The hidden cost of delay in a credit loan portfolio

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Overview

- Accumulated/discounted aggregate losses of a credit loan portfolio
- Shot noise process
- Delay between default occurrence and partial (or full) payment
- Hidden cost of delay in a portfolio and its illustration

Accumulated aggregate losses

- Accumulated value of aggregate losses up to time t is

$$L(t) = \sum_{i=1}^{N(t)} X_i e^{\delta(t-s_i)}$$

where X_i, $i = 1, 2, \cdots$, is the amount of money that banks suffers when i-th borrowers defaults, which are assumed to be independent and identically distributed with the distribution function $H(x)$ ($0 < x < U$), U is the maximum amount of loan to which banks allow borrowers to have their loans. δ is the instantaneous rate of interest, s_i's are time points at which losses occur ($s_i < t < \infty$) and $N(t)$ is the number of losses up to time t, which follows the Poisson process with arrival rate ρ.

Accumulated aggregate losses with delay

- In practice, the case that the loss amounts would be paid immediately after default occurrences is highly rare. Considering the delay between default occurrence and final settlement, denoted by τ $(0 \leq \tau < \infty)$, which is independent of s, accumulated value of aggregate losses paid with delay is given by

$$L_{\mathcal{T}}(t) = \sum_{i=1}^{N(t)} X_i e^{\delta(t-s_i-\tau_i)}.$$

- And its expectation is given by $E\left\{L_{\mathcal{T}}(t)\right\} = E_{\mathcal{T}}\left[E\left\{L(t-\tau)\right\}\right].$

Accumulated aggregate losses with delay and recovery

- Also in reality, the eventual loss amounts paid at settlement can be different from the loss amounts at default. As the worst case, the final payment could be 0 but in most case, the bank can recover the part (or whole) of losses after the liquidation of borrowers' assets. So if we consider the recovery rate, κ_i which is an independent, identical random variable, accumulated aggregate losses paid with delay and recovery is given by

$$L_{\tau,\kappa}(t) = \sum_{i=1}^{N(t)} \kappa_i X_i e^{\delta(t-s_i-\tau_i)}.$$

- And its expectation is given by $E\left\{L_{\tau,\kappa}(t)\right\} = E_{\tau}\left[E\left\{L_{\kappa}(t-\tau)\right\}\right].$

Discounted aggregate losses with delay and recovery

- Multiply $e^{-\delta t}$ both sides in the above equation, it becomes the discounted value of aggregate losses up to time t, denoted by $L^{0}_{\tau,\kappa}(t) = e^{-\delta t} L_{\tau,\kappa}(t)$ and its expectation is given by

$$E\left\{L^{0}_{\tau,\kappa}(t)\right\}.$$

Shot noise process

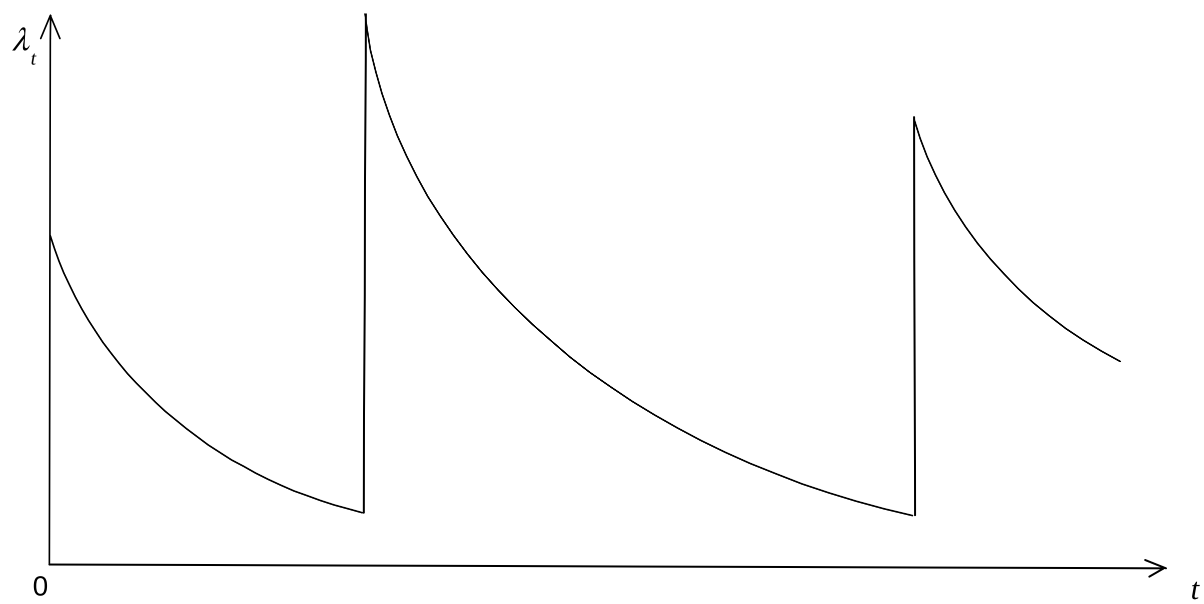
- We will adopt the shot noise process used by Cox & Isham (1980):

$$\lambda_t = \lambda_0 e^{-\delta t} + \sum_{\substack{\text{all } i \\ s_i \leq t}}^{N(t)} y_i e^{-\delta(t-s_i)}$$

  where:

λ_0	initial value of λ
y_i	jump size of primary event, where $E(y_i) < \infty$
s_i	time at which primary event i occurs, where $s_i < t < \infty$
δ	exponential decay
ρ	the rate of primary event arrival.

Graph illustrating shot noise process



Duality of the accumulated losses processes and shot noise

- Set $-\delta$ to δ and substitute y with X then it becomes $\lambda(t) = \lambda(0)e^{\delta t} + \sum X_i e^{\delta(t-s_i)}$ and assume that $\lambda(0)$, that can be considered as the total losses up to present time 0, is 0, then interestingly, we can see that it is equivalent to L_t, that is the accumulated value of aggregate losses, i.e.

$$L(t) = \sum_{i=1}^{N_t} X_i e^{\delta(t-s_i)}$$

- The decay rate δ is now is the instantaneous rate of interest.

The generator of the process $(L(t),\, t)$

- The generator of the process $(L(t), t)$ acting on a function $f(l, t)$ belonging to its domain is given by

$$
\begin{aligned}
& A\ f(l,t) \\
= \;& \lim_{dt \downarrow 0} \frac{E[f(L(t+dt), t+dt) \mid L(t) = l] - f(l,t)}{dt} \\
= \;& \frac{\partial f}{\partial t} + \delta l \frac{\partial f}{\partial l} + \rho \left[\int_0^U f\,(l+x,,t)\, dH\,(x) - f\,(l,t) \right].
\end{aligned}
$$

Loss size, delay and recovery distributions

- Let us assume that the loss size distribution follows truncated exponential, i.e. $h_U(x) = \left(\frac{1}{1-e^{-\alpha U}}\right) \alpha e^{-\alpha x}$, $0 < x < U$ and $\alpha > 0$

- and the delay between default occurrence and final settlement follows exponential distribution, i.e. $j(\tau) = \beta e^{-\beta x}$, $\tau > 0$ and $\beta > 0$

- and the recovery rate follows Beta distribution, i.e.

$$u(\kappa) = \frac{\Gamma(\gamma+\xi)}{\Gamma(\gamma)\Gamma(\xi)} \kappa^{\gamma-1} (1-\kappa)^{\xi-1}, \ 0 < \kappa < 1, \ \gamma > 0, \ \xi > 0.$$

The expectation of discounted aggregate losses

- The expectation of discounted value of aggregate losses is given by

$$E\left\{L^{0}(t)\right\} = \left(\frac{\rho}{1-e^{-\alpha U}}\right)\left\{\frac{1-e^{-\alpha U}(1+\alpha U)}{\alpha}\right\}\bar{a}_{t|}$$

where $\bar{a}_{t|} = \frac{1-e^{-\delta t}}{\delta}$.

The expectation of discounted aggregate losses with delay and full recovery

- The expectation of discounted value of aggregate losses paid fully with delay between default occurrence and final settlement is given by

$$
\begin{aligned}
E\left\{L_{\tau}^{0}(t)\right\} &= \left(\frac{\rho}{1-e^{-\alpha U}}\right)\left\{\frac{1-e^{-\alpha U}(1+\alpha U)}{\alpha}\right\}\left(\frac{\beta}{\beta+\delta}\right)\bar{a}_{t|} \\
&\quad -\left(\frac{\rho}{1-e^{-\alpha U}}\right)\left\{\frac{1-e^{-\alpha U}(1+\alpha U)}{\alpha}\right\}\frac{1}{(\beta+\delta)}e^{-\delta t}
\end{aligned}
$$

where $\beta \geq \frac{1}{t}$.

The expectation of discounted aggregate losses with delay and partial recovery

- The expectation of discounted value of aggregate losses paid partially with delay between default occurrence and final settlement is given by

$$
\begin{aligned}
& E\left\{L^{0}_{\tau,\kappa}(t)\right\} \\
= \; & \left(\frac{\rho}{1-e^{-\alpha U}}\right)\left\{\frac{1-e^{-\alpha U}(1+\alpha U)}{\alpha}\right\}\left(\frac{\gamma}{\gamma+\xi}\right)\left(\frac{\beta}{\beta+\delta}\right)\bar{a}_{t|} \\
& -\left(\frac{\rho}{1-e^{-\alpha U}}\right)\left\{\frac{1-e^{-\alpha U}(1+\alpha U)}{\alpha}\right\}\left(\frac{\gamma}{\gamma+\xi}\right)\frac{1}{(\beta+\delta)}e^{-\delta t}.
\end{aligned}
$$

The predictor of the hidden cost of delay with full recovery

- Let us define the predictor of the hidden cost of delay with full recovery as

$$E\left\{L^0(t)\right\} - E\left\{L^0_\tau(t)\right\} = \left(\frac{\rho}{1-e^{-\alpha U}}\right)\left\{\frac{1-e^{-\alpha U}(1+\alpha U)}{\alpha}\right\}$$
$$\times\left[\left\{1-\left(\frac{\beta}{\beta+\delta}\right)\right\}\bar{a}_{t|} + \frac{1}{(\beta+\delta)}e^{-\delta t}\right].$$

The predictor of the hidden cost of delay with partial recovery

- And define the predictor of the hidden cost of delay with partial recovery as

$$E\left\{L^0(t)\right\} - E\left\{L^0_{\tau,\kappa}(t)\right\} = \left(\frac{\rho}{1-e^{-\alpha U}}\right)\left\{\frac{1-e^{-\alpha U}(1+\alpha U)}{\alpha}\right\}$$
$$\times\left[\left\{1-\left(\frac{\gamma}{\gamma+\xi}\right)\left(\frac{\beta}{\beta+\delta}\right)\right\}\bar{a}_{t|} + \left(\frac{\gamma}{\gamma+\xi}\right)\frac{1}{(\beta+\delta)}e^{-\delta t}\right].$$

Numerical examples

- Now let us illustrate the calculations of hidden cost of delay, assuming that $\alpha = 0.00001$, $\beta = 3$, $\gamma = 10$, $\xi = 10$, $\delta = 0.05$, $\rho = 5$, $U = 1,000,000$ and $t = 1$.

Example 1

- The calculations of the predictors of the hidden cost of delay are shown in Table 1:

Table 1

$E\left\{L^0(t)\right\} - E\left\{L^0_\tau(t)\right\} = \left(\frac{\rho}{1-e^{-\alpha U}}\right)\left\{\frac{1-e^{-\alpha U}(1+\alpha U)}{\alpha}\right\} \times \left[\left\{1-\left(\frac{\beta}{\beta+\delta}\right)\right\}\bar{a}_{t\|} + \frac{1}{(\beta+\delta)}e^{-\delta t}\right]$	163, 860
$E\left\{L^0(t)\right\} - E\left\{L^0_{\tau,\kappa}(t)\right\} = \left(\frac{\rho}{1-e^{-\alpha U}}\right)\left\{\frac{1-e^{-\alpha U}(1+\alpha U)}{\alpha}\right\} \times \left[\left\{1-\left(\frac{\gamma}{\gamma+\xi}\right)\left(\frac{\beta}{\beta+\delta}\right)\right\}\bar{a}_{t\|} + \left(\frac{\gamma}{\gamma+\xi}\right)\frac{1}{(\beta+\delta)}e^{-\delta t}\right]$	325, 670

<u>Example 2</u>

- The calculations of the predictor of the hidden cost of delay at each value of δ and β with partial recovery are shown in Table 2:

Table 2

	$\delta = 0.03$	$\delta = 0.05$	$\delta = 0.07$
$\beta = 10$	$271,090$	$268,610$	$266,200$
$\beta = 3$	$328,650$	$325,670$	$322,740$
$\beta = 1$	$488,780$	$481,730$	$474,880$

Example 3

- The calculations of the predictor of the hidden cost of delay at each value of γ and ξ with partial recovery are shown in Table 3:

Table 3

$\gamma = 1.5$ and $\xi = 28.5$	$471,300$
$\gamma = 10$ and $\xi = 10$	$325,670$
$\gamma = 14$ and $\xi = 6$	$260,950$