternative approach, insurers could therefore be allowed to allocate their assets which carry most risk to the free surplus. Assuming that the assets with the highest risk are equity, property and investments in foreign currencies, this approach would imply lowering the market values of the assets held in these asset types in the formulas used for calculating  $Solv^{(equity)}$ ,  $Solv^{(prop)}$  and  $Solv^{(forex)}$  respectively. The resulting market values for these types may, of course, not be negative, while the maximum allowable decrease in the sum is equal to  $MV^A - MV^L - Solv^{other}$ . If, in addition, part of the fixed-interest securities has to be allocated to the free assets, then the market value of this asset type can be lowered as well when calculating the solvency requirement. In this case fewer assets could be allocated to duration bands  $i$  that show expected cash flow surpluses ( $S^{(fix)(i)} > 0$ ), starting from those bands for which  $ABS\{dur^{(i)} * \Delta r^{(i)}\}$  is highest<sup>v</sup>.

Since the first approach does not require assumptions regarding the type of assets that represent the free assets (*free surplus*), this approach is very straightforward, objective and transparent. The main advantage of the second approach is that it results in an (even) lower solvency requirement for market risks, as it maximises the discount. However, the second approach is much less transparent than the first approach, particularly if the company has relatively low investments in riskier assets like equity, property and investments in foreign currencies. Due to increasing pressure to improve transparency, we believe that, at least, public disclosure of the way free assets are defined and ignored is necessary. Supervisors may even prefer to prescribe a methodology for this in order to 'guarantee' the comparability of outcomes for different companies.

## 6. Summary and conclusions

- In measuring market risks both assets and liabilities must be valued at their actual 'market values'. The market value of the liabilities can be approximated by using the concept of the 'replicating asset portfolio', defined as follows: *The replicating asset portfolio (only) replicates the liability cash flows that are ('risk'-) adjusted for the systematic non-financial risks, while volatility due to diversifiable non-financial risks (for instance, volatility risk as a consequence of mortality) is fully ignored.*
- We believe that the liability cash flows/replicating asset portfolio should be valued by discounting the cash flows by the risk-free spot yields. Supervisors should calculate and prescribe these rates in order to prevent different insurers using different spot yields.
- Market risks basically comprise two types, namely
  - I. Risks due to uncertainty of the composition of the replicating asset portfolio, resulting in uncertainty of its market value (and therefore of its sensitivity to changes of asset yields).
  - II. Given the (assumed) replicating asset portfolio, the volatility of the difference between the market value of the actual asset portfolio and the asset portfolio that replicates the liabilities, due to changes of asset yields.
- The first type of market risk is (generally) the most complex, as these risks (generally) relate to options which are embedded in the liabilities (such as minimum investment return guarantees). Due to the non-existence or limited availability of financial instruments to hedge these options fully, it may be difficult to value these options. Consequently, it may also be difficult to estimate the potential change in the market value of these options within the next year.
- Additionally, it may be impossible to hedge fully the (risk-adjusted) liability cash flows in the very long term, since these cash flows may have a duration that is (much) longer than the duration of (risk-free) fixed-interest securities obtainable on the capital market.
- The problems relating to market risks of type I are basically due to incompleteness of the capital market. Hopefully, these problems will be solved 'automatically' in the future, when new instruments are introduced. Nevertheless, for the time being, we stress the need to approximate and include the market value of embedded options, either by (calibrated) Black-Scholes types of models or by stochastic simulations. Developing a generic factor-based approach to valuing all these options may be possible, but is generally difficult.

- On the other hand, the solvency requirements for market risks of type II are (generally) not difficult to define. In this respect, we propose to consider market risks related to equity and property investments and assets denominated in foreign currency as stand-alone asset risks. For these asset portfolios, solvency requirements can be set which are equal to their actual market value times an assumed percentile change that may occur within the next year.
- Solvency requirements for market risks (mismatch risks) which bear a relation to the difference between the market value of the fixed-interest securities and the market value of the liabilities (apart from embedded options) can be defined by applying a more or less advanced form of (modified) duration analysis. We prefer independent duration analyses for a number of duration bands, because such an approach allows for non-parallel shifts in the spot yield curve. Moreover, it would render superfluous a separate buffer for very long-term liabilities, for which no matching assets are available in the capital market (type I risk).
- Solvency requirements for mismatches between the options embedded in liabilities and derivatives must be calculated in a manner which is consistent with the way the solvency requirements for the underlying assets are defined.
- With respect to correlations, we propose (implicitly) assuming zero correlations between equity, property and foreign currency investments and a correlation equal to one between these assets, on the one hand, and derivatives, on the other hand. With respect to the different spot yields, we propose applying independent duration analyses for a number of duration bands (zero correlations), while adding up the resulting solvency requirements (of course, the sum total corresponds to correlations equal to one).
- Finally, a discount may be applied to the resulting solvency requirement for market risks, since market risks that bear a relation to free assets may be ignored. We have described two possible approaches to calculating such a discount. Which method is preferred strongly depends on requirements with regard to transparency, 'fairness' and comparability of outcomes for different companies.

We therefore end up with the following formula:

$$\text{Solv}^{\text{MR}} = \sqrt{(\text{Solv}^{(\text{equity})2} + \text{Solv}^{(\text{prop})2} + \text{Solv}^{(\text{fix})2} + \text{Solv}^{(\text{forex})2}) + \text{Solv}^{(\text{deriv})} - \text{Reduction}}$$

where	$\text{Solv}^{(\text{equity})}$	= solvency requirement regarding equity investments
	$\text{Solv}^{(\text{prop})}$	= solvency requirement regarding property investments
	$\text{Solv}^{(\text{forex})}$	= solvency requirement regarding assets in foreign currency
	$\text{Solv}^{(\text{fix})}$	= $\sum_i \text{Solv}^{(\text{fix})(i)}$ = solvency requirement regarding fixed-interest securities minus liabilities (excluding embedded options)
	$\text{Solv}^{(\text{fix})(i)}$	= solvency requirement for the $i^{\text{th}}$ duration band
	$\text{Solv}^{(\text{deriv})}$	= solvency requirement for the difference between the market value of the derivatives and the market value of the options embedded in the liabilities
	Reduction	= discount for free assets

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<sup>i</sup> This also includes situations where policy benefits, for instance pensions within life insurance, are indexed to adjust for price or wage inflation (either 'unconditionally' or 'conditionally' depending on the available capital). In such cases, inflation risk is incurred. Inflation risk relating to health and non-life insurance benefits or future internal expenses are ignored here, since they are considered to be special types of 'trend risks' and 'operating risks' respectively.

<sup>ii</sup> Alternatively a so-called deflator approach may be useful (see, for instance, Jarvis *et al.*, 2001). However, this methodology is still very much under development.

<sup>iii</sup> The Dutch insurance supervisor (PVK) has described such insurance liabilities recently as 'soft' liabilities. A possible way to handle them may be to assess them in a less quantitative and more qualitative way within the second pillar of supervision (the 'supervisory review process').

<sup>iv</sup> More specifically, if it is assumed that the possible change of the risk-free rate(s) is equal to  $\Delta r$ , then the same change should be assumed when measuring the possible change in the market value of the embedded options and the assets that are supposed to hedge these options.

<sup>v</sup> There will generally be no need to also transfer (part of) the amount of derivatives to the free assets. Otherwise, this could be prohibited by the supervisor, for instance for reasons of transparency.