

## **An Examination of Insurance Pricing and Underwriting Cycles**

*Madsen, Chris K., ME, ASA, CFA, MAAA  
Pricing Leader, GE Frankona Re  
Grønningen 25, 1270 Copenhagen K, Denmark  
Phone: +45 33 97 95 02, Fax: +45 33 97 94 41  
E-Mail: chris.madsen@ercgroup.com*

*Pedersen, Hal W.,  
Warren Centre for Actuarial Studied and Research  
Faculty of Management  
University of Manitoba  
Winnipeg, Manitoba R3X 1J8  
Canada  
Phone: +1 204 474 9529  
E-Mail: pedersn@ms.umanitoba.ca*

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### ***Abstract***

This paper lays out a theory of a "price of risk" as defined in this paper and suggests that this price links all risky financial transactions. In particular, the paper details insurance pricing in terms of this price of risk and supports this with an analysis of the performance of the property and casualty insurance industry for the past fifty years.

The performance of the property and casualty insurance industry is measured by the industry's combined ratio, essentially losses and underwriting expenses divided by premiums, the volatility of which has long been a mystery to the industry itself. The theory presented here is extended to also provide a model for the underwriting cycle, that is, the apparent cyclical nature of the combined ratio.

While the theory in the paper is incomplete and is at most a reduced-form perspective, it is hoped that the ideas can be used to develop insights into the insurance pricing process.

### ***Key Words***

Price of risk, risk-adjusted value of insurance, insurance pricing, option pricing, underwriting cycle, property and casualty insurance, general insurance, market cycle, implied volatility

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## ***Introduction***

Just a few years ago, there was plenty of capital willing to take seemingly enormous risks. The shift in investor and speculator mentality since then is simply astounding. Investors' appetite for risk (or lack thereof) permeates through the entire economy and affects the price of all risky transactions. Thus, any theory, that can facilitate our understanding of cycles in investor risk-aversion, should further our understanding of how different financial transactions are linked and priced.

## ***Risky Financial Transactions***

Financial markets basically exist to facilitate risk transfers. To generalize this concept, we define - for now - the price of risk to be the price for one standard unit of risk. This is not precisely the same "market price of risk" that one normally talks about in arbitrage-free models.

### **Equation 1: Price of Risk**

Price of Risk = Price for one standard unit of risk

In the simplest of terms, there are two ways to be a buyer of risk:

- Pay today => Get uncertain cash flows in the future
- Get today => Pay uncertain cash flows in the future

Both of these are buyers of risk, but the nature of the transactions has dramatically different effects on how the price of risk influences the profitability of the transaction. The first of the bullets is the traditional equity or fixed income investor<sup>1</sup>. The second is the traditional insurer. Both are buyers of risk, but the differences high-lighted above have significant consequences in terms of how these players respond to changes in the market price of risk.

In particular, when the riskiness of the cash flows increases, the equity or fixed income price will drop if the price of risk is unchanged. Conversely, in insurance, when the riskiness of cash flows increases, the price increases. Further, when the price of risk increases, equity prices drop, but the price of insurance rises.

The standard present value of cash flow model shown below fails to account for this behavior.

### **Equation 2: Present Value of Cash Flows**

$$P = \sum_{t=1}^{\infty} \frac{1}{1+r} \cdot E[CF_t^*]$$

In this equation,  $r$  is the discount rate,  $P$  is the price paid or received for the risk transfer, and  $CF_t^*$  is the *uncertain* cash flow at time  $t$ .

## ***Developing the Price of Risk***

It is relatively easy to get a sense for the price of risk over time. When equity prices soar, for example, there may be good fundamental reasons. But often, the price increase goes above the fundamental justification. In other words, the price of risk begins to fall as investors are willing to pay more and more for the same Dollar of risky transaction. This sentiment carries through to all financial transactions.

When the price of risk increases:

- ❑ Prices of equities fall
- ❑ Prices of corporate bonds fall (bond spreads widen)
- ❑ Prices of options rise
- ❑ Prices of insurance rise

When the price of risk decreases:

- ❑ Prices of equities rise
- ❑ Prices of corporate bonds rise (bond spreads narrow)
- ❑ Prices of options fall
- ❑ Prices of insurance fall

### ***What is Insurance?***

Insurance is a financial transaction on losses. At the time a policy is written, the insurer has a certain estimate of what the expected loss is (reserve). Over time, this estimate develops as more information becomes available. Thus, the expected loss develops over time much like the price of a stock develops over time. In insurance, using a log-normal distribution to describe aggregate losses is nothing new. The log-normal is often used as a simplifying assumption when detailed data is not available. It is convenient, although not essential, for us to assume that losses have a log-normal distribution. Under this assumption we may then use the usual formulas for limited expected values.

One way to view the buying of insurance is to consider it as buying a call option on losses. Selling insurance is the equivalent of selling a call option on losses. The insured has an option on losses, that pays only if losses exceed a certain specified threshold (in the case of a simple deductible). The insurer is generally short naked calls (naked because the insurer does not have a cash flow stream to hedge the payments they must make for the call they have written).

We suggest that option-pricing theory and an assumption about relationships between changes in risk aversion in the financial and insurance markets can be used to gain some insight into insurance pricing. We explore this concept in the following sections.

### ***Adding the Market Price of Risk to the Black-Scholes Model***

#### **Equation 3: Black-Scholes Option Pricing Model**

$$C = S \cdot N(d_1) - K \cdot e^{-r\Delta t} \cdot N(d_2),$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{s^2}{2}\right) \cdot \Delta t}{s \cdot \sqrt{\Delta t}}$$

$$d_2 = d_1 - s \cdot \sqrt{\Delta t}$$

$C$  is the price of a call option with strike price  $K$  on an underlying security with current price  $S$ . The risk-free rate is denoted by  $r$  and the time to option expiration is  $\Delta t$ .  $s$  is the standard deviation of equity price returns over a time period of  $\Delta t = 1$ .

To use the Black-Scholes option pricing model, we must first clarify how the various parameters are estimated in an insurance environment.

$C$  is the price of insurance with deductible  $K$  on losses with current expected discounted value of  $S$ . The risk-free rate is denoted by  $r$  and the insurance term is  $\Delta t$ .  $s$  is the standard deviation of the change in expected discounted losses over a time period of  $\Delta t = 1$ .

We will use the coefficient of variation (ratio of the standard deviation to the mean) as a standardized measure of riskiness. The riskier a cash flow, the higher its coefficient of variation.

#### Equation 4: Coefficient of Variation

$$\mathbf{a} = \frac{StdDev}{Mean}$$

We now define the market price of risk as the price investors are willing to pay per unit of riskiness as measured by the coefficient of variation,  $\mathbf{a}$ . Thus, we relate the volatility of returns,  $s_t$ , to a standardized “market price of risk”,  $\mathbf{I}_t$ . We can use this relationship to determine the market price of risk from existing data such as equity option prices. In particular, we use the Chicago Board Options Exchange (CBOE) volatility index or VIX as a proxy for the  $s_t$  of equity returns as measured by the S&P 500 index<sup>ii</sup>. The coefficient of variation,  $\mathbf{a}$ , for equities is based on long-term volatility divided by long-term return. This is shown in Equation 6.

#### Equation 5: The Market Price of Risk

In general,

$$s_t = \mathbf{I}_t \cdot \mathbf{a}$$

⇕

$$\mathbf{I}_t = \frac{s_t^{S\&P500}}{\mathbf{a}^{S\&P500}}$$

We can use the S&P500 to find our price of risk at time  $t$ :

$$\mathbf{I}_t = \frac{s_t^{S\&P500}}{\mathbf{a}^{S\&P500}} = \frac{s_t^{S\&P500}}{\left( \frac{s^{S\&P500}}{\mathbf{m}^{S\&P500}} \right)}$$

Once the market price of risk,  $\mathbf{I}_t$ , is determined from equity option prices, we need a method for determining our loss volatility parameter to use in the equation. This work is laid out in Equation 7.

### Equation 6: Determination of Implied Insurance Volatility Parameter

We want to find the implied insurance volatility parameter, which we can then use to price the insurance policy. The volatility we refer to is that of the "loss return", or simply, the volatility of the expected change in the discounted loss over a given time period

$$s_{\frac{L_t - L_0}{L_0}} = s_{\frac{L_t}{L_0}} = \frac{s_{L_t}}{L_0},$$

where

$L_0$  is the expected discounted losses at time 0 (no variability)<sup>iii</sup>.

$L_t$  is the expected discounted losses at time t (unknown at time zero).

The coefficient of variation associated with this volatility is

$$\mathbf{a}^{Loss} = \frac{\left(\frac{s_{L_t}}{L_0}\right)}{\left(\frac{\mathbf{m}_{L_t} - L_0}{L_0}\right)} = \frac{s_{L_t}}{\mathbf{m}_{L_t} - L_0} = \frac{s_{L_t}}{(1+r) \cdot L_0 - L_0} = \frac{s_{L_t}/L_0}{r},$$

where

$r$  is the risk-free rate. This is because  $L$  is the discounted estimate of losses at time t.

From this, we can solve for the insurance volatility parameter,  $s_t^{Loss}$ , at time t:

$$s_t^{Loss} = \mathbf{I}_t \cdot \mathbf{a}^{Loss}$$

The coefficient of variation measures volatility relative to its expected value. We can adjust the Black-Scholes formula to handle this (Equation 8):

### Equation 7: Black-Scholes Option Pricing Model

$$C = S \cdot N(d_1) - K \cdot e^{-r\Delta t} \cdot N(d_2),$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{s^2}{2}\right) \cdot \Delta t}{s \cdot \sqrt{\Delta t}} = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{(\mathbf{I} \cdot \mathbf{a})^2}{2}\right) \cdot \Delta t}{\mathbf{I} \cdot \mathbf{a} \cdot \sqrt{\Delta t}}$$

$$d_2 = d_1 - \mathbf{I} \cdot \mathbf{a} \cdot \sqrt{\Delta t}$$

$\mathbf{I}$  is the market price per standardized unit of volatility or, simply, the market price of risk.

$\mathbf{a}$  is the coefficient of variation.

### Utility Theory

Gerber and Pafumi<sup>iv</sup> show how various utility functions can be applied to insurance pricing. They also extend their research to show how utility theory can be extended to derive Black-

Scholes option prices. Based on the detail we have described here so far, we should thus be able to continue building our framework incorporating some of Gerber's and Pafumi's work. This attractive for many reasons:

- It provides greater support for our framework
- Due to the simplicity and elegance of some of Gerber's and Pafumi's results, we can find more intuitive and simple ways to price
- It may be intuitively easier to explain an economic theory for insurance pricing on the basis of utility theory as opposed to options theory.

We will focus on one aspect of their analysis, the exponential utility function. Here, they show that the premium for an insurance policy can be determined based on the following equation:

**Equation 8: Premium Determination Based on Exponential Utility Function**

$$P = \mathbf{m}_{Loss} + \frac{a}{2} \cdot \mathbf{s}_{Loss}^2,$$

where  $a$  is the risk aversion parameter.

This equation possesses the characteristics and behavior we set forth initially for insurance pricing.

***Adding the Market Price of Risk to Utility Theory***

We add the price of risk as a factor to risk aversion parameter. This is consistent with our view that investor risk aversion changes over time with the price of risk. It may also be a function of other things, such as wealth. We will ignore the wealth effect for now, as we will continue to use the exponential utility function.

**Equation 9: Premium Calculation Using Exponential Utility and Price of Risk**

$$P_t = \mathbf{m}_{Loss} + \frac{a \cdot \mathbf{l}_t}{2} \cdot \mathbf{s}_{Loss}^2$$

Assuming independence of losses over time, we can extend this to multiple time periods, which will be a useful result.

**Equation 10: Premium Calculation from Equation 10 with Multiple Time Periods**

$$P_t = \mathbf{m}_{Loss} \cdot \Delta t + \frac{a \cdot \mathbf{l}_t}{2} \cdot \mathbf{s}_{Loss}^2 \cdot \Delta t$$

The above does not explicitly consider a deductible, but when we need to consider a deductible, the mean and variance parameters above can be estimated based on net losses. Alternative, we can break Equation 11 into two policies: One covers all losses ( $P_t$ ), and the other covers losses below the deductible,  $K$  ( $P_{1,t}$ ). The price of a policy with deductible  $K$  ( $P_{2,t}$ ) must necessarily be the difference between buying a policy with no deductible and

selling one that covers losses up to K. If this were not so, there would be an arbitrage opportunity.

**Equation 11: With Deductible, K<sup>v</sup>**

$$P_{2,t} = P_t - P_{1,t} = (\mathbf{m}_{Loss} - \mathbf{m}_{Loss_1}) \cdot \Delta t + \frac{a \cdot \mathbf{I}_t}{2} \cdot (\mathbf{s}_{Loss}^2 - \mathbf{s}_{Loss_1}^2 - COV_{Loss, Loss_1}) \cdot \Delta t$$

Note, that we have subtracted a covariance term. This is essential, since the losses are correlated. They cover the same risk, though different aspects of it. It is at first interesting to observe that a positive covariance decreases price, but it makes sense, since we are buying one cover and selling the other. The more correlated they are, the more they hedge each other and the lower the premium.

Before we move on, let us consider the premium portfolio problem. What happens when a company writes more than one policy and losses are correlated in some form? Continuing with our current frame-work, we look at two policies in Equation 13 below:

**Equation 12: Writing Two Policies On Correlated Losses**

$$P_t = P_1 + P_2 = (\mathbf{m}_{Loss_1} + \mathbf{m}_{Loss_2}) \cdot \Delta t + \frac{a \cdot \mathbf{I}_t}{2} \cdot (\mathbf{s}_{Loss_1}^2 + \mathbf{s}_{Loss_2}^2 + 2COV_{Loss_1, Loss_2}) \cdot \Delta t$$

This, of course, suggests that when one writes a policy in a correlated portfolio, one must charge as laid out in Equation 14.

**Equation 13: Writing A Policy in a Correlated Portfolio<sup>vi</sup>**

$$P_1 = (\mathbf{m}_{Loss_1}) \cdot \Delta t + \frac{a \cdot \mathbf{I}_t}{2} \cdot (\mathbf{s}_{Loss_1}^2 + COV_{Loss_1, OtherLosses}) \cdot \Delta t$$

A positive covariance increases the premium and a negative decreases it.

Extending this to the company level, a company's premium is simply the following:

**Equation 14: Insurance Portfolio**

$$P_{Company} = \left( \sum_{i=1}^N \mathbf{m}_{Loss_i} \right) \cdot \Delta t + \frac{a \cdot \mathbf{I}_t}{2} \cdot \left( \sum_{i=1}^N \sum_{j=1}^N COV_{Loss_i, Loss_j} \right) \cdot \Delta t$$

In the case where all losses are independent, this simplifies to Equation 10.

We can use Equation 15 to find the optimal level of premium a company should write for a given level of risk, if losses are correlated. In establishing a frame-work for insurance pricing, this is outside the scope of this paper, and we leave this particular topic as a fascinating one for future research.

***Determining the Market Price of Risk***

In a February 1, 1999 A.M. Best report, A.M. Best stated that "A.M. Best believes the property/casualty underwriting cycle has been replaced by a permanent 'down market'". If this brought reminiscences of Irving Fisher's<sup>vii</sup> "Stocks have been replaced by what looks like a permanently high plateau", it was appropriate. The price of risk had been dropping for much of the 90's much like it did in the 20's. During the 90's, this meant soaring equity prices and poor insurance prices.

Margins may erode and markets may become more efficient, reward new technology and theory, but it is extremely unlikely that there is a thing such as a permanent down market. It is equally unlikely that a permanent up market exists.

As underwriters are well aware, the financial markets are cyclical. Sometimes, risk-takers get compensated well for taking risk and sometimes they do not. This is true for all financial transactions whether in insurance or investing. In the late 90's, investors bid many speculative issues way in excess of their fundamental value. This essentially pushed the price of risk to nothing or perhaps into negative territory. In other words, no one was getting compensated for taking risk, as the market place did not perceive any real risk.

Conversely, after September 11<sup>th</sup>, 2001<sup>viii</sup>, the perception of risk was great and equity markets fell and insurance prices rose as the price of risk rose.

We will examine our model from Equations 8 and 11 to see if they can help describe some of the cyclical behavior that property and casualty insurers have been exposed to. In order to do this, we must first determine the market price of risk.

We know from equity option pricing that option prices often do not reflect the long-term volatility of equity returns. Thus, the Black-Scholes formula is often reversed and solved for the volatility given prices. This volatility is called the implied volatility. Using the generalization from Equation 8, the implied volatility can also be separated into standardized volatility and market price of risk.

The following is an excerpt from the Chicago Board Options Exchange website:

*"One measure of the level of implied volatility in index options is CBOE's volatility index, known by its ticker symbol VIX. VIX, introduced by CBOE in 1993, measures the volatility of the U.S. equity market. It provides investors with up-to-the-minute market estimates of expected volatility by using real-time OEX index option bid/ask quotes. This index is calculated by taking a weighted average of the implied volatilities of eight OEX calls and puts. The chosen options have an average time to maturity of 30 days. Consequently, the VIX is intended to indicate the implied volatility of 30-day index options. It is used by some traders as a general indication of index option implied volatility. Implied volatility levels in index options change frequently and substantially."*

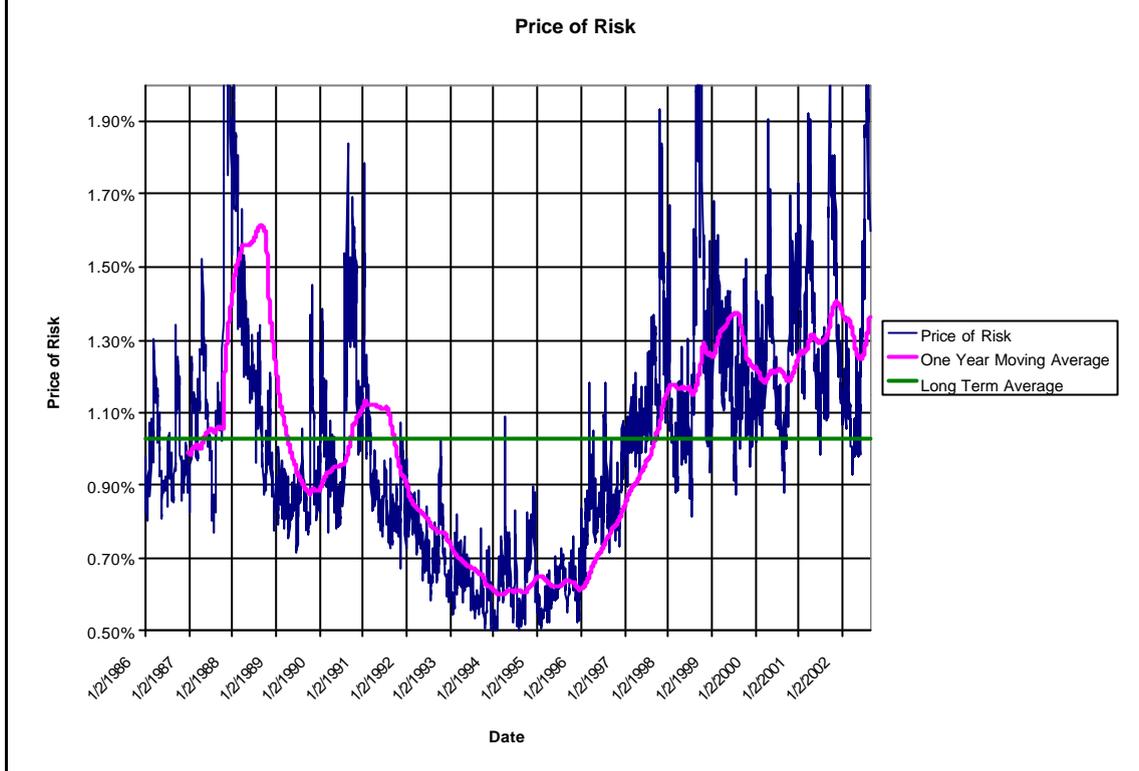
Using the VIX as implied volatility for the equity market, we can - by assuming long-term monthly expected equity *price* return and volatility - solve for the market price of risk.

**Equation 15: Coefficient of Variation for Equities**

$$a = \frac{s^{S\&P500}}{m^{S\&P500}} = \frac{20.90\%}{1.01\%} = 20.66$$

This means we can develop the following graph for **I** which, in effect, is just a rescaled graph of the VIX index:

**Graph 1: The Price of Risk**



From this graph, we can clearly see that for most of the 90's, the price of risk was depressed. This would seem to suggest that it was a good time to buy options and a bad time to sell them. Remember that selling insurance may be interpreted as selling options. Of course, interest rates also play a role in option pricing. We already know with the benefit of hindsight that the 90's were a tough time for many property and casualty insurers because of low prices. We now develop an insurance price index based on the above information and examine how it stacks up against historical experience.

### ***Insurance***

Consider now an insurance policy. The ground-up losses have a standard deviation of three times the expected loss (Example 1a).

#### **Example 1a: Loss Parameters**

Expected Loss: \$100,000  
Standard Deviation of Losses: \$300,000  
Risk-free Rate: 5.75%

This means

$$a = \frac{\left( \frac{300,000}{100,000} \right)}{0.0575} = 52.174$$

We insure losses that exceed \$50,000 (Example 1b).

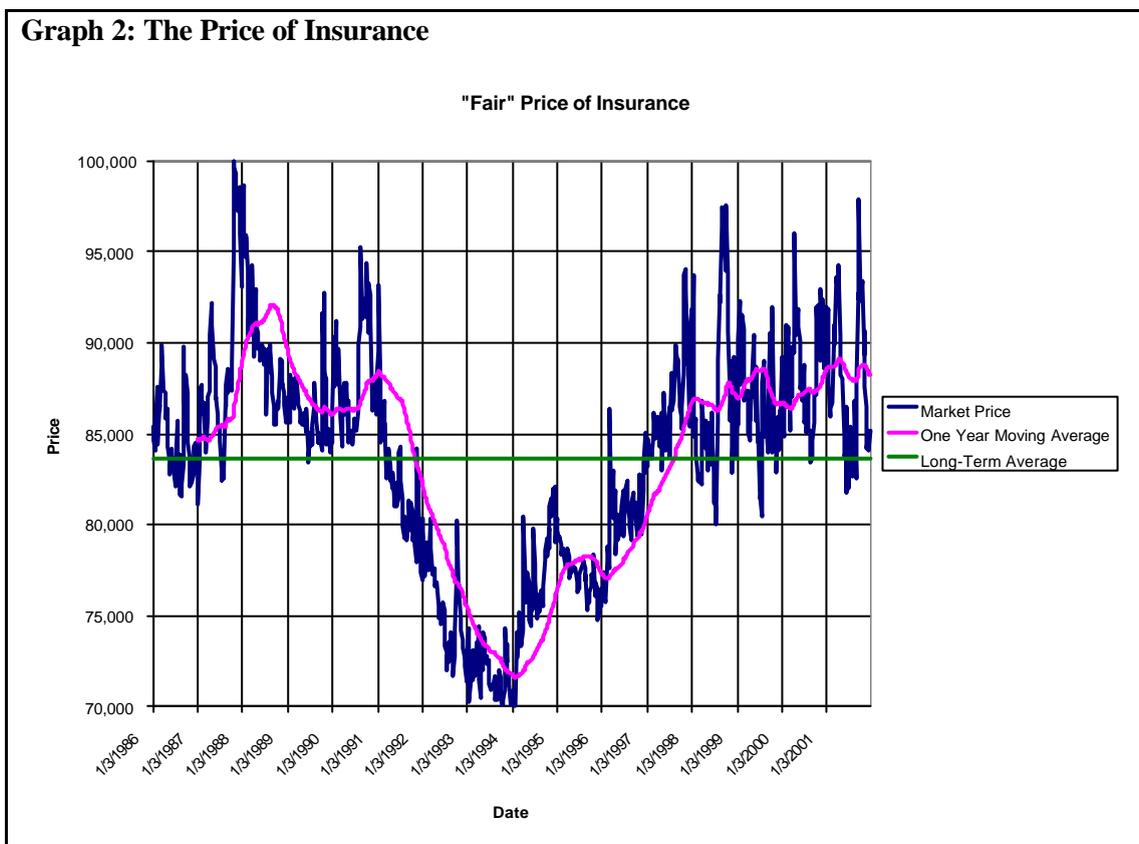
### Example 1b: Terms of Policy

Duration: 1 Year

Deductible: \$50,000

Given that we know the price of risk over time, we can now use the Black Scholes model to determine the price of the insurance policy. Since the price of risk fluctuates over time, the price of insurance will too. Graph2 looks much like Graph1, although there are some subtle differences. In particular, the previous chart only looked at the price of risk and did not consider the effect of interest rates. Interest rates are accounted for in the chart below. A feature of the model we are using is that the model prices insurance by adjusting the volatility of the losses used for pricing in response to changes in the VIX. Intuitively, as the VIX can be used as a proxy for changes in the attitude toward risk, we are allowing the pricing of insurance to reflect these changing attitudes as well.

Graph 2: The Price of Insurance



We can also use our utility theory model to mirror the results above. In fact, for certain values of the risk aversion parameter, the models are very similar.

Insurance prices are cyclical. We already know this and we need look no further than combined ratios to see this.

The combined ratio is defined in Equation 17 below.

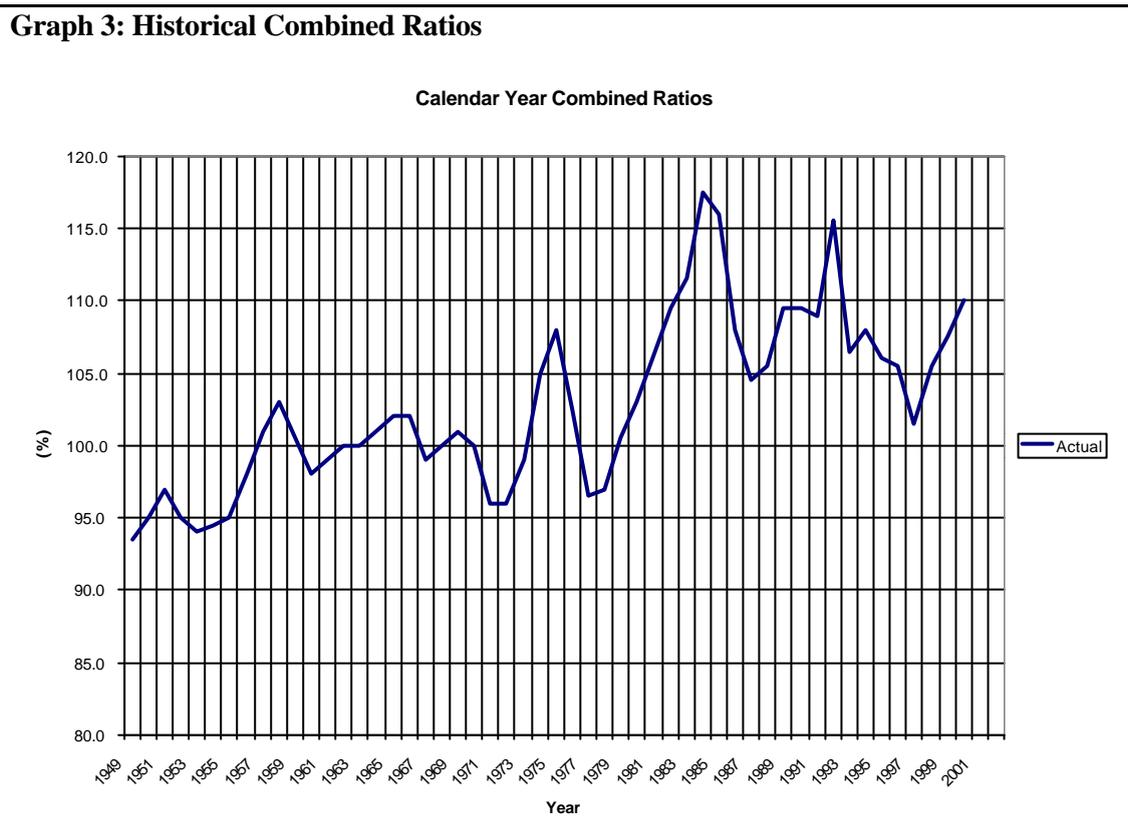
### Equation 16: Combined Ratio

$$CR_t = \frac{L_t}{PE_t} + \frac{E_t}{PW_t}, \text{ where}$$

$CR_t$  is the combined ratio at time t,  
 $L_t$  is the incurred loss at time t,  
 $E_t$  is the incurred expenses at time t,  
 $PE_t$  is the earned premium at time t, and  
 $PW_t$  is the written premium at time t.

Incurring losses are paid losses plus changes in loss reserves.

In Graph 3 below, we show the historical values of the calendar year combined ratio.



We can try to model this time series with a simple auto-regressive model as proposed by Kaye D. James in her discussion of underwriting cycles<sup>ix</sup> from 1980. She specifically proposes a model of the form:

**Equation 17: K.D. James Model**

$$CR_t = CR_{t-1} - .9115 \cdot CR_{t-2} + .9132, \text{ where}$$

$CR_t$  is the combined ratio at time t.

K.D. James used data from 1950 through 1980 to fit her model. She cited the following statistics on the model:

$$SE = .01597$$

$$R^2 = 72.09\%$$

$$t - value_{CR_{t-1}} = -7.006$$

However, in trying to reproduce results using the same period of data and the same data as far as we can tell, we get a different best-fit model:

**Equation 18: Re-fitted K.D. James Model**

$$CR_t = CR_{t-1} - .5487 \cdot CR_{t-2} + .5456$$

$$SE = .01989$$

$$R^2 = 47.97\%$$

$$t - value_{CR_{t-1}} = -5.081$$

If we allow for a more general time series model (without restricting the  $CR_{t-1}$  parameter to be one), second order auto-regressive, we can fit the following model. This time we use all data (1950-2000)

**Equation 19: Second Order Auto-Regressive Time Series Model**

$$CR_t = 1.011 \cdot CR_{t-1} - .2098 \cdot CR_{t-2} + .2069$$

that has the following statistics

$$SE = .03167$$

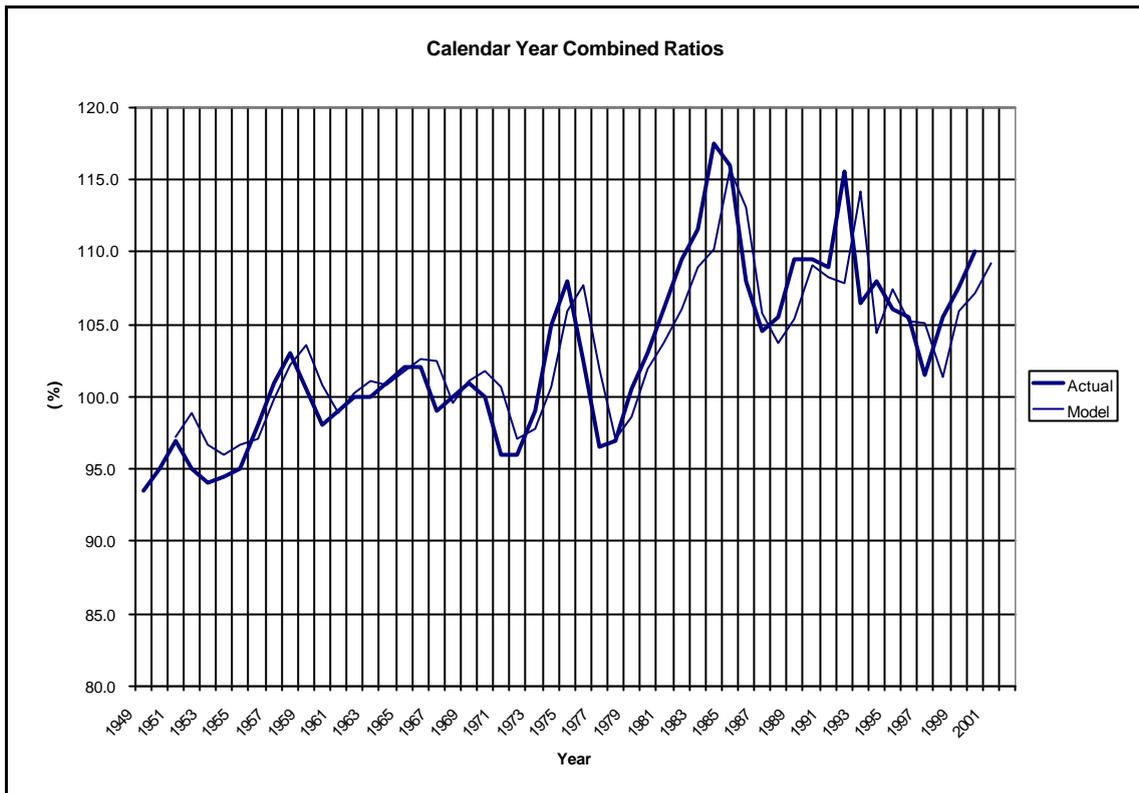
$$R^2 = 71.04\%$$

$$t - value_{CR_{t-1}} = 7.021$$

$$t - value_{CR_{t-2}} = -1.484$$

This model is graphed in Graph 4 below along with the actual data.

**Graph 4: Combined Ratios - Actual and Modeled**



While an AR(2) model is excellent at modeling the general characteristics of a time series over time, it has a major short-coming in that it is essentially a moving average of the time series it is set up to model. This means it will tend to overestimate the combined ratios in a down-trend and underestimate them in an up-trend.

In the next section, we will explore if the insurance pricing index, that we developed, can help explain better the historical behavior of combined ratios.

### ***Putting It Together***

Using our previously developed view of the insurance world as selling naked call options on losses, we can fit a time series model as before, but include our insurance index or call option price. We develop the following model for industry calendar year combined ratios:

#### **Equation 20: Combined Ratio Model**

$$CR_t = .2027 + .08836 \cdot CR_{t-1} + .3979 \cdot CR_{t-2} + .4258 \cdot C_{t-1}, \text{ where}$$

$CR_t$  is the combined ratio at time t, and

$C_t$  is the call option price at time t divided by 1,000.

The data used to fit this model was 1988-2000, since the implied volatility data for previous periods was not readily available. The statistics for this model were:

$$SE = .02555$$

$$R^2 = 55.56\%$$

$$t\text{-value}_{CR_{t-1}} = 0.3722$$

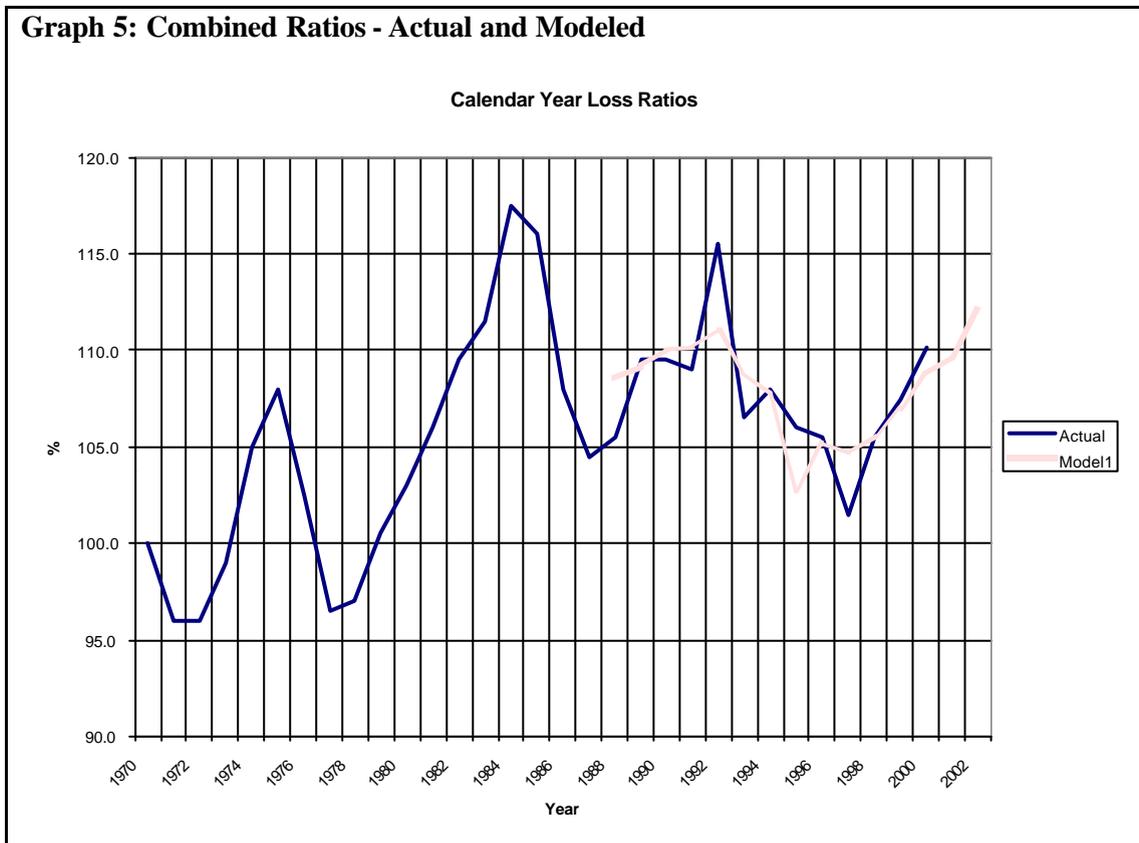
$$t\text{-value}_{CR_{t-2}} = 1.573$$

$$t\text{-value}_{C_{t-1}} = 3.064$$

While at first it may seem that the R-squared here is lower than in the K.D. James model, it is not. They are fitted on different time periods and it turns out that the period 1988-2000 is much more difficult to fit! The generalized K.D. James model fitted on the same data as above yields only an R-squared of 9.19%.

Note, that our insurance index ( $C_t$ ) is the most significant variable and that the parameter is positive suggesting that the industry combined ratio rises with our pricing index. This is the exact opposite of what should be happening if the industry was properly reflecting the market price of risk.

**Graph 5: Combined Ratios - Actual and Modeled**



### Discussion

What is particularly interesting and counter-intuitive about Equation 21 is the fact that when the call option price rises, so does the combined ratio. *This seems to suggest that when insurers can and should raise prices, they do not or at least not by enough. They are, in fact, selling under-priced options.*

Insurance is generally priced as discounted cash flows. This means that the higher the interest rate, the lower the price. *But for options the relationship is opposite.* The higher the interest rate, the higher the price of the option<sup>x</sup>.

Insurers rationalize that they can charge less for insurance when interest rates rise, since they can invest at higher yields. But this fails to account for the fact that when interest rates rise, losses - in effect - are more likely to hit the attachment point (generally due to higher inflation). In other words, to move only the interest rate without adjusting losses and the attachment point is like getting or giving a free lunch. *When interest rates rise, the price of insurance – all other things remaining equal - should rise as well.*

As for the second component of option pricing, implied volatility, insurers do raise prices when implied volatility increases dramatically. This is in reaction to supply and demand, as implied volatility is much easier to read in the market place than small changes in interest rates. After September 11<sup>th</sup>, 2001, everyone was much more concerned with insurance all of a sudden. The same was the case with Hurricane Andrew in the early 90's. Insurers correctly use these opportunities to raise prices. It is questionable whether they raise them by enough. We know this from the market place and not from the data, so we must conclude that over a full year, the effects of the poor decisions on pricing with respect to interest rates, have a greater influence on the well-being of the insurance industry.

In short, it appears the insurance industry reacts properly to changes to implied volatility but may in fact do the exact opposite of what they should do when interest rates change.

If this assertion is true, we should be able to "correct" for this behavior and model the combined ratio cycles. In order to test this, we produce two new data series. The first represents traditional insurance pricing and is a present value of expected cash flows. In this case, we assume an even payout that occurs over five years. Thus, the preliminary traditional pricing index is

**Equation 21: Traditional Insurance Pricing Index**

$$TP_t = \frac{1}{r_t} - \frac{1}{r_t \cdot (1 + r_t)^N}$$

where  $N$  is the number of years of payout.

The option pricing index is simply the price of the call option at time  $t$ . Both indexes are normalized by dividing by their respective values at 1/1/1986. 1/1/1986 has no special significance other than that it is the beginning of the time series and it is, importantly and conveniently, very close to the means of the time series.

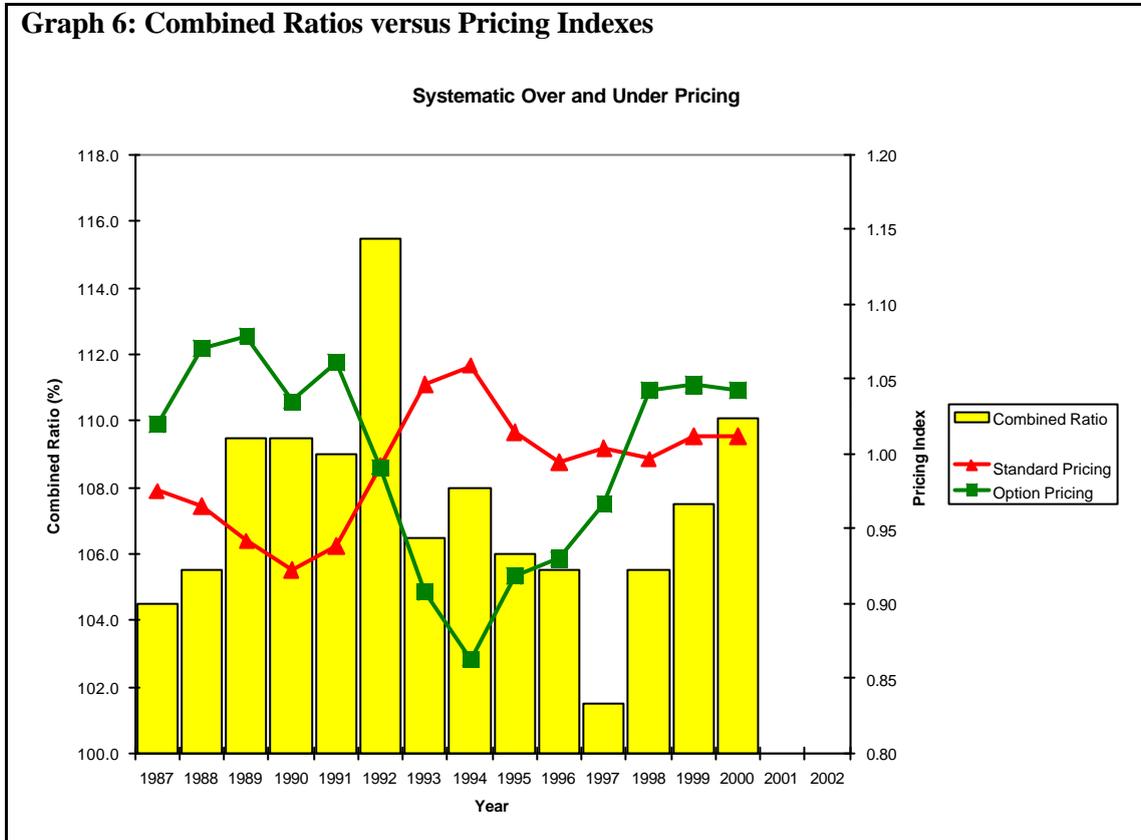
**Equation 22: Insurance Option Pricing**

$$TP_t^* = \frac{TP_t}{TP_0}, OP_t^* = \frac{C_t}{C_0}$$

In order to get the final pricing index for the year  $t$ , we take the average of the normalized indexes for the prior year. This makes sense, since most renewals occur at the beginning of the year and thus, are based on the prior year's data.

The industry is over-pricing when the traditional pricing index is above the option pricing index. The industry is under-pricing when the traditional pricing index is below the option pricing index.

This leads to Graph 6 below.

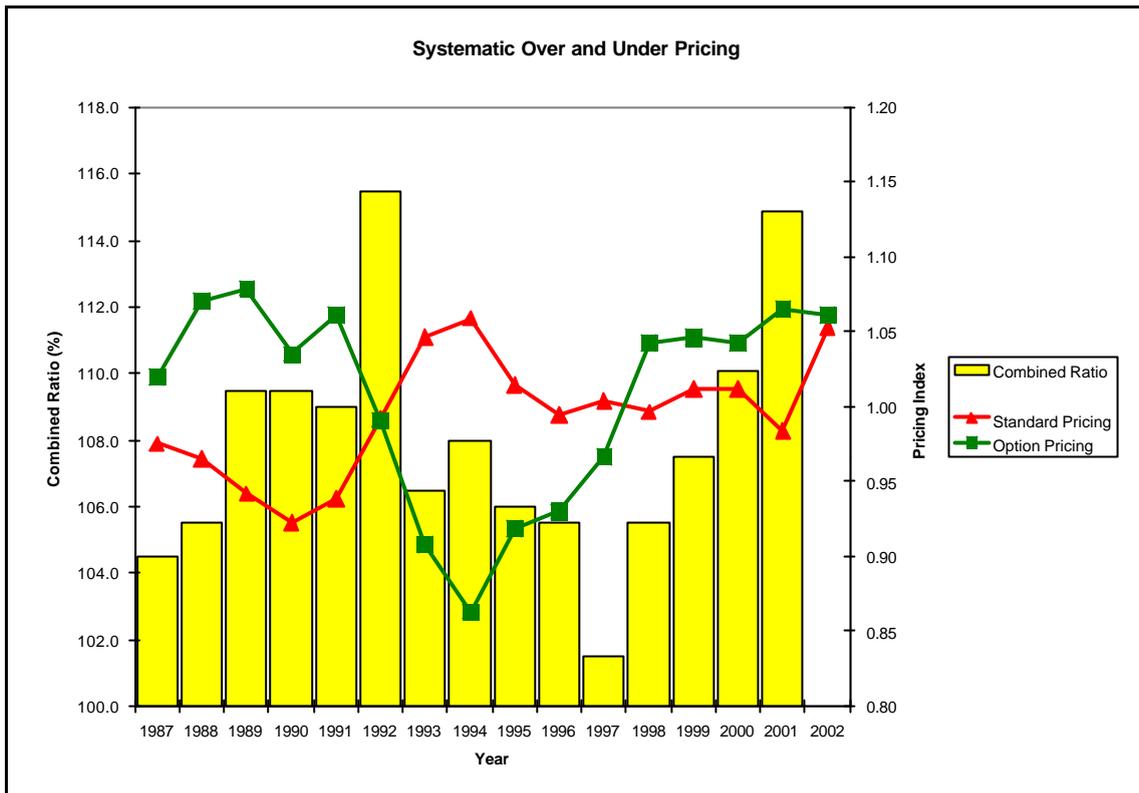


As can be seen, we have had three cycles during the past 14 years. Combined ratios worsened up through 1992, then improved until 1997 and have since worsened again. The model presented here explains all three trends. From 1987 to 1992, the industry was under-pricing, so combined ratios worsened. From 1992 to 1997, the industry overpriced, so combined ratios improved. Starting in 1997, the industry again under-priced and combined ratios worsened.

Note, that the green line (marked with squares) matches the underwriting cycle almost perfectly and gives one year advance notice.

If we bring the data forward to the time of writing, we see the following:

**Graph 7: Same as Graph 6, but with 2001 and 2002 Forecast**



The model properly accounts for the poor underwriting results of 2001 (out of sample) and though prices have increased in 2002, it would not be surprising if 2002 also is a poor year for property and casualty insurers. Indeed, all initial indications are that this will be the case.

Using Equation 12 and a value for  $a$  of 0.00001, we can generate an almost identical graph. This value suggests that after the first million USD, utility is pretty constant. Of course, with the exponential utility function, the value of  $a$  that roughly matches the values for Black Scholes depends on the loss parameters. When the expected loss and standard deviation grow by a factor of 10, the  $a$  that matches falls by a factor of 100. This is due to the characteristics of variance, and as such is an undesirable feature. Ideally, we want the risk aversion parameter,  $a$ , to be a constant for different sizes of losses.

### Assumptions

The assumptions made here are similar to those made by Black and Scholes.

- No payouts before the end of the term.
- Markets are efficient (meaning cash flows can be replicated with other securities).
- No commissions are charged for transactions.
- Interest rates remain constant and are known.
- Losses are log-normally distributed.

It is true that these assumptions - in particular, the second and third assumptions - are somewhat less valid in an insurance market which is, almost without exception, over-the-counter. Most transactions are direct and negotiated person to person. An efficient market does not exist.

But this was also true of equity options prior to the Black Scholes model. While it will clearly take some time, there is no reason to believe that the insurance market will not eventually

blend in with other financial transactions. We have witnessed some of this with catastrophe bond issues and other securitizations, but these concepts have not caught on yet. This is mostly due to the "buyer's market" that has existed in insurance for much of the past decade. When people are willing to sell goods at below fair market value, there is little incentive to pursue fair market value. As prices increase and players charging too little are eliminated, the incentive for fair prices will once again be established.

Regulators are a wild-card. Clearly, barriers to entry, mandated pricing and similar regulatory issues work against efficient insurance markets. Large fees and transaction costs also work against efficiency. Any transition is necessarily slow and incremental, but if our findings here have any relevance, it is just a matter of time.

### ***Put-Call Parity***

In insurance there are no put options per se and that impacts the liquidity and the efficiency of the market. It is not easy to replicate an insurance payoff.

In option theory, being long a stock with price  $S$ , short a call on the stock with strike price  $K$ , and long a put with strike price  $K$ , is equivalent to the discounted strike price. In other words, holding a stock, a put and being short a call is equivalent to holding cash. This is because the options offset the stock payoffs completely. The equation is shown in Equation 17.

#### **Equation 23: Put-Call Parity**

$$C - P + dK = S$$

This also means that we can replicate the stock return with a call, a put and some cash.

In insurance it is not so easy. We do not really have instruments that we can replicate cash flows with. An insurance put option means that if losses came in less than expected, there is a positive payoff from the option. We already know that there is a market for insurance call options, since that is what insurance is. But who would be interested in insurance put options? The exact same entities that are buying call options.

Insurers should be interested in selling loss put options as it is a way to get some extra cash that can offset higher than expected losses. Insured entities should be interested in buying them as it gives them cash back when losses are less than expected. In fact, this gives insurers another incentive to sell put options. A cash incentive below expected losses may make smaller and administratively costly claims less likely to occur.

If such a market develops, it would serve to increase efficiency and, as such, better pricing.

### ***Applying the Theory in Practice***

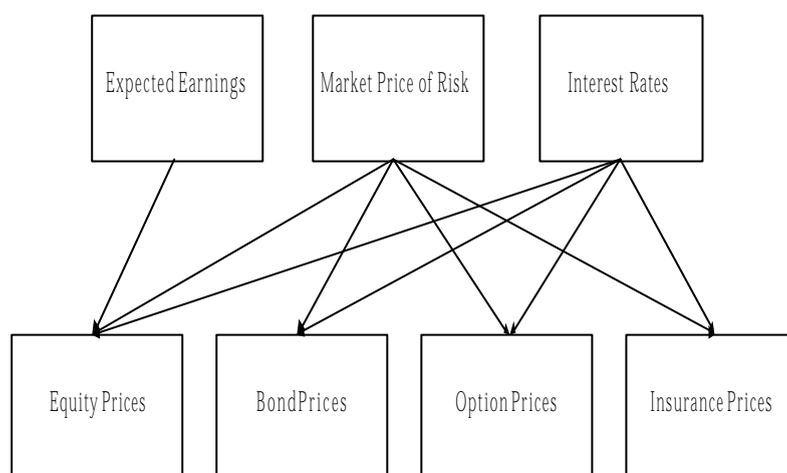
The examples laid out above describe in detail how to use the pricing model. Even if a practitioner does not believe in the pricing method and theory laid out here, the practitioner may still benefit from the model of the underwriting cycle. Of course, to the extent that practitioners use the pricing model and markets become more efficient, it changes the underwriting cycle, but it will be many years.

If the coverage to be priced is a layer, then simply calculate two options prices, as the price of the layer must necessarily be the difference between the two options prices.

If we look at the property and casualty market in segments, there may be pockets of opportunity and perhaps greater insights can be gained from doing such as study. For the time being, we leave that as outside the scope of this paper and suggest it as an area of future study.

### ***Economic Rationale***

The basic rationale is as laid out in the following graphic. The Price of Risk, as defined in this paper, permeates through all financial transactions.



In efficient markets, there must be one price for identical products. Risk can be viewed as one such product and its market price is a component of all risky financial transactions.

### ***Conclusion***

There are several implications of the ideas we have presented here. Firstly, a fairly general theory of the price of risk was developed to create another perspective on the bridge between the asset pricing world and the insurance world. Secondly, this theory was extended to create an insurance pricing model, which in turn was used to model the underwriting cycle. Thirdly, the findings from the underwriting cycle model suggested that the insurance cycle could potentially be explained by systematic over and under pricing by insurers. Indeed, based on the data available, this appears to be the case. The extension of this is that if insurers price the options they are granting correctly, the underwriting cycle, as we know it today, will largely disappear.

The insurance world changes gradually. Indeed, past prophecies of sudden changes in the insurance market have not been realized. Such gradual change is natural, in part because companies are hesitant to undertake new pricing practices if it means sitting out a whole renewal season.

There are more ideas to explore here. The pricing model should be set on a full economic framework, which explains why changes in volatility for pricing purposes are reasonable. The

idea of an integrated financial framework has continued to gain ground. It is our hope that the ideas we have proposed will provide the basis for other classes of insurance pricing models.

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<sup>i</sup> Fixed income is risky at some level. Though a coupon is specified, there is no certain guaranty that the company, or the government for that matter, will not default.

<sup>ii</sup> VIX is actually based on eight S&P 100 options. In general, when dealing with implied volatility, one must consider the volatility smile. We avoid this by using a market based measure based on several options.

<sup>iii</sup> This is an area that could be explored further. In keeping the discounted expected loss constant, we are possibly changing the "true" loss through time. The correct discounted loss to use should perhaps be a risk adjusted discounted value. If we were to implement this, we would also achieve the desirable feature that our option price changes when there is no deductible. But it also complicates the analysis greatly, and makes it more difficult to track. We leave this as an area for further study.

<sup>iv</sup> See references

<sup>v</sup> Since we are cutting the distribution at K, it is not clear that variance is the best risk measure, but we are attempting to specify a framework here. The framework can be expanded to more advanced measures of utility or options pricing, but that is beyond the scope of this paper.

<sup>vi</sup> Technically, there are an infinite number of ways to split the covariance. Without going into further detail here, we borrow a concept from Game Theory known as the Shapley Value, which has many

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desirable qualities. Splitting twice the covariance by assigning one times the covariance to each product reflects this property.

<sup>vii</sup> Irving Fisher, a Professor of Economics at Yale University, made those remarks in 1929 a few weeks before the October crash.

<sup>viii</sup> On September 11<sup>th</sup>, 2001, the World Trade Center and Pentagon terrorist attacks occurred.

<sup>ix</sup> See references.

<sup>x</sup> The Black-Scholes option pricing model tells us that the price of an option is the expected loss with no insurance minus the present value of the attachment point. The higher the risk-free interest rate, the lower the present value of the attachment point, and thus, the higher the price of the option.