Fair valuation of life insurance contracts:
The interaction between assets and liabilities

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Abstract

In this paper we have taken the work of Briys and De Varenne (1997) and the papers of Grosen and Jorgenson (2000, 2001) as a starting point. The paper of Briys and De Varenne only deals with the risk free world, which is used to value embedded options. The real world is not modeled and therefore risk/return profiles cannot be derived. We have expanded the Briys and Varenne model in such a way that this real world can also be analyzed. We changed the Grosen and Jorgenson model to allow for stochastic interest rates resulting in a term structure that is used for valuation purposes. The model then becomes more practical applicable. Combining the theoretical framework with the estimated parameters, the empirical study shows that it is necessary that the equity holder has enough space to participate in this contract. Small values of equity are very profitable from the point of view of the policyholder and are less attractive seen from the position of the equity holder. Increasing the time horizon and lowering the offered return increases the profits for the equity holder. However in the scenario presented in this paper this is not enough to generate a risk/return profile that can be characterized as interesting for both parties.

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1. Introduction

Institutions like pension funds and insurance companies face a world of severe competition in which return is of primary importance. However, return and risk always stick together. Risk is disliked and carefully looked at, not only by the institutions but also by the regulator. In order to meet the return requirements of the equity holders on the one hand and to stay in line with the solvency requirements of the regulator on the other hand an integrated approach in which both risk and return are taken into account is necessary. The outcome of this approach is a balanced policy in which return is attractive and risk is acceptable. Asset Liability Management (ALM) is the appropriate tool to design such a policy. One of the basic problems is how to quantify risk. Quantifications often used are the probability of under funding and downside deviation (see Sortino and Van der Meer, 1991). Although these quantifications have value, they yield no information about the real financial value of risk. Briys and De Varenne (1997) developed a basic ALM-model in which risk is valued against market prices.

The basic idea behind their model is that the features of liabilities can be modeled and priced within an option framework. Grosen and Jorgenson (2000,2001) used the same concept but they value the liabilities by means of a fixed interest rate. In their point of view the liabilities are more an accounting value than a market value and therefore is their approach less applicable in determining fair values. In the Briys and De Varenne model there is one important proposition in the model namely that there is only one moment in time that is of importance and that is the moment of expiration of the contract. What happens in between is of no importance. In Grosen and Jorgenson (2001) there is an extension in the sense that funding levels in between have to have minimal values. But – as mentioned before - the framework is set up in a book value environment and therefore less useful in a market value context.

The aim of the article presented here is to extend both the Briys and De Varenne model as the Grosen and Jorgenson model by introducing intertemporal risk levels in a market value context. This extension implies that it is no longer sufficient to model the economic world only under the risk neutral probability measure Q (see e.g. Harrison and Kreps (1979)) but also under the real world probability measure P (see e.g. Jarrow and Turnbull (2000)). Under this measure P it is possible to simulate the institutions real financial position on every moment in time between 0 and T, the moment of expiration of the contract.

The outline of the paper is as follows. Section 2 describes the general framework in which assets, liabilities and equity are modeled. The valuation of the liabilities is based on the concept of fair valuation. In Section 3 estimation of the model as proposed in the previous section takes place. Therefore a term structure model based on the Vasicek-model is used. Estimation of the parameters in the model is based on De Munnick and Schotman (1994). The dynamics of the stock market are taken from Briys and De Varenne (1997). Furthermore, the numerical procedure to simulate the institutions financial position through time is described. In Section 4 a wide range of numerical simulation experiences are presented and discussed. Finally, Section 5 concludes with a summary of the results and with topics for further research.

2 The general framework

In this section the insurance contract under investigation is described. The contract is issued at t=0 and matures at t=T so we are interested in the value of the contract over the time interval [0,T]. The contract under investigation is kept simple for reasons of simplicity and has the same features as the contract used by Briys and De Varenne (1997). The characteristics of this contract are that the policyholder has to pay a contribution L_0^C. The policyholder is offered a payout L_T^* at expiration date, implying a base return $r_{offered}$ for the length of the contract, which implies that $L_0^C = L_T^* e^{-r_{offered}T}$. On top of the base return the policyholder is entitled to a share of $\alpha\delta$ of the net financial revenues. This participation coefficient δ can be viewed as making up for the difference between the base return and the return on a risk-free zero-coupon bond with the same time-to-maturity T. This embodies the required risk premium by the policyholders holding risky life insurance contracts. Indeed, shareholders have a limited liability and if the company is declared insolvent at maturity date they simply walk away. Furthermore it is assumed that there is no cross subsidizing, so that the contribution paid is equivalent to the market value (called the fair value) of the contract L_0 at that moment. The shareholders fair value, the equity, equals E_0. Adding the policyholders' contribution L_0^C to the shareholders value E_0 results in the value of the firm, described by A_0. The proportion of the policyholders' contribution with respect to the total value A_0 is equal to the leverage ratio α.

The resulting balance sheet at the issuing date (t=0) is presented in Table 1.

Table 1 The institutions balance sheet at time t=0

Assets		Liabilities and equity	
Assets	A_0	Liabilities	$L_0 = L_0^C = \alpha A_0$
		Equity	$E_0 = (1-\alpha) A_0$
Total	A_0	Total	A_0

In order to be able to determine the fair value of the contract over the whole interval [0,T] we distinguish the issue date (t=0), the maturity date (t=T) and the period in between. The first moment under consideration is the maturity date T. At this point there is no uncertainty and the value of the contract is known with certainty. Based on the possible outcomes at maturity, the present value of the contract at the issue date can then be derived. Finally, the intermediate value of the contract is determined.

2.1 The contract and the balance sheet at maturity

Valuation of the contract is straightforward once we know how the final value of the assets is distributed between the policyholder and the equity holder. The policyholder is paid first. If the value of the assets A_T is smaller than or equal to L_T^*, then the policyholder receives all the assets. After that, the equity holder is paid up to a return that is equal to the base return on the policy. The latter break-even point is reached if the value of the assets equals L_T^*/α. If the value of the assets exceeds L_T^* /α a fraction of δα of the remainder is distributed to the policyholder and a fraction of 1-δα to the equity holder.

To determine the value of the contract at maturity, we consider three situations. In the first situation the return on the assets has been sufficient to yield a return for the equity holders that is greater than or equal to the base return. In the second case, the return on the assets has been high enough to

If the value of the assets at the maturity date, A_T, is equal to L_T^* /α, the offered return equals the asset return exactly. There is no bonus and the equity holders receive the same return as the policyholders. If A_T exceeds L_T^* /α the difference is labeled the bonus surplus. The policyholder receives a fraction $\delta\alpha$ of this bonus surplus. Because negative values are not allowed for a bonus, the payout then equals:

$$B_T = \delta\alpha \max\left[0, A_T - \frac{L_T^*}{\alpha}\right] \tag{1}$$

A closer look at the liability side shows that there are three possible situations. In the first situation things have worked out very well in a sense that $A_T > L_T^*$ /α. The policyholder gets its agreed payout L_T^* and a bonus. The final payout to the policyholder L_T can be written as:

$$\begin{aligned} L_T &= L_T^* + B_T \\ &= L_T^* + \delta\alpha \max(0, A_T - \frac{L_T^*}{\alpha}) \\ &= \delta\alpha A_T + (1-\delta) L_T^* \end{aligned} \tag{2}$$

For the equity holder the final payout E_T is then defined as:

$$\begin{aligned} E_T &= A_T - L_T \\ &= (1-\delta\alpha) A_T - (1-\delta) L_T^* \end{aligned} \tag{3}$$

In the second situation it holds that $L_T^* / \alpha > A_T > L_T^*$. There are enough assets to fulfill the committed payout but there are no additional funds for a bonus payout. The payout to the policyholder is then:

$$L_T = L_T^* \tag{4}$$

and for the equity holders remains

$$E_T = A_T - L_T^* \tag{5}$$

In the third situation $A_T < L_T^*$. The policyholder receives the available amount of funds A_T and for the equity holder there are no funds anymore. This implies that:

$$\begin{aligned} L_T &= A_T \\ E_T &= 0 \end{aligned} \tag{6}$$

The final payout to the policyholder can than be written as:

$$\begin{aligned} L_T &= \min\left[A_T, L_T^*\right] + B_T \\ L_T &= L_T^* - \max\left[0, L_T^* - A_T\right] + \delta\alpha \max\left[0, A_T - \frac{L_T^*}{\alpha}\right] \end{aligned} \tag{7}$$

Note that $\max\left[0, L_T^* - A_T\right]$ is the pay-off of a put option on A_T with exercise price L_T^* and $\max\left[0, A_T - \frac{L_T^*}{\alpha}\right]$ is the pay-off of a call option on A_T with exercise price $\frac{L_T^*}{\alpha}$.

For the equity holder equivalent equations can be derived:

$$E_T = A_T - L_T$$

$$= A_T - L_T^* + \max\left[0, L_T^* - A_T\right] - \delta\alpha \max\left[0, A_T - \frac{L_T^*}{\alpha}\right]$$

$$E_T = \max\left[0, A_T - L_T^*\right] - \delta\alpha \max\left[0, A_T - \frac{L_T^*}{\alpha}\right] \qquad (8)$$

2.2 Theoretical market value of liabilities and equity at origination

At origination the insurance contract specifications have to be defined. After having done this, standard valuation techniques using a risk-neutral measure Q can value the liabilities at origination in the following way:

$$L_0 = L_T^*\ P(0,T) - Put(A_0, L_T^*) + \delta\alpha Call(A_0, \frac{L_T^*}{\alpha}) \qquad (9)$$

in which $P(t,T)$ is the time-t price of a zero-coupon risk-free bond with T-t years time-to-maturity[3], $Put\ (A_0,\ L_T^*)$ is the price of an European put option, with exercise price L_T^* and the value of the assets A_0 at t=0 and $Call\ (A_0,\ L_T^*/\alpha)$ is the price of a European call option, with exercise price L_T^*/α and the value of the assets A_0 at t=0. At t=0, $P(0,T)$ is known and it is equal to the price of a zero-coupon bond with a time-to-maturity of T years. Hence $P(0,T) = e^{-r_0(T)T}$ where $r_0(T)$ is the T-year zero-coupon interest-rate at t=0.

Similarly the fair value of the contract of the equity holder at t=0 can be written as:

$$E_0 = Call(A_0, L_T^*) - \delta\alpha Call(A_0, L_T^* / \alpha) \qquad (10)$$

2.3 Parameter restrictions due to the absence of cross subsidizing

As we have seen δ can be viewed as a risk premium in order to compensate the policyholder for the risk he is carrying. This means that the contribution paid by

[3] The special case where t=0 is used in the formula presented here. Later on we also need the notation $P(t,T)$ for t>0.

the policyholder has to be in line with value of the contract. Seen from the point of view of the equity holder (*1-*δ) is to compensate him for the risk of losing all the invested funds.

If the contribution of the policyholder and the equity holder are equal to the market value of the contract, the contract is characterized as being fair. This implies, as stated before, that $L_0 = L_0^C$. If one assumes that one cannot influence the development of A_t – which is usaul in real practice - , the final payout at the maturity of the contract only depends on the parameters chosen within the insurance contract namely δ, α and $r_{offered}$. If the offered rate $r_{offered}$ and α are fixed the remaining parameter δ can be calculated by:

$$L_0 = L_0^C \Leftrightarrow$$

$$L_T^* P(0,T) - Put(A_0, L_T^*) + \delta\alpha Call(A_0, \frac{L_T^*}{\alpha}) = L_0^C$$

which leads to:

$$\delta = \frac{L_0^C - L_T^* P(0,T) + Put(A_0, L_T^*)}{\alpha Call(A_0, \frac{L_T^*}{\alpha})} \tag{11}$$

or equivalently:

$$\delta = \frac{L_T^* (e^{-r_{offered}T} - P(0,T)) + Put(A_0, L_T^*)}{\alpha Call(A_0, \frac{L_T^*}{\alpha})} \tag{12}$$

If the offered return is lower than the zero coupon yield there has to be profit sharing (δ>0) in order to achieve fairness. An offered return that is larger would result in δ<0 so then no fair contract is possible.

2.4 Fair values of liabilities and equity during the life of the contract

The above equations define and describe the fair value of the assets and the liabilities at origination and at maturity. What we are additionally interested in is the intertemporal value of equity and liabilities at time t (0<t<T). The theoretical framework presented in section 2.2 and the additions made in section 2.3 provide the necessary tools to come to the intertemporal valuation, which results in:

$$L_t = L_T^* P(t,T) - Put(A_t, L_T^*) + \delta\alpha Call(A_t, \frac{L_T^*}{\alpha}) \quad (13)$$

$$E_t = Call(A_t, L_T^*) - \delta\alpha Call(A_t, \frac{L_T^*}{\alpha}) \quad (14)$$

In these formulas, *P(t,T)* and A_t refer to the price of a T-t period risk-free zero-coupon bond and the value of the assets at time t. The general valuation framework is now completed. In the next section we impose more structure on the model to describe the evolution of $P(t,T)$, $Put(A_t, L_T^*)$, and $Call(A_t, \frac{L_T^*}{\alpha})$ over time.

3. Asset Modeling

In Section 2 a general framework for the valuation of the liabilities and equity of a very simple insurance company was presented. As explained in Section 2.4, the fair value of the liabilities depends on the parameters (α,δ,$r_{offered}$), the price of a risk-free zero-coupon bond *P(t,T)*, the value of the assets at time t, A_t and the parameters governing the processes for *P(t,T)* and A_t. The model in Section 2 is general in the sense that the functional forms is not yet specified for the call- and put option in the fair value of the liabilities. Therefore this section focuses on two issues.

Firstly we discuss the model we use for the evolution of *P(t,T)* and A_t over time. Secondly we discuss the functional forms that are used to derive the value of the remaining components in the fair value of the liabilities, i.e. the call- and put option within the liability equation. These two issues have a close relationship because the first issue requires models that describe the evolution of *P(t,T)* and A_t under the real-world measure *P* whereas the second issue is solved using the risk-neutral measure Q. Estimation of the parameters will be the topic of the next section.

3.1 Asset Modeling under *P*

The most common approach to model the term structure of interest rates starts with the specification of a time series process to describe the behavior of the short term interest rate $r_t(0)$ over time. We follow this approach to model the process of $P(t,T)$ and assume that the Vasicek model is a good representation for the behavior of the short term interest rate. Furthermore it is assumed that the asset portfolio follows a Brownian motion. The extension to the Briys and De Varenne model is that not only two moments in time are examined, namely t=0 (which would only require processes under the risk neutral measure Q) and t=T (when the value of the assets and liabilities are known), but that the whole time interval [0,T] is taken into account. Because of this extension it is no longer sufficient to model only under the measure Q, but also under the measure P. Under the above defined assumptions the processes for the instantaneous short-rate and the value of the assets in the real-world measure P are:

$$
\begin{aligned}
dr_t(0) &= a(\mu - r_t(0))dt + \sigma dW_t^P \\
dA_t &= \mu_A A_t dt + \sigma_A A_t \left(\rho dW_t^P + \sqrt{1-\rho^2}\, dZ_t^P\right)
\end{aligned}
\qquad (15)
$$

Within the short-rate equation a is the mean reversion parameter, μ the unconditional mean of $r_t(0)$, σ the volatility of the short-rate and dW_t^P the geometric Brownian motion under the real-world measure P. Within the asset equation μ_A en σ_A denote respectively the annualized expected return on assets and their annualized volatility and dZ_t^P is again a geometric Brownian motion under the real-world measure P. The coefficient ρ represents the correlation between the total value of assets and the interest rate. For given values of $r_0(0)$ and A_0 and the parameters $(a, \mu, \sigma, \mu_A, \sigma_A, \rho)$, these equations govern the process of $r_t(0)$ and A_t, respectively. In order to price the liabilities and equity specified in

(13) and (14) it is no longer sufficient to work within the real world *P* but valuation in the risk neutral world *Q* is necessary.

3.1 Asset Modeling under *Q*

In this paragraph we introduce the concept of risk neutral interest rates and the relationship between this concept and the valuation of zero coupon bonds and the call- en put options. In a risk neutral world the expected return on all bonds is the risk-free rate. If we assume that observable interest rates move according to the Vasicek model stochastic process above, we have to specify the stochastic process under the risk neutral measure *Q*. It can be shown (see e.g. De Munnik and Schotman, 1994) that this risk neutral process for the short rate as of time t - denoted by the tilde - can be formulated as follows.

$$d\tilde{r}_t(0) = [a(\mu + \frac{\lambda\sigma}{a} - \tilde{r}_t(0))]dt + \sigma\, dW_t^Q \qquad (16)$$

or

$$d\tilde{r}_t(0) = a(\tilde{\mu} - \tilde{r}_t(0))dt + \sigma dW_t^Q \qquad (17)$$

with

$$\tilde{\mu} = \mu + \frac{\lambda\sigma}{a} \qquad (18)$$

λ is the market price of risk[4] and the starting value of the risk-neutral process is equal to the short-rate at time t, i.e. $\tilde{r}_t(0) = r_t(0)$.

In the Vasicek model it can be shown that *P(t,T)* is an exponential function of the short-rate with:

$$P(t,T) = e^{-r_t(T-t)(T-t)} = e^{-F(T-t)r_t(0)-G(T-t)} \qquad (19)$$

[4] Using this definition of the market price of risk we stick to Vasicek (1977). De Munnik and Schotman (1994) use a different definition for the market price of risk.

where $r_t(T-t)$ is the *(T−t)*-period yield-to-maturity at time t and *F(T−t)* and *G(T−t)* are known functions of the remaining time-to-maturity of the bond and the parameters *a,* $\tilde{\mu}$ and *σ* as specified next.

$$F(\tau)=\frac{1}{a}\left(1-e^{-a\tau}\right) \tag{20}$$

$$\begin{aligned} G(\tau) &= \left(\tilde{\mu}-\frac{\sigma^2}{2a^2}\right)(\tau-F(\tau))+\frac{\sigma^2}{4a^2}(2F(\tau)-F(2\tau)) \\ &= \left(\tilde{\mu}-\frac{\sigma^2}{2a^2}\right)(\tau-F(\tau))+\frac{\sigma^2}{4a}F^2(\tau) \end{aligned} \tag{21}$$

The (T−t)-period interest rate is defined as

$$r_t(T-t)=-\frac{1}{T-t}\ln[P(t,T)] \quad (22)$$

In the Vasicek Model this reduces to:

$$r_t(T-t)=-\frac{1}{T-t}[-r_t(0)F(T-t)-G(T-t)]=\frac{F(T-t)}{T-t}r_t(0)+\frac{G(T-t)}{T-t}$$

As time to maturity gets infinitely long, for the corresponding interest-rate $r_t(\infty)$ it holds that

$$r_t(\infty)=\tilde{\mu}-\frac{\sigma^2}{2a^2}\equiv r(\infty). \tag{23}$$

Now for $G(\tau)$ it holds that it can be rewritten as

$$G(\tau)=(\tau-F(\tau))r(\infty)+\frac{\sigma^2}{4a}F^2(\tau)\,. \tag{24}$$

In general, $r(\infty)$ is positive, $\tau-F(\tau)$ is positive for positive τ and the mean reversion parameter a is positive. Hence $G(\tau)$ is positive for positive τ. A positive a also implies that $F(\tau)$ is positive for positive τ. When $r_t(0)$ is positive, all this implies that risk-free bond prices are in between zero and one.

To describe the term structure of risk-free interest rates following from the Vasicek model we first rewrite the expression for $P(t,T)$:

$$P(t,T) = \exp\left[-F(T-t) r_t(0) - (T-t-F(T-t)) r(\infty) - \frac{\sigma^2}{4a} F^2(T-t) \right] \quad (25)$$

and hence

$$r_t(T-t) = \frac{F(T-t)}{T-t} r_t(0) + \left(1 - \frac{F(T-t)}{T-t}\right) r(\infty) + \frac{\sigma^2}{4a} \frac{F^2(T-t)}{T-t} \quad (26)$$

When determining the value for the call- and put options in (13) and (14) Black and Scholes (1973) and Merton (1973) show that

$$Put(A_t, L_T^*) = -A_t N(-d_1) + P(t,T) L_T^* N(-d_2) \quad (27)$$

$$Call(A_t, L_T^*) = A_t N(d_1) - P(t,T) L_T^* N(d_2) \quad (28)$$

where

$$d_1 = \frac{\ln\left(\frac{A_t}{P(t,T) L_T^*} \right) + \bar{\sigma}^2(t,T)(T-t)/2}{\bar{\sigma}(t,T)\sqrt{(T-t)}} \quad (29)$$

$$d_2 = d_1 - \bar{\sigma}(t,T)\sqrt{(T-t)} \quad (30)$$

$$\begin{aligned} &\bar{\sigma}^2(t,T) \\ &= \sigma_A^2 + \left(\frac{2\rho\sigma_A\sigma}{a} + \frac{\sigma^2}{a^2} \right)\left(1 - \frac{F(T-t)}{T-t}\right) + \frac{\sigma^2}{2a^2}\left(\frac{F(2(T-t))}{T-t} - 2\frac{F(T-t)}{T-t} \right) \quad (31) \\ &= \sigma_A^2 + \left(\frac{2\rho\sigma_A\sigma}{a} + \frac{\sigma^2}{a^2} \right)\left(1 - \frac{F(T-t)}{T-t}\right) - \frac{\sigma^2}{2a} \frac{F^2(T-t)}{T-t} \end{aligned}$$

Note that these expressions explicitly involve the parameters under the risk-neutral measure *Q*. However, implicitly, the parameters under the real-world measure are involved in the computations as well because they determine the values for A_t and, through $r_t(0)$, *P(t,T)* that appear on the right-hand-side of the equations (27) and (28).

4. Data and Parameter Estimation

The empirical problem is to estimate the key parameters ($a, \mu, \tilde{\mu}, \sigma, \mu_A, \sigma_A, \rho$) of the various models. The usual procedure is time series estimation. Estimation of a mean reversion parameter for the short-term interest rate will be very difficult with time series data. The alternative is to use cross-sectional estimation procedures using bond prices at a single instant in time. Both procedures have their merits and drawbacks. Estimation of a mean reversion parameter for the short term interest rate will be very difficult with time series data, because one requires many observations spanning a large number of years in order to estimate the mean reversion parameter in even the simples model with only first order dynamics[5]. But a long time series creates its own problems, since the empirical model has to cope with structural breaks. Furthermore, a time series model that fits the short-term interest rate well, does not necessarily provide a good fit for the yield curve. Cross sectional estimation has two advantages, since the estimated model will provide the best possible fit to the term structure, and because we do not need more data than just one day, if enough different bonds are traded on the market. Cross-section data thus have our preference because they incorporate all information and they are forward looking instead of backward looking as in time-series. One problem is that the parameter estimates are often nonsensical (negative variances, explosive models, etc.). Another problem is that the parameters are not very stable over time. Ideally, if the simple one-factor models for the term structure were true, the differences between time series and cross sectional estimates should be small. Another problem is that the short rate is not observable. The shortest available maturity is a one-moth interest rate. For time series estimation we need a discrete time representation of the spot rate process. Since the yield on all discount bond is a simple linear function of the instantaneous risk free rate, the time series process of the one moth rate can be derived from the discrete process of the instantaneous spot rate in the Vasicek model. As shown by De Munnik and Schotman (1994) this discrete process can be formulated as follows.

[5] See Campbell and Perron (1991) for more details.

$$r_{t+\tau}(s) = b_0 + b_1[r_t(s) - b_0] + \eta_{t,t+\tau}(s)$$

with

$$b_0 = r(\infty) + \frac{1-e^{-as}}{as}(\mu - r(\infty)) + \sigma^2 \frac{\left(1-e^{-as}\right)^2}{4a^3 s}$$

$$b_1 = e^{-a\tau}$$

$$E_t[\eta_{t,t+\tau}(s)^2] = \sigma^2 w_1(s)^2 \frac{1-e^{-2a\tau}}{2a}$$

$$w_1(s) = \frac{1-e^{-as}}{as}$$

Not all the parameters of the Vasicek model can be identified. The identifiable parameters are a and σ^2. Neither the unconditional mean μ, nor the infinite maturity yield $r(\infty)$ can be identified. Therefore the time series estimates are not sufficient to construct cross sectional term structures or to price bond options. As De Munnik and Schotman (1994) suggest we use cross sectional data to estimate the parameters (a, $\tilde{\mu}$, σ) of the Vasicek model. They are estimated by minimizing the unweighted squared difference between the observed prices and the theoretical prices. Because the government bonds are a sum of cash flows, the theoretical price is simply the sum of these discounted cash flows. The discount factors follow directly from $P(t,t+\tau)$ where τ is the number of periods at which the future cash flows take place. However, as noted in De Munnik and Schotman (1994), the resulting objective function is generally flat in σ. Therefore we first estimate σ using time-series data. Again we use the approach described in section 2.3 of De Munnik and Schotman (1994) which amounts to an AR(1) model for the 1-month interest rate. In the model for the short-rate, the parameter $\tilde{\mu}$ is now the only remaining parameter to estimate. It follows from the intercept in the aforementioned AR(1) regression. As De Munnik and Schotman (1994) show, using 1-month interest rates, one can estimate b_0, a, $\tilde{\mu}$, σ and an estimate for μ follows.

To estimate the parameters μ_A and σ_A we use historical time-series on A_t. Note that this series depends on the asset mix. For a given asset mix and time-series on the individual assets it is straightforward to estimate μ_A and σ_A. Details can be found in the Appendix.

The final parameter to estimate is ρ. This parameter governs the correlation between the Brownian-motions in the continuous-time processes for the short-rate and the assets. Two issues have to be dealt with in estimation. Firstly, the short-rate is not observed. However, given the parameter estimates, a time-series for the short-rate can be estimated, see the appendix for more details. Secondly, we observe data in discrete-time. This also affects the equations on which estimation of the parameter ρ is based. Again, see the appendix for more details.

4.1 The data

When estimating the parameters using data, we use cross-section data related to 28 February 2002, provided by JP Morgan on government bonds in their EMU bond index. This index includes government bonds issued by all EMU countries. We first examine the data graphically. When plotting the yield-to-maturities for each country, substantial differences between the countries prevail. Therefore we choose to use a single country. Germany is a first candidate because it is often used when representing the EMU. However, some of the German bonds are special in the sense that they are used in Bund-futures. For these bonds, the yield-to-maturities are lower than for bonds without this special feature. In addition to these explainable outliers the German data also exhibit some additional outliers in the 2.5−3 year and the over 26 year time-to-maturity range, see Figure 1.

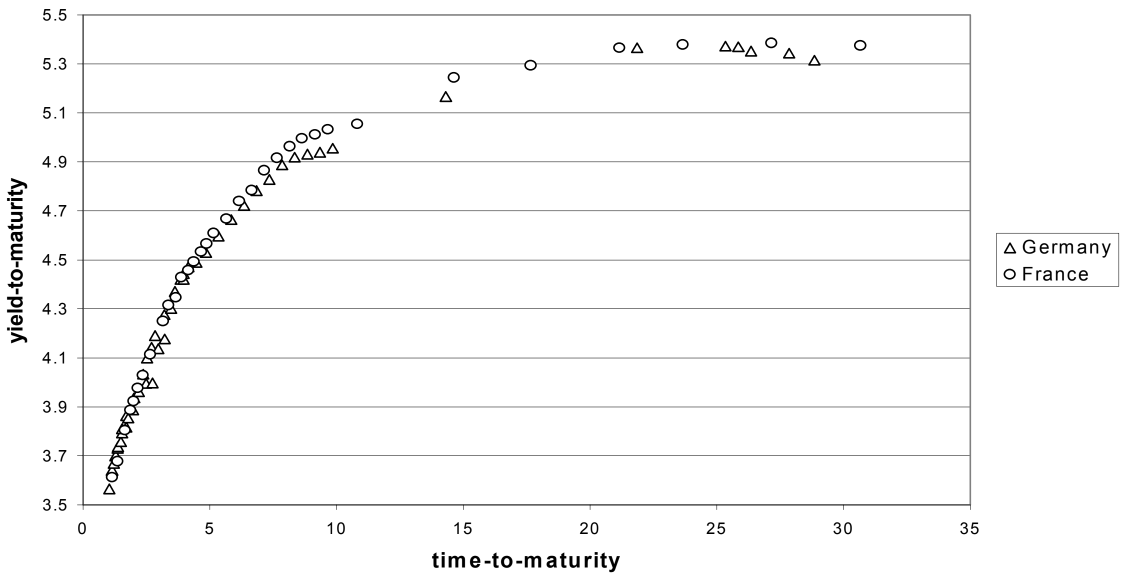


Figure 1: Yield-to-maturity versus time-to-maturity for Germany and France.

To avoid any subjective deletion of outliers on the basis of our graphical inspection, we will use the cross-section on French data to estimate the parameters. Differences between the French yield-to-maturities and German yield-to-maturities are small; the maximum difference is about 8 bp.

For the short-term interest-rate we use 1-month interbank-offered rates, as in De Munnik and Schotman (1994). We started using the Paris Interbank-Offered Rate (PIBOR) and compared them with the Frankfurt Interbank-Offered Rate (FIBOR). The results are displayed in Figure 2.

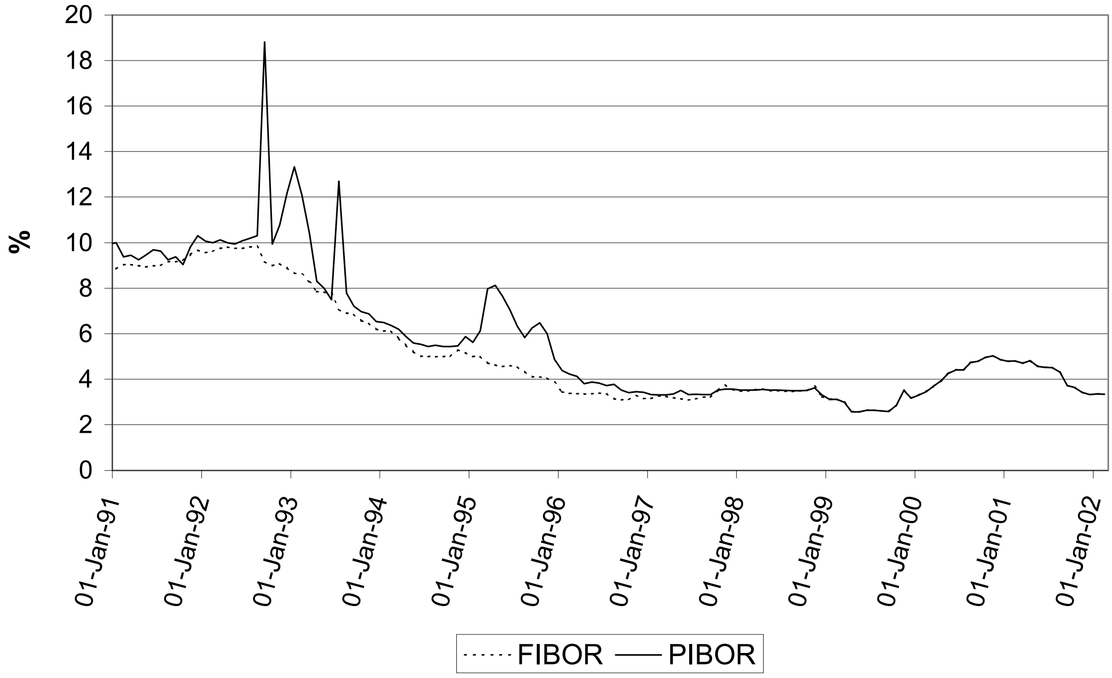


Figure 2: PIBOR versus FIBOR

At the end of 2002, in the first half of 1993 and during 1995, the French rates seem to exhibit different behavior than the German rates. This could be due to the instability in the EMS in those periods with threats of devaluation of the French Franc. As of 1998 the rates are close (within 5 bp). The German rates exhibit much smoother behaviour and therefore we decide to use the FIBOR rates.

Given the choice for the cross-section on the French government bonds we can further classify this sub sample in terms of liquidity. JP Morgan classifies bonds as benchmark (most liquid), active and traded (least liquid). The number of bonds with benchmark liquidity is usually small compared to the total number of bonds for a given country. For France, the number of bonds are 5, 18 and 10, respectively. Comparing the results on the basis of the estimation error on the prices of the bonds we conclude that excluding the traded bonds improves the pricing errors substantially so those are left out. Hence, our cross-section estimation results are based on 23 French government bonds.

4.2 Parameter estimation

We performed the time-series estimation technique based on daily, weekly and monthly data on the 1-month FIBOR. This results in negative or extremely low (maximum 1.1%) estimates for the unconditional mean of the short rate under *P*. Therefore we decided to use the sample average as the parameter estimate for the unconditional mean in the AR(1)-model and estimated the AR(1) parameter after subtracting this mean from the data. In terms of R-squared this does not make a difference.

The parameter a is chosen from the cross-section regression. The estimate for σ based on the time-series estimates is insensitive to changes in the estimates used for a. Therefore we do not modify the time-series estimate for σ and use it directly in the cross-section optimization.

Table 1: Estimation results using time-series data and using sample averages to estimate the unconditional mean in the AR(1) process

frequency	Daily	Weekly	Monthly
a	0.0823	0.0912	0.1405
σ	0.0056	0.0067	0.0067

Note that we do not use the estimates for the parameter a reported in Table 1 but they are reported for comparisons with the estimates on the basis of the cross-section data. Because the estimates based on the weekly and monthly frequency are both 0.0067 we use it as our estimate for σ.

Table 2: Estimation results using cross-section data

parameter	estimate
a	0.4630
$\tilde{\mu}$	0.0562
σ	0.0067
$r_0(0)$	0.0291

The unconditional mean of the short-rate, μ, is equal to 0.0524 so that the drift adjustment in the risk-neutral world is equal to 0.004 or 40 basis points.

5.1 Simulation results

After having selected the model structure and having estimated the required parameters the fundament of the simulation model has been created. In this paragraph the practical implementatiom of the model is the central issue. The predefined estimation results are used as a starting point, which means *that* $a=0.463$, $\tilde{\mu}=0.0562$, $\sigma=0.0067$ *and* $r_0(0)=0.0291$. Given these parameters the yield-curve can be derived. The shape of this curve is shown in the next figure.

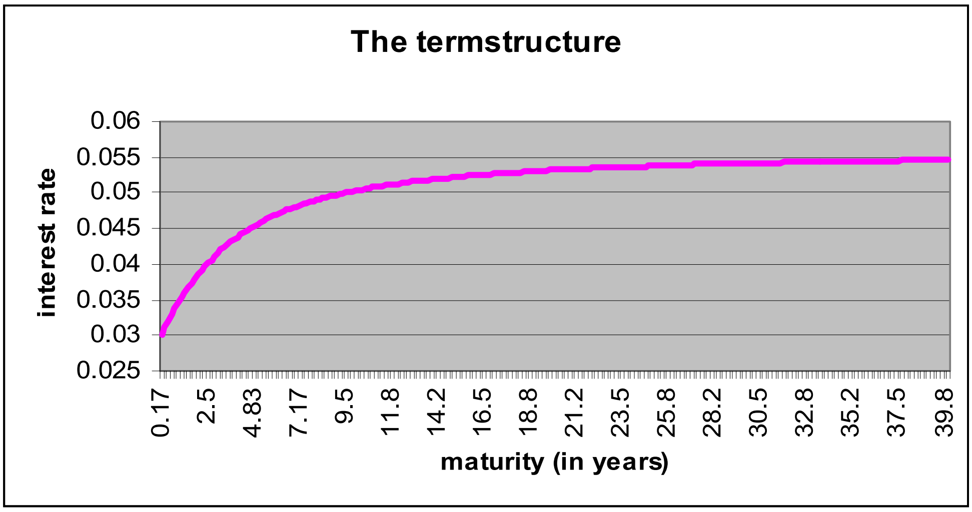


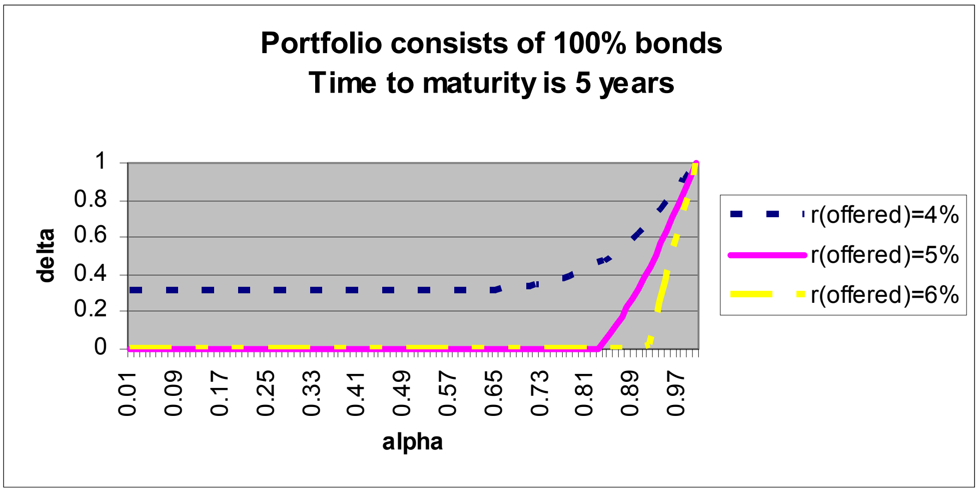
Figure 1 The used yield curve

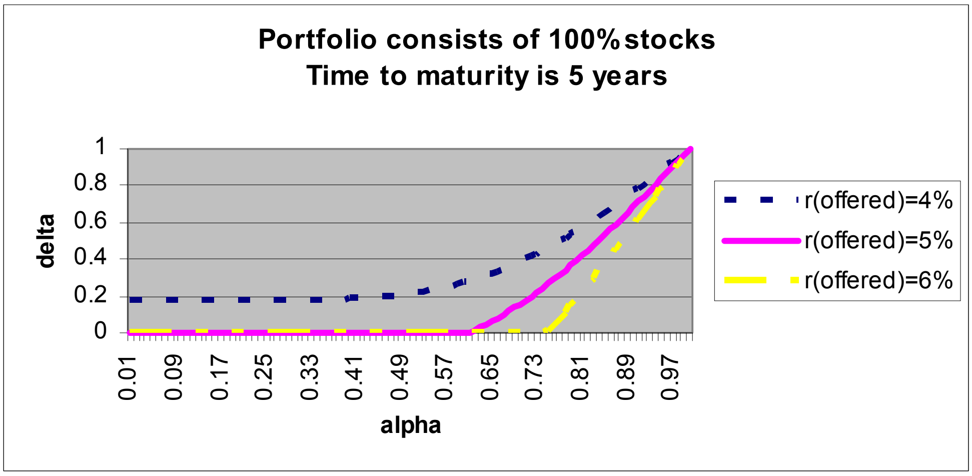
Given the asset model defined in (15) three kind of portfolios are closer looked at, namely a portfolio that consists of 1) 100% bonds, 2) 100% stocks and 3) 40% bonds and 60% stocks. The estimation results for the required parameters μ_A, σ_A, and ρ_A are presented in the following table. The estimation of μ_A and σ_A are based on Dimson (2003). The value of ρ_A is derived from monthly data concerning the German Stock Market index (period 30-11-1990 until 28-02-2002) and the German Government Bond Market Index over the equivalent period.

Table 3: Estimation results based on different portfolios

Portfolio	μ_A	σ_A	ρ_A
100% bonds	0.054	0.085	-0.086
100% stocks	0.092	0.165	-0.031
40% bonds, 60% stocks	0.075	0.114	-0.05

Furthermore we assume that there are three offered return rates. The first one is equivalent with the ten-year interest rate, for the second one the ten-year rate is increased by 1%-point and for the third contract the ten-year rate is decreased by 1%-point. In order to be able to evaluate the insurance contract, assumptions about δ, α and $r_{offered}$ have been made. Given the predefined value of $r_{offered}$ the relation between δ en α as described in (12) can be shown graphically.





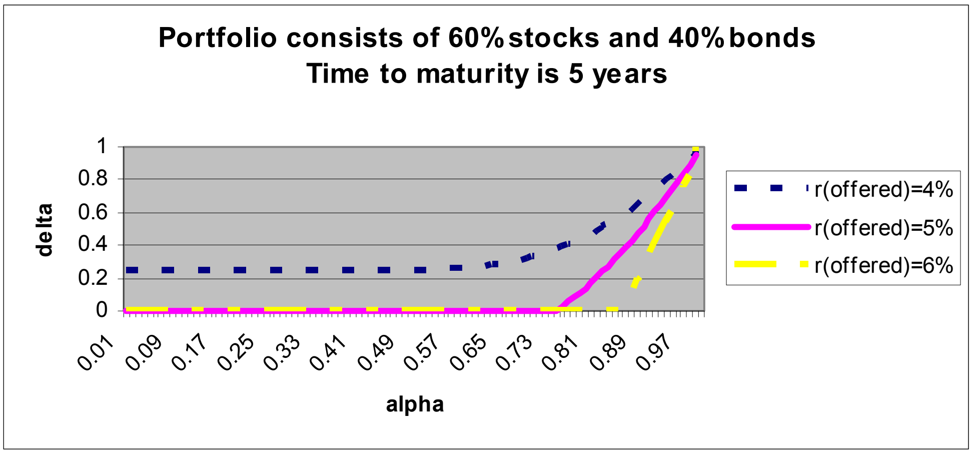


Figure 2 *The relation between δ en α for a predefined $r_{offered}$ and a contract maturity of 5 years*

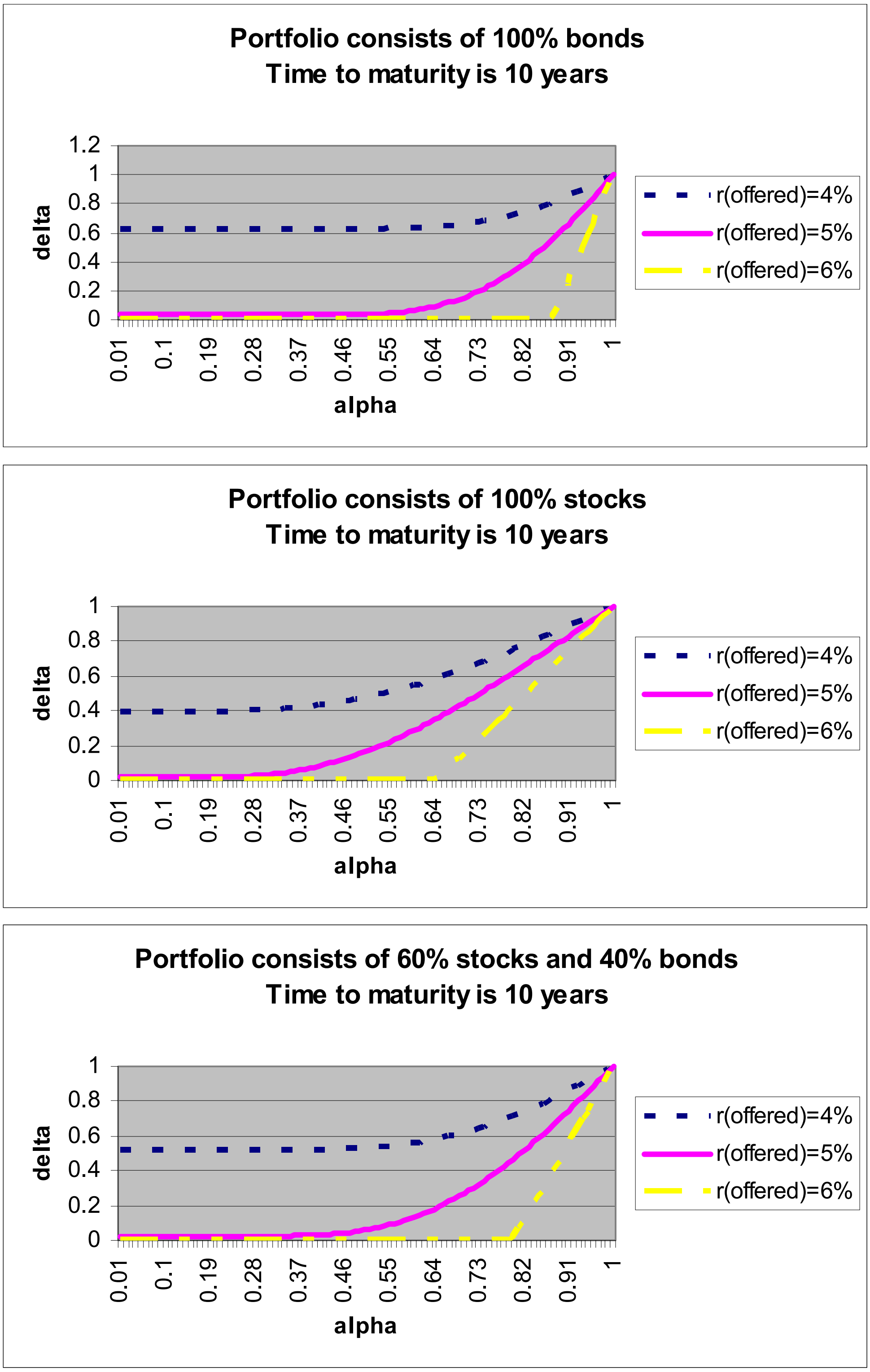


Figure 3 ***The relation between δ en α for a predefined $r_{offered}$ and a contract maturity of 10 years***

After having described the deterministic relationship of delta and alpha for a specific contract it is interesting to look at the profitability. Based on the assumption shown before with concern to return rates the risk/return profile within a stochastic context is presented.

Table 4: Risk/return profile of a contract for different portfolios

$\alpha = 0.95$, $\delta = 0.91$, $r_{offered} = 0.04$, $T = 10$

	100% bonds, 0% stocks			100% stocks, 0% bonds			60% stocks, 40% bonds		
interval	Assets	Liabilities	Equity	Assets	Liabilities	Equity	Assets	Liabilities	Equity
-100	0.0	0.0	26.7	0.0	0.0	20.4	0.0	0.0	17.4
-90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-65	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-30	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0
-25	0.0	0.0	0.1	0.0	0.0	0.1	0.0	0.0	0.1
-20	0.0	0.0	0.2	0.0	0.0	0.1	0.0	0.0	0.1
-15	0.0	0.0	0.4	0.0	0.0	0.2	0.0	0.0	0.2
-10	0.0	0.0	0.7	0.0	0.0	0.3	0.0	0.0	0.4
-5	0.0	0.0	1.1	0.6	0.4	0.5	0.0	0.0	0.6
0	2.7	1.7	1.7	6.0	5.1	0.8	2.8	2.0	1.1
5	43.0	45.3	6.9	22.1	23.6	3.4	26.3	27.9	4.4
10	49.0	49.1	30.0	34.1	35.7	16.8	47.8	49.9	22.0
15	5.2	3.8	23.8	25.0	24.7	21.3	20.6	18.5	27.2
20	0.1	0.0	7.5	10.0	8.7	18.9	2.4	1.6	18.8
25	0.0	0.0	0.7	2.1	1.6	11.4	0.1	0.0	6.5
30	0.0	0.0	0.0	0.2	0.2	4.5	0.0	0.0	1.1
35	0.0	0.0	0.0	0.0	0.0	1.1	0.0	0.0	0.0
40	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0
45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
>50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
*mean**	*5.3*	*5.4*	*-20.2*	*8.3*	*8.2*	*-9.0*	*7.2*	*7.1*	*-7.5*
*standard deviation**	*2.8*	*2.5*	*48.5*	*5.7*	*5.3*	*46.5*	*3.9*	*3.6*	*42.9*

** In contrast to the predefined μ_A the mean is based not on the arithmetic mean but the geometric mean. The standard deviation is the standard deviation of this geometric mean over the 50.000 simulations.*

The above table shows that the position of the policyholder looks advantageous with concern to the position of the equity holder. The policyholder is granted with a higher return and faces a lower risk. Although the contract is based on the fair value principle, one might not expect an equity holder to join in a contract as pre specified. What should be changed to make the contract more attractive to the equity holder? To answer this question different scenarios are closer looked at. The first move is to lower the value of alpha. This implies that the equity holder gets a greater share of future bonuses and because of the larger amount of money invested, the probability of losing the whole investment should be smaller. Furthermore one might expect that therefore the average return will increase and the standard deviation will decrease. The next table underpins these expectations.

Table 5: Risk/return profile of a contract for different portfolios

$\alpha = 0.80$, $\delta = 0.72$, $r_{offered} = 0.04$, $T = 10$

	100% bonds, 0% stocks			100% stocks, 0% bonds			60% stocks, 40% bonds		
interval	Assets	Liabilities	Equity	Assets	Liabilities	Equity	Assets	Liabilities	Equity
-100	0.0	0.0	10.4	0.0	0.0	12.3	0.0	0.0	7.8
-90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-65	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-35	0.0	0.0	0.1	0.0	0.0	0.1	0.0	0.0	0.1
-30	0.0	0.0	0.2	0.0	0.0	0.1	0.0	0.0	0.1
-25	0.0	0.0	0.3	0.0	0.0	0.2	0.0	0.0	0.2
-20	0.0	0.0	0.5	0.0	0.0	0.4	0.0	0.0	0.3
-15	0.0	0.0	1.1	0.0	0.0	0.6	0.0	0.0	0.7
-10	0.0	0.0	2.0	0.0	0.0	1.1	0.0	0.0	1.3
-5	0.0	0.0	3.5	0.6	0.2	1.8	0.0	0.0	2.1
0	2.7	0.3	6.6	6.0	2.5	2.9	2.8	0.5	3.7
5	43.0	50.4	14.5	22.1	27.9	6.7	26.3	31.9	8.7
10	49.0	47.9	35.9	34.1	41.5	18.6	47.8	55.3	26.0
15	5.2	1.4	20.9	25.0	21.8	22.7	20.6	11.8	28.5
20	0.1	0.0	3.8	10.0	5.4	18.3	2.4	0.4	16.0
25	0.0	0.0	0.1	2.1	0.7	10.1	0.1	0.0	4.1
30	0.0	0.0	0.0	0.2	0.0	3.4	0.0	0.0	0.4
35	0.0	0.0	0.0	0.0	0.0	0.7	0.0	0.0	0.0
40	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0
45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
>50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
mean	*5.3*	*5.4*	*-5.2*	*8.3*	*7.8*	*-1.9*	*7.2*	*6.7*	*1.0*
standard deviation	*2.8*	*1.8*	*33.0*	*5.7*	*4.4*	*37.8*	*3.9*	*2.7*	*30.3*

Although there is a severe increase in return and a large decrease in the standard deviation, these values are still not that attractive that an equity holder may join in such a product.

Maybe a decrease in the offered rate will help? Of course it will, but the impact is not enough to be beat the alternative in which alpha is decreased as the following table shows.

Table 6: Risk/return profile of a contract for different portfolios

$\alpha = 0.95$, $\boldsymbol{\delta = 0.96}$, $\boldsymbol{r_{offered} = 0.03}$, $T = 10$

	100% bonds, 0% stocks			100% stocks, 0% bonds			60% stocks, 40% bonds		
interval	Assets	Liabilities	Equity	Assets	Liabilities	Equity	Assets	Liabilities	Equity
-100	0.0	0.0	16.0	0.0	0.0	15.3	0.0	0.0	11.3
-90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-65	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-30	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0
-25	0.0	0.0	0.1	0.0	0.0	0.1	0.0	0.0	0.1
-20	0.0	0.0	0.2	0.0	0.0	0.1	0.0	0.0	0.1
-15	0.0	0.0	0.3	0.0	0.0	0.2	0.0	0.0	0.2
-10	0.0	0.0	0.5	0.0	0.0	0.2	0.0	0.0	0.3
-5	0.0	0.0	0.9	0.6	0.4	0.5	0.0	0.0	0.6
0	2.7	1.7	1.5	6.0	5.1	0.7	2.8	2.0	0.9
5	43.0	45.1	15.3	22.1	23.7	6.6	26.3	28.0	8.5
10	49.0	48.6	42.8	34.1	35.1	20.6	47.8	48.8	29.9
15	5.2	4.5	20.2	25.0	24.7	24.6	20.6	19.3	30.9
20	0.1	0.0	2.1	10.0	9.1	18.5	2.4	1.9	14.4
25	0.0	0.0	0.0	2.1	1.8	9.3	0.1	0.0	2.7
30	0.0	0.0	0.0	0.2	0.2	2.8	0.0	0.0	0.2
35	0.0	0.0	0.0	0.0	0.0	0.5	0.0	0.0	0.0
40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
>50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
mean	*5.3*	*5.4*	*-9.8*	*8.3*	*8.2*	*-4.4*	*7.2*	*7.1*	*-2.0*
standard deviation	*2.8*	*2.6*	*39.7*	*5.7*	*5.4*	*41.2*	*3.9*	*3.7*	*35.4*

Another alternative is to increase the time horizon. By increasing the expiration date one expects that the probability of generating a return as least as great as the offered rate, increases. The effect however is not that much that expanding the expiration date makes the offer for the equity holder attractive as can be seen form the next table.

Table 7: Risk/return profile of a contract for different portfolios

$\alpha = 0.95$, $\boldsymbol{\delta} = \mathbf{0.97}$, $r_{offered} = 0.04$, $\boldsymbol{T} = \mathbf{20}$

	100% bonds, 0% stocks			100% stocks, 0% bonds			60% stocks, 40% bonds		
interval	Assets	Liabilities	Equity	Assets	Liabilities	Equity	Assets	Liabilities	Equity
-100	0.0	0.0	23.0	0.0	0.0	13.4	0.0	0.0	11.1
-90	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-70	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-65	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-55	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-15	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0
-10	0.0	0.0	0.2	0.0	0.0	0.1	0.0	0.0	0.1
-5	0.0	0.0	0.5	0.0	0.0	0.2	0.0	0.0	0.2
0	0.3	0.2	1.2	1.7	1.4	0.4	0.4	0.3	0.6
5	44.0	44.7	12.8	19.8	20.3	4.9	21.8	22.2	6.4
10	54.6	54.1	55.8	46.2	47.0	34.2	62.9	63.6	48.8
15	1.1	1.0	6.5	27.3	26.7	34.0	14.7	13.8	30.4
20	0.0	0.0	0.0	4.7	4.4	11.4	0.3	0.2	2.5
25	0.0	0.0	0.0	0.2	0.2	1.2	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
>50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
mean	*5.3*	*5.4*	*-17.8*	*8.2*	*8.2*	*-4.2*	*7.1*	*7.1*	*-3.0*
standard deviation	*2.0*	*1.9*	*44.9*	*4.0*	*3.9*	*37.9*	*2.7*	*2.7*	*34.4*

The signal of the above tables is that there is one important parameter as far as the equity holder is concerned namely the value of alpha. One might expect that the lower this value is, the more the equity holder profits from future returns. In order to see if this proposition holds, the next table is included.

Table 8: Risk/return profile of a contract for different portfolios

$r_{offered} = 0.04$

100% bonds			mean			*standard deviation*		
alpha	delta	T	assets	liabilities	equity	assets	liabilities	equity
0.25	0.62	10	5.3	5.3	5.3	2.8	1.4	3.4
0.50	0.62	10	5.3	5.3	4.9	2.8	1.4	6.1
0.75	0.68	10	5.3	5.3	-1.6	2.8	1.7	27.1
0.25	0.83	20	5.3	5.4	5.2	2.0	1.4	2.3
0.50	0.83	20	5.3	5.4	4.2	2.0	1.4	9.8
0.75	0.87	20	5.3	5.4	-3.7	2.0	1.5	29.8

100% stocks			mean			*standard deviation*		
alpha	delta	T	assets	liabilities	equity	assets	liabilities	equity
0.25	0.40	10	8.3	6.5	8.6	5.7	2.5	7.1
0.50	0.47	10	8.3	6.8	7.2	5.7	2.9	17.4
0.75	0.69	10	8.3	7.6	0.2	5.7	4.1	34.4
0.25	0.62	20	8.2	7.4	8.2	4.0	2.8	6.5
0.50	0.71	20	8.2	7.6	6.2	4.0	3.1	17.4
0.75	0.85	20	8.2	8.0	1.2	4.0	3.5	29.6

60% stocks, 40% bonds			mean			*standard deviation*		
alpha	delta	T	assets	liabilities	equity	assets	liabilities	equity
0.25	0.52	10	7.2	6.1	7.4	3.9	2.0	4.6
0.50	0.53	10	7.2	6.2	7.4	3.9	2.0	8.6
0.75	0.66	10	7.2	6.5	3.0	3.9	2.5	26.1
0.25	0.73	20	7.1	6.7	7.3	2.7	2.1	3.1
0.50	0.76	20	7.1	6.7	6.7	2.7	2.1	9.5
0.75	0.85	20	7.1	6.9	3.0	2.7	2.3	23.1

From this table we learn that real low levels of alpha lead to equity returns that look interesting from a point of view from both the policyholder as the equity holder. Furthermore we can see that there is a huge decline in return for the equity holder if alpha has a level above the 50%. This is because the value of delta increases more than proportional with concern to the increase of alpha. This implies that the policyholder gets an increasing part of the generated bonus, which increases of course the return rate of the policyholders and decreases the return generated by the equity holders.

5.2 Conclusions

In this paper we have taken the work of Briys and De Varenne (1997) and the papers of Grosen and Jorgenson (2000, 2001) as a starting point. The paper of

Briys and De Varenne only deals with the risk free world, which is used to value the embedded options. The real world is not modeled and therefore risk/return profiles cannot be derived. We have expanded the Briys and Varenne model in such a way that this real world can also be analyzed. We changed the Grosen and Jorgenson model so that not only a fixed interest rate can be used to determine market values, but that the whole term structure is used for valuation purposes. This makes the model more practical applicable. As the empirical study shows it is necessary within the presented theoretical framework that the equity holder has enough space to participate in this contract. Large values of alpha are very profitable from the point of view of the policyholder and are less attractive from the position of the equity holder. Increasing the time horizon and lowering the offered return increases the profits of the equity holder. However in the scenario presented in this paper this is not enough to generate a risk/return profile that can be characterized as interesting for both parties.

Appendix

In this appendix we provide more details on the estimation of the parameters (μ_A, σ_A, ρ). To estimate ρ we use a time-series of the instantaneous short-rate $r_t(0)$. Because $r_t(0)$ is not observed we first explain how to estimate a historical time-series for it. Our derivation is based on (8) in De Munnik en Schotman (1994), which is repeated as (26) in the main text. It states that

$$r_t(T-t) = w_1(T-t)r_t(0) + (1-w_1(T-t))r(\infty) + w_2(T-t)\sigma^2$$

So if we define $\tau = T - t$, then

$$r_t(0) = \left(r_t(\tau) - (1-w_1(\tau))r(\infty) - w_2(\tau)\sigma^2\right) / w_1(\tau)$$

$$w_1(\tau) = \frac{1-e^{-a\tau}}{a\tau}$$

$$w_2(\tau) = \frac{(1-e^{-a\tau})^2}{4a^3\tau}$$

In our case, τ=1/12, $w_1(\tau)$=0.981, $w_2(\tau)$=0.0433 and σ^2=0.00004. Then it holds that $r_t(0)$ is approximately equal to the 1-month FIBOR divided by $w_1(\tau)$ so $r_t(0)$ is roughly 2% higher than the 1-month FIBOR.

To estimate μ_A, σ_A and ρ we use that in discrete-time, the processes are as follows:

$$r_{t+\tau}(0) = e^{-a\tau}[r_t(0) - b] + b + u_{t+\tau}$$

$$\ln(A_{t+\tau}) = \ln(A_t) + (\mu_A - 0.5\sigma_A{}^2)\tau + v_{t+\tau}$$

$$u_{t+\tau} = \sigma e^{-a\tau}\int_0^\tau e^{as}dW_s$$

$$v_{t+\tau} = \sigma_A \int_0^\tau (\rho dW_s + \sqrt{1-\rho^2}dZ_s)$$

It then follows that

$$E_t\{u_{t+\tau}\} = 0, E_t\{v_{t+\tau}\} = 0$$

$$Var_t\{u_{t+\tau}\} = \frac{\sigma^2}{2a}(1-e^{-2a\tau})$$

$$Var_t\{v_{t+\tau}\} = \sigma_A^2\tau$$

$$E_t\{u_{t+\tau}v_{t+\tau}\} = \frac{\rho\sigma\sigma_A}{a}(1-e^{-a\tau})$$

where E_t and Var_t denote the expectation and variance conditional on the information available at time t.

Now μ_A and σ_A can be estimated using the equation for $\ln(A_{t+\tau})$ together with the fact that the conditional variance of $v_{t+\tau}$ is equal to $\sigma_A{}^2\tau$. Values for the residuals $u_{t+\tau}$ and $v_{t+\tau}$ follow from the above equations when replacing the parameters by their estimates. An estimate for the parameter ρ then follows from the final equation. The left hand side can be estimated using the aforementioned residuals. Because τ is known and a, σ and σ_A are estimated consistently, a consistent estimator for ρ follows.

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