

The Value of non Enforcable Future Premiums in Life Insuranceⁱ

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Abstract:

The valuation of future premiums in life insurance must take into account that the premiums are not enforceable by the insurance company: the policyholder can stop to pay the premiums at his discretion. This can be a valuable embedded option in the policy. The valuation can be based on the classical actuarial method, using a fixed percentage of decrements. It is also possible to assume that the policyholder behaves economically rational and apply a contingent claim analysis for the valuation. The current proposals (DSoP) for the new accounting standards for insurance contracts leaves the choice between both methods open. In this paper a model is set up to analyze this feature of insurance contracts. Both methods will be compared, and it will be seen how mortality and policy conditions can influence the rational behavior of the policyholder. It becomes clear that the release of profits from a contract will be influenced significantly by the model used to value the future premiums.

Keywords:

Life Insurance; embedded options; mortality; policyholder behavior;

The Value of Future Premiums in Life Insurance

There is a growing interest for the valuation of life-insurance policies using principles from financial economics, often called fair value. Fair value for life insurance can be regarded as the introduction of techniques from the financial economics in actuarial problems. Fair value is defined in the DSoP (3.15) as "the amount for which an asset could be exchanged or a liability settled between knowledgeable, willing parties in an arm's length transaction", IASB (2002).

The implementation of fair value will lead to changes in the values used for the balance sheet and in the profit and loss account (P&L). A lot of attention goes to the calculation of the fair value of life insurance liabilities because this is a major item on the balance sheet and typical for insurance operations. As there are no observed market values for the insurance liabilities, the value must be calculated using a model that is consistent with market prices and valuation techniques (for instance: no arbitrage). For policies with no future premiums this seems relatively easy: only the future outgoing payments due to the paid up policy must be valued. These payments are contingent on the policyholder's future lifetime, not on the future state of the economy (disregarding the possibility to lapse).

With premium paying policies the situation is more complicated, as the future insurance benefits also depend on the future premium income due to the same policy. The policyholder has often the right to stop paying premiums: in that case the policy is converted in a paid-up policy (this conversion will be called "to pup" in the rest of the article) and the insured amount will subsequently be decreased using the tariff of the original policy. In some cases this represents a valuable option to the policyholder, especially if the interest rates in the market are much higher than the guaranteed interest rates in the policy. In the literature this option is often ignored or overlooked. This makes the valuation of the future premiums and benefits more difficult, as they are dependent on the state of the economy. The modeling of these embedded pup-options is quite complicated: If a policy is puped there is no right to recommence in the future with the original premium-tariff. So the pup-option of a policy only can be exercised once.

This article addresses the issue of the future premiums in fair value calculations. Other policyholder options are neglected on purpose in order to clarify the analysis. The inclusion of the right to lapse a policy is straightforward but does not give additional important insight. Two models are analyzed; the first, classical, model assumes a fixed behavior of policyholders independent from the economy. The second, new, model assumes economic rationality for the decision to pay the premiums. The models include mortality in its analysis, adding this typical life insurance feature in order to come closer to the actuarial world.

The paper is organized as follows. First a short oversight of relevant literature is given, and then the institutional framework for the right to stop paying premiums is set out. Then follows the model as used for the analysis and a numerical example of the model. Hereafter we give some applications in the field of mortality selection and the introduction of a penalty when the policy is puped. After this we give some considerations on the "real" behavior of the policyholder and a replicating investment strategy. We conclude with some final remarks.

The International Accounting Standards Board (IASB) for listed life-insurance companies in the European Union will define the new regulations. The current Draft Statements of Principles (DSoP) clearly go into the direction of Fair Value valuation principles (even if this is done under other names like Entity Specific Value). In the second place, some insurance

supervisors (like the Dutch and Danish) are also moving in the direction of the use fair value principles for the evaluation of the financial position of insurance companies. This not only can be attributed to a desire of consistency with the new IASB rules, but also is mainly caused by a desire to have tools for risk measurement with more power than the present accounting. The current accounting rules can point to dangerous situations when it is already too late to react. For instance a permanent downward shift in interest rates will only be seen in the long term, when the old bonds have to be reinvested and the coupons are lower than previously.

The valuation of embedded options in financial products assumes often rational behavior of economic agents, even if experience can show that the real behavior of the clients differs from that. This is the standard procedure for the valuation of American put options. For mortgages similar problems are encountered, as prepayment options are in a lot of jurisdictions valuable to the client. The valuation of the prepayment option using fixed behavior can get very misleading when the behavior of clients change (for instance due to more aggressive competition) or when it is based on experience of the past, when the past experience did not include a sharp fall in interest rates.

In general actuaries rely more on projections based on own experience than on models assuming economic rationality. Smid and Wolthuis (2001) introduce in their model of a life insurance company a lapse frequency that is only dependent on t , not on the economic environment at that time. This procedure is also followed in Gerber (1990), who introduces a decrement force only dependent on time rather than the economic environment.

Bouwknegt and Pelsser (2002) show for a simplified policy with profit sharing how a fair value of this liability can be calculated. The “certain” cash flows (premiums, guaranteed benefits) are valued with the term structure of interest as derived from government bonds. The valuation of the profit sharing is done using risk neutral valuation techniques. For the particular profit sharing under consideration it is possible to use swaptions with well-known analytical valuation formulae. The premium payments are (implicitly) considered to be “sure”, and to simplify the analysis the authors disregard common life insurance features as mortality and embedded options like the right to lapse or put a policy.

The option to lapse a variable annuity is considered by Milevsky and Salisbury (2001). Their analysis both includes mortality and the economy. However, the product they analyze has an important feature uncommon to a lot of insurance markets: an annuity that can be lapsed. They conclude that the option to lapse the annuity can be very valuable to the policyholder, and for the insurance company it is important to consider the option with great care. The option has an American character, as the policyholder has the right to lapse (only one time) during the whole period of the annuity. Their analysis to lapse the policy differs from the analysis in this paper, as it only considers the interest rates at a certain time, not including the option to lapse in the future. In the situation in this article the interest rate environment is more complex because the interest rates of future (premium-) moments are also included in the analysis.

Grosen and Jorgensen (2000) consider a stylized single premium policy without mortality and with some kind of profit sharing. They value the profit sharing and show that the option to lapse the policy can be very valuable in certain circumstances. The value of the lapse option is highly dependent from the guaranteed interest rate, the profit sharing and the market interest rate. The models we study are different, as they exclude profit sharing but include the right to

pay premiums in the future at the current tariff, it also includes mortality. Also in this paper an arbitrage free stochastic interest rate environment is included.

Wallace (2002) compares US-GAAP and fair value accounting. She demonstrates that the release of profit will differ significantly under fair value compared to classical accounting rules. She considers a liability consisting of cash outflows. She underlines the possibility to define the initial value of a contract to equalize the initial single premium, by adjusting the market value margin (MVM). The issue of economic rational behavior is not addressed.

In the financial literature there exist products with a comparable one-sided right to pay the premium like a normal life insurance policy: the installment optionsⁱⁱ. With this option the client pays the premium for his option in installments. If he stops paying premiums the option cannot be exercised anymore. So if the option gets far out of the money it may be interesting to stop paying the installments. It will be clear that the total premium of the option will be higher if the right to stop paying it is included. The issuer of the option (=receiver of the premium) will not receive the premium in situations that are very advantageous for him. In the other cases the option will be continued, so he loses on one side. The valuation of these (American) kind of options is rather complex, as similarly both the underlying option and the premiums must be considered, in their mutual relation. For normal option valuation only the option itself is considered. Compared to the here presented analysis the main difference with the aforementioned options lies in the inclusion of typical insurance features like mortality, the existence of paid up policies. The principles of valuation however can be the same.

The most important contribution of this paper lies in the combination of both “classical” actuarial accounting that is used for setting the premiums and the calculation of the paid up value of the policy with financial economic techniques for modeling the behavior of the policyholder. In doing so it tries to bridge the difference between both sides. A second contribution is in the emphasis that is laid on the one sided character of premium payments, as the policyholder has the option to stop it as his discretion. This right can be very important and is often overlookedⁱⁱⁱ. In this way the paper extends the analysis of embedded options in life insurance. A third contribution is in the view that the own knowledge of the health of a policyholder now also is seen as a kind of option, because it uses information asymmetry. A fourth point is that the emergence of profit in the here presented way differs dramatically from what at the moment is thought to be the expected profitability under Fair Value accounting^{iv}.

Institutional Framework

In most countries (for instance The Netherlands) the premiums for a life insurance policy are not directly enforceable by the insurance company. The policyholder has the right to stop paying premiums if he wishes so; the insurance company can not go to the court and enforce him to continue to pay. The main reason for this is that the life insurance policy is seen as a kind of investment from the policyholder, and like a savings account or other forms of investment, there is no claim on the policyholder: he is free to spend his money as he wants.

In the current version of the DSoP there are two views on the way to treat policyholder behavior. Article 4.35 states: *"...should assume that policyholders will exercise lapse decisions in the way that is least favorable to the insurer "*. This is consistent with the financial economics approach as presented in this article. However, in 4.36 another possible approach is mentioned: *"An insurer will estimate the probability of different lapse decisions...and then use the expected present value..."*. This approach differs fundamentally from the previous one, as the future expected behavior of the policyholder is central in the

analysis, this method is the actuarial approach to the problem. Other articles in the DSoP elaborate on the 4.36 approach as for instance the present value of future premiums must be disclosed. This is impossible with the method using economic rationality of the policyholder.

It is our view that, especially for supervisory purposes, the "unfavorable-behavior" approach should be applied, because it is inherently safe and it does diminish the possibility to influence the reserve setting process by using favorable assumptions. Using the earlier mentioned fair value definition, it is defensible to use the "best-estimate behavior" approach because in a transaction between willing parties the non rational behavior of the policyholders will probably be included in the transaction price.

General Model

In the model the following variables are used:

P	:	the annual premium
SP_y^{tariff}	:	the single premium as calculated on the tariff for the insurance of one unit for a person with age y
IA	:	the insured amount
$\ddot{a}_{x:m }$:	the value as calculated on the tariff of an m -year annuity (payments in advance) for a person with age x
x	:	the age of the insured at the start of the policy
n	:	the maximal duration of premium payments
PU_m	:	the paid up insured amount of the policy just before the m^{th} premium payment
ΔPU_m	:	the increase in paid up insured amount due to the m^{th} premium payment
q_y^{BE}	:	the best estimate one-year mortality rate for a person with age y
α	:	the correction for mortality
q_y^{tariff}	:	the one-year mortality rate for a person with age y as used in the tariff
${}_m p_y^{\text{BE}}$:	the best estimate m -year survival probability for a person with age y
$Z_{n,t}$:	the zero interest rate for an n -year zero loan with no credit risk at time t
$D_{n,t}$:	the value of a n -year discount bond with no credit risk at time t
${}_m E_y^{\text{BE}}$:	the single premium for one unit insured of a pure endowment, payable over m years for a person now aged y at time t as calculated on market rates and best estimate mortality probabilities
$SP_{y,t}^{\text{BE}}$:	the single premium for one unit insured amount as calculated on market rates for interest and with the best estimate mortality for a person aged y at time t
β_m	:	the reduction in the paid up insured amount if the m^{th} premium is the first premium that is not paid
s_y	:	the probability that the policy will be pupped for a person aged y
${}_m r_y^{\text{BE}}$:	the best estimate probability that the policy still is in force after m years for a policyholder with age y
$VB_{m,t}$:	the value of the future policy benefits as measured at time t just before the m^{th} premium payment
$VP_{m,t}$:	the value of the future premiums just before the m^{th} premium payment as measured at time t
$TR_{m,t}^{\text{IF}}$:	the technical reserves just before the m^{th} premium payment as measured at time t for an in-force policy, measured on the basis of fixed policyholder behavior

$TR_{m,t}^{PU}$:	the technical reserves for a paid up policy that is puped after the m^{th} premium payment as measured at time t, measured on the basis of fixed policyholder behavior
$VF_{m,t}$:	the value of the future premiums minus the related benefits just before the m^{th} premium payment as measured at time t
$PP_{m,t}$:	the value of the m^{th} premium payment at time t disregarding the value of future premium payments
$FV_{m,t}$:	the value of the right to continue paying premiums in the future just after the m^{th} premium payment at time t
$r(s)$:	the instantaneous interest rate at time s
$RV_{m,t}$:	the value of the reduction in insured amount if the m^{th} premium is the first premium that is not paid
$VP_{m,t}$:	the value of the right to pay the m^{th} premium at time t
$RT_{m,t}^{IF}$:	the technical reserves just before the m^{th} premium payment as measured at time t for an in-force policy, measured on the basis of dynamic policyholder behavior

Tariff calculations

The calculations of the values, insured amounts and premiums are performed on classical actuarial methods, using tariff rates that can differ from market rates and with a general mortality assumption. For simplicity of the analysis only interest and mortality are considered, disregarding costs, morbidity and profit sharing, though these factors can be included.

The annual premium for the insurance will be:

$$P = (SP_x^{tariff} \cdot IA) / \ddot{a}_{x:\overline{n}|}$$

So at time m, just before the m^{th} premium payment, the paid up insured amount is:

$$PU_m = (SP_{x+m}^{tariff} \cdot IA - \ddot{a}_{x+m:\overline{n-m}|} \cdot P) / SP_{x+m}^{tariff}$$

The increase in the paid up insured amount in year m (ΔPU_m) is financed with the m^{th} premium payment:

$$\Delta PU_m = P / SP_{x+m}^{tariff} = PU_{m+1} - PU_m$$

Finally, the full insured amount IA will be financed with the total of n premiums:

$$\sum_{i=1}^n \Delta PU_i = IA$$

This analysis is fully performed using classical actuarial formulae. It is important to note that the time t does not enter in the formulae. All calculations can be performed at the start of the policy; there is no influence of the real economic environment after that date on the values calculated above. For the value of PU this is reasonable because it is a contractual right.

Mortality and interest

The model uses a single parameter for the best estimate mortality compared to the tariff mortality α . If α differs from 100% (i.e. the tariff mortality is different from the best estimate mortality) this can be attributed to some considerations of the life insurance company issuing the policy. These considerations can include an aggressive pricing policy or the use of conservative estimates in order to maximize profit. It is also possible that the policyholder has more knowledge of his individual mortality probabilities. A small α (for instance 0,1) points to a very healthy person, a large α (for instance 2) indicates serious health problems.

$$q_{x+m}^{BE} = \alpha q_{x+m}^{tariff}$$
$${}_m p_x^{BE} = \prod_{i=1}^m (1 - q_{x+i-1}^{BE})$$

The value of a certain payment in m-years time as measured at time t is the discount bond $D_{m,t}$:

$$D_{m,t} = (1 + z_{m,t})^{-m}$$

The most elementary actuarial single premium is defined in an analogous manner compared to the classical actuarial formulae:

$${}_m E_{x,t}^{BE} = D_{m,t} \cdot {}_m p_x^{BE}$$

Using the real market interest rate as observed at time t, this is a clear breach with the classical model and makes the connection with the financial economy.

For the sake of simplicity we restrict ourselves to a pure endowment, extensions to more complex types of insurance can blur the view of the underlying processes.

$$SP_{m,t}^{BE} = {}_{n-m} E_{x+m,t}^{BE}$$

Reduction of the puped policy

In the model we include the possibility that the insurer applies a reduction to the paid up amount when the policy actually is puped, for instance calculated on a Zillmerized basis. The reason for this can be that in his way the initial costs are (partly) recovered, or it is a compensation for the loss of possible future profitability. In the model this reduction is given by the factor β . The value of this reduction takes into account the reduction in insured amount and the then prevailing market rates for interest and the actual mortality. The value of the reduction when the m^{th} premium is the first not to be paid at time t is:

$$RV_{m,t} = \mathbf{b}_m \cdot PU_m \cdot SP_{m,t}^{BE}$$

Fixed behavior

In this section we will consider a model that assumes a fixed behavior of the policyholder, independent from the economy. The behavior of the policyholders is assumed to be fixed in time, a certain percentage of the policyholders will pup their policy in a year. In most embedded value calculations a fixed lapse and pup-rate are assumed for the projection of future cash flows. For the policy under consideration we assume only the possibility to pup the policy exists, with a predetermined pup-probability of s_{x+i} for a person aged x+i.

For convenience we introduce the variable ${}_i r_x$ that defines the probability that a policy that is in force at age x is still in force (hence not puped) in year $i+x$:

$${}_m r_x^{BE} = \prod_{j=1}^m (1 - q_{x+j-1}^{BE} - s_{x+j-1})$$

In the lifetime of the policy the value of the benefits as calculated in year m at time t (just before paying the m^{th} premium) $VB_{m,t}$ is

$$VB_{m,t} = D_{n-m,t} \cdot \left(\begin{aligned} &PU_m \cdot {}_{n-m}P_{x+m}^{BE} + \sum_{i=0}^{n-m-1} \Delta PU_{m+i} \cdot {}_i r_{x+m}^{BE} \cdot {}_{n-m-i}P_{x+m+i}^{BE} \\ &- \sum_{i=1}^{n-m-1} PU_{m+i+1} \cdot \mathbf{b}_{m+i+1} \cdot {}_i r_{x+m}^{BE} \cdot s_{x+m+i} \cdot {}_{n-(x+m+i+1)}P_{x+m+i}^{BE} \end{aligned} \right)$$

The value of the future premiums $VP_{m,t}$ just before the m^{th} premium payment is

$$VP_{m,t} = P \cdot \sum_{i=0}^{n-m-1} (D_{i,t} \cdot {}_i r_{x+m}^{BE})$$

The technical reserves of the in force policy at time t just before the m^{th} premium payment $TR_{m,t}^{IF}$, is

$$TR_{m,t}^{IF} = VB_{m,t} - VP_{m,t}$$

If the policy is puped just before the m^{th} premium, the value of the paid up policy at time t is

$$TR_{m,t}^{PU} = PU_m \cdot {}_{n-m}E_{x+m,t}^{BE}$$

The initial technical reserves at time t after the signing of the contract, before the first premium is received, $TR_{0,t}^{IF}$ is:

$$TR_{0,t}^{IF} = VB_{0,t} - VP_{0,t}$$

If the tariff is profitable, $TR_{0,t}^{IF}$ is negative. In this way the expected profit of the policy is released in one time. Note that a profitable policy initially leads to a negative liability, thus an asset. This is often considered to be undesirable, because an asset is a claim that is enforceable. The future premiums are not enforceable by the company, clearly contradicting the characteristics of an asset.

The value of the future, not yet insured, benefits minus the premiums is $VF_{m,t}$

$$VF_{m,t} = VB_{m,t} - PU_m \cdot {}_{n-m}E_{x+m} - VP_{m,t}$$

The value of the technical reserves at time $t+m$ for an in-force policy can now be rewritten as

$$TR_{m,t}^{IF} = TR_{m,t}^{PU} + VF_{m,t}$$

If the tariff would be profitable for the company at time $t+m$ the value of $VF_{m,t}$ is positive, otherwise it is negative. If the policy is puped when the tariff is profitable, a loss to the

company arises equal to $VF_{m,t}$. In this case the technical reserves of the in-force policy does not cover the value of the paid up policy, so creating a dangerous situation in case of discontinuity of the company.

Dynamic behavior

In the previous section we assumed that the policyholder behavior was known in advance and independent from the economic environment. In this section we will take a look at a model that assumes that the policyholder acts dynamically in his own best interest in response to the economic environment. If we assume that the policyholder behaves in an economic rational way, the value of the policy must include the right of the policyholder to make decisions dependent on the state of the economy. This is fully consistent with the valuation principles as used in the valuation of (embedded) derivatives, rather than the classical actuarial approach assuming a fixed behavior like we discussed in the previous section.

The analysis of dynamic behavior of the policyholder assumes economic rationality of the policyholder with regard to the decision to continue his policy or to pup it. Therefore the valuation is split in two parts: the value of the paid up amount due to previous premium payments and the value of the current and future premium payment. The latter exists of the present value (at market rates) of the additional paid up insured amount and the value to make the same decisions with later premium payments. The single premium in this part of the analysis uses the real expected mortality that off course can differ from the mortality in the tariff.

The direct value of the additional insured amount can be set of to the price of it, and that is the classical premium P as previously calculated. The value of future premiums and guarantees is excluded from this variable, so the value of the m^{th} premium payment at time t $PP_{m,t}$ is:

$$PP_{m,t} = \Delta PU_m \cdot SP_{m,t}^{BE} - P$$

The value of the decision to pay premiums in the future is defined using a recursive relation, as in year $t+1$ the same decision must be made like in year t . It would be wrong to assume that the decision to continue the premiums at time t would imply that all the further payments will be performed. The formula also takes into account one-year discounting and one year mortality.

$$FV_{m,t} = {}_1P_{x+m}^{BE} \cdot E_t^Q \left(e^{-\int_t^{t+1} r(s) ds} VP_{m+1,t+1} \right)$$

This formula uses the risk neutral probability measure Q . The function $r(s)$ has a double role, as it both is the discounting of the future cash flows and defines these cash flows themselves.

The value of the m^{th} premium payment includes the right to pup the policy. If the premium is paid then there is an addition to the insured amount that is set of against the premium that must be paid. Only when the premium is paid you have the right to continue making the same decision a year later. If the premium is not paid the costs will be the reduction in the paid up insured amount with the factor β . The model states that the policyholder will choose the maximum of both values. The choice is made just before every premium payment, so this option is a Bermudan style option.

$$VP_{m,t} = \max(PP_{m,t} + FV_{m,t}; -RV_{m,t})$$

This formula introduces the pup-behavior in the model. The policyholder only will pup his policy if that is in his economic advantage. When the policy is in force, the value of the technical reserves at time t, $RT_{m,t}^{IF}$, is the sum of the value of the paid up insured amounts and the value of the next premium payment

$$RT_{m,t}^{IF} = TR_{m,t}^{PU} + VP_{m,t}$$

If there is no reduction in paid up insured amount when the policy is puped ($\beta=0$), then $VP_{m,t}$ can not become negative. The technical reserves $RT_{m,t}^{IF}$ will not be lower then the paid up value of the policy. Future profitability of the premiums is excluded from the technical reserves, as it would lead to a reduction in the reserves. Future losses however are included and lead to an increase in the reserves. In this way the value of old guarantees will gradually increase the reserves, not due to a board- decision, but rather due to the market price of these guarantees. The value of the guarantees is recognized even if these guarantees are out of the money.

If $\beta>0$ then it is possible that $VP_{m,t}<0$: whatever the policyholder decides he will lose money in certain circumstances. If he stops paying premiums the value of his paid up insured amount will be reduced, leaving him with a loss. If he continues to pay premiums the tariff rate can lead to a lower value for money then if he invests in the market. Only in this case the value of the technical reserves are lower then the value of the paid up policy (before reduction).

Also it becomes apparent that it is not possible that a new policy directly will lead to a profit (becoming an asset on the insurers balance sheet). The payment of the first premium can yield a profit, but only the profit from that payment will be taken into account. If the tariff however is loss making to the insurer the whole loss will be taken directly when the policy is sold.

Numerical example

Consider a pure endowment policy with insured amount $IA=100.000$, the policy expires after five years ($n=5$). In the event the insured dies before that date there will be no payment from the insurance company to the insured person or his survivors. The policy is financed with equal annual premium payments as long as the policy is in force. The tariff of the policy is based on a technical interest of 5% and an annual mortality rate (equal for all ages) of 1%.

This leads to an annual premium of 16.705,72. The development of the paid up insured amount can be seen in table 1.

Table 1: Development of the paid up insured amount

M	ΔPU_m	PU_{m+1}
1	22 420.01	22 420.01
2	21 138.86	43 558.87
3	19 930.93	63 489.80
4	18 792.02	82 281.81
5	17 718.19	100 000.00

The initial ($t=0$) term structure of interest is depicted as a zero coupon curve using the Nelson-Siegel^V interpolation with parameters $\tilde{a}_0=0,062538$; $\tilde{a}_1=-0,013053$; $\tilde{a}_2=-0,034068$; $\tau=2,5$.

Fixed behavior

The policy cash flows are fully known in advance, as the behavior is independent from the future state of the economy. The technical reserves for the in-force policy are dependent on the actual interest rates that are observed during the lifetime of the policy.

Using the standard assumptions ($\alpha_m=100\%$; $\beta_m=0$; $s_x=2\%$) and using the aforementioned initial term structure of interest and the tariff as mentioned above, we perform the reserve calculations for a simple policy where the premiums are paid until the end of the policy. In the model it is assumed that part of the policy is lapsed, it is clear that this is not possible for this simple policy. Tables 2.1, 2.2 and 2.3 show for three different interest rate paths in the future the value of the technical reserves based on the model with a fixed pup-behavior. The interest rate is depicted as "restzero", that is the zerorate in the term structure with the same maturity as the policy.

Table 2.1: Technical reserves for a path (1) (fixed)

m	$Z_{n-m+1,t+m-1}$	$TR_{m,t+m-1}^{PU}$	$VF_{m,t+m-1}$	$TR_{m,t+m-1}^{IF}$
1	4,68%	0,00	317,14	317,14
2	6,80%	16.410,56	-2.827,68	13.582,88
3	7,46%	33.792,63	-2.341,85	31.450,78
4	6,39%	54.765,54	-733,63	54.031,92
5	8,05%	75.160,10	-521,09	74.639,02

Table 2.2: Technical reserves for a path (2) (fixed)

m	$Z_{n-m+1,t+m-1}$	$TR_{m,t+m-1}^{PU}$	$VF_{m,t+m-1}$	$TR_{m,t+m-1}^{IF}$
1	4,68%	0,00	317,14	317,14
2	3,09%	19.035,03	2.628,36	21.663,39
3	3,14%	38.468,27	1.582,34	40.050,61
4	3,67%	57.826,20	568,61	58.394,82
5	3,29%	78.822,77	267,61	79.090,38

Table 2.3: Technical reserves for a path (3) (fixed)

m	$Z_{n-m+1,t+m-1}$	$TR_{m,t+m-1}^{PU}$	$VF_{m,t+m-1}$	$TR_{m,t+m-1}^{IF}$
1	4,68%	0,00	317,14	317,14
2	5,56%	17.242,50	-1.119,71	16.122,79
3	4,87%	36.525,01	-61,90	36.463,11
4	3,67%	57.826,20	568,61	58.394,82
5	3,29%	78.822,77	267,61	79.090,38

The three paths are chosen in order to show a high (1), low (2) and mixed (3) future interest development. In all the tree paths the policy shows an initial loss of 317,14, because the tariff interest rate is higher than the interest rates in the initial term structure. The high path (1) shows that the technical reserves get low compared to the other paths because of two reasons. First, the high interest rates gives a lower value to the already insured paid up value of the policy in column $TR_{m,t+m-1}^{PU}$. Secondly, column $VF_{m,t+m-1}$ shows that the value of the future expected premiums is high for the company, as the tariff contains a lower return to the customer. Note that puping of the policy becomes dangerous for the company, because the technical reserves in column $TR_{m,t+m-1}^{IF}$ are not sufficient to cover the paid-up policy. The low path (2) gives exactly the opposite picture of the high path, as the low path has higher technical reserves and because the losses due to the future premiums with a relatively high

guarantee are directly reserved for. The mixed (3) path shows that it is possible that during the lifetime of the policy the value attributed to the future premiums changes of sign.

Dynamic behavior

The model for the future interest rates is based on the Hull White^{vi} trinomial tree. This model was chosen as it is arbitrage free, widely known, easy to implement in a Microsoft Excel spreadsheet and relatively easy to understand. It is as well very practical for the implementation of decision trees, as one node describes a full zero-curve. A major disadvantage of the Hull White model is the possibility of negative interest rates in the future. In the current literature on interest rate derivatives more complex interest rate models are widely used. This paper however focuses not on the exact price of an interest rate derivative but rather is about the inclusion of future possible interest rates in the price of an insurance contract. The interest rate tree is built using steps of one month; this leads to some discretization errors that seem to be acceptable. Further parameters used in the model are $a=0,1$ and $\sigma=0,01$.

The technical reserves $RT_{0,t}$ for this policy (=initial loss to the company) is 780,48. For the three previously analyzed paths, the value of the technical reserves and the value of the current premium payment at that moment is shown in tables 3.1, 3.2 and 3.3. The situation is fully comparable with the previous analyzed situation, as the policyholder here also continues to pay his premiums, even if that is not rational. The same parameters for the policy are used as in the previous analysis. The variable $RT_{m,t}$ can be calculated as

$$RT_{m,t} = TR_{m,t}^{PU} + \max(PP_{m,t} + FV_{m,t}; 0).$$

Table 3.1: Technical reserves for a path (1) (dynamic)

m	$Z_{n-m+1,t+m-1}$	$TR_{m,t+m-1}^{PU}$	$PP_{m,t+m-1}$	$FV_{m,t+m-1}$	$RT_{m,t+m-1}$
1	4,68%	0	169,04	611,94	780,48
2	6,80%	16 410,56	- 1 232,90	3,60	16 410,56
3	7,46%	33 792,63	-1 243,46	0,16	33 792,63
4	6,39%	54 765,54	-495,95	2,36	54 765,54
5	8,05%	75 160,10	-521,09	0	75 160,10

Table 3.2: Technical reserves for a path (2) (dynamic)

m	$Z_{n-m+1,t+m-1}$	$TR_{m,t+m-1}^{PU}$	$PP_{m,t+m-1}$	$FV_{m,t+m-1}$	$RT_{m,t+m-1}$
1	4,68%	0	169,04	611,94	780,48
2	3,09%	19 035,03	1 241,60	1 433,89	21 710,52
3	3,14%	38 468,27	895,94	706,49	40 070,70
4	3,67%	57 826,20	409,96	168,70	58 404,86
5	3,29%	78 822,77	267,61	0	79 090,38

Table 3.3: Technical reserves for a path (3) (dynamic)

m	$Z_{n-m+1,t+m-1}$	$TR_{m,t+m-1}^{PU}$	$PP_{m,t+m-1}$	$FV_{m,t+m-1}$	$RT_{m,t+m-1}$
1	4,68%	0	169,04	611,94	780,48
2	5,56%	17 242,50	-448,51	83,96	17 242,50
3	4,87%	36 525,01	6,77	131,93	36 663,72
4	3,67%	57 826,20	409,96	168,70	58 404,86
5	3,29%	78 822,77	167,61	0	79 090,38

In the high path (1) only the first premium is rational, the other premiums are loss making for the policyholder. This can be seen as $\max(\text{PP}+\text{FV};0)=0$ for all the premiums except for the first one. Path 2 is in a low interest rate environment and the tariff interest guarantee is in the money as can be seen from the positive $\text{PP}+\text{FV}$. This is the reason that all the premiums are profitable to the client. Just before the second premium payment the value of the technical reserves are for a large part (12%) due to the possibility to pay the following premium. Table 3.3 demonstrates that for a mixed path (3) it is possible that irrational behavior (the second premium payment is irrational) can become profitable to the policyholder, if he is lucky.

There is a clear relation between $\text{VF}_{m,t}$ in the fixed behavior model and $\text{VP}_{m,t}$ ($=\max(\text{PP}_{m,t}+\text{FV}_{m,t};0)$) in the dynamic behavior model. If the value of the future premiums in the fixed model is negative, than the value of the coming premium in the dynamic model will (in general) be nil, as the dynamic model assumes that the policyholder will not pay the premium if it is loss making for him. The fixed model assumes that the policyholder is not aware of the loss he is making and will continue to pay.

Mortality selection

The tariff of the policy assumes a certain mortality pattern for the insured. In general the insured will know more about his health than the insurance company, creating an asymmetric situation. Often insurance companies will try to encounter this problem with a procedure to ask for health checks when the policy is sold. However, during the policy the health of the insured can change and that makes it for the policyholder possible to decide what to do with the policy dependent on the own health situation. The insurer can not react to this, insurance is meant to give this protection.

Fixed behavior

The fixed model does not allow for the possibility that the knowledge of the own mortality probabilities influences the behavior of the policyholder. Though the relation between the mortality factor and the initial reserves will not be linear, there will be a steady increase in the initial reserves when the mortality factor goes down. In table 4 the relation between the mortality factor and the initial reserves are given for the fixed model. The other assumptions are the standard assumptions ($\beta_m=0$, $s_x=2\%$).

Table 4: Initial technical reserves for different mortality levels (fixed)

α	$\text{TR}_{0,t}$
0%	2.692,13
50%	1.489,01
100%	317,14
200%	-1.934,95
400%	-6.086,89

The relation between α and $\text{TR}_{0,t}$ is clear: a higher mortality (higher α) for a pure endowment is in the interest of the company, as the probability that the policy must be paid out decreases. The negative $\text{TR}_{0,t}$ indicates a high initial profit of the policy.

Dynamic behavior

In the policy considered here it is clear that the policyholder will pup the policy if his health situation deteriorates very much. On the other side: if the policyholder feels very healthy he will be more inclined to pay the premiums, as he hopes he will "outperform" the mortality rate in the tariff.

The initial technical reserves of the policy are clearly influenced by the real mortality versus the tariff mortality. In table 5 the initial value of the policy to the policyholder (before the first premium payment) is given for different α . The other parameters are the standard ones.

Table 5: Initial technical reserves for different mortality levels (dynamic)

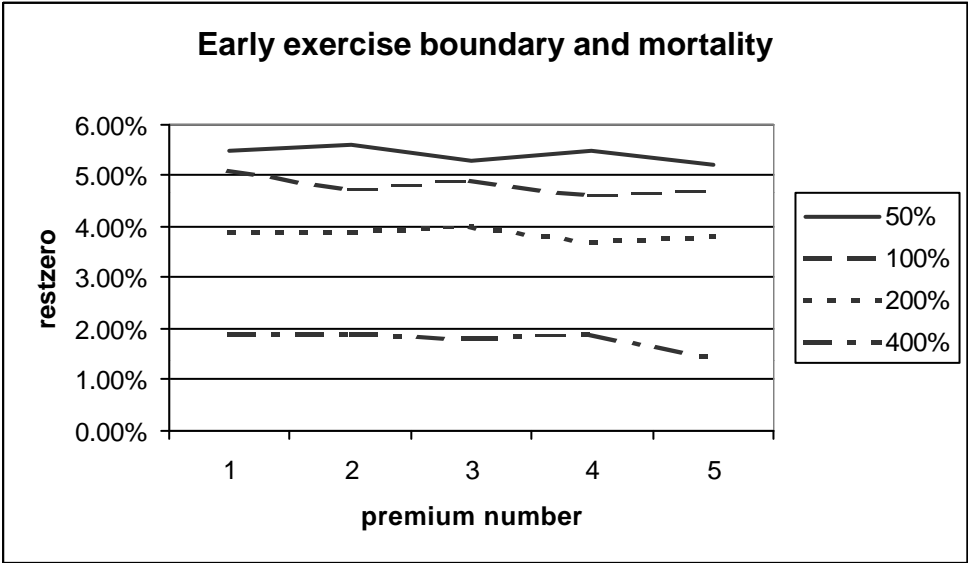
α	$RT_{0,t}$
0%	2 809,73
50%	1 725,37
100%	780,48
200%	0,00
400%	0,00

The table makes clear that if the insured has a mortality rate that is double the rate from the tariff ($\alpha=200\%$) then the policy is not profitable for him. However, if he is very healthy ($\alpha=50\%$) then the policy is good for him. If he is fully sure that he will survive the whole period ($\alpha=0\%$) then the policy even generates a higher profit.

The relation between the fixed and the dynamic model is clear. The positive values of the technical reserves are in the same magnitude for $\alpha=0\%$; 50% and 100%. For higher α the rational behavior of the policyholder is different from the expected behavior, because he will not pay the premiums when this is loss making for him, leading to a nil technical reserve for the dynamic behavior when the reserves becomes negative in the fixed model.

The optimal behavior from the policyholder can be drawn in a graph with on the horizontal axis the premiums and on the vertical axis the interest rate level. For different mortality levels α a line is drawn that is known as the early exercise boundary. Under the line it is rational to continue paying premiums, above the line it is better to pup the policy. Graph 1 for instance makes clear that with $\alpha=200\%$ it is for the third premium rational to pay the premium only when the interest rate for the zero for the rest of the policy-period is below 4%.

Graph 1: Early exercise boundary and mortality α



The graph illustrates that the interest dependency of the policyholder behavior changes with other factors, i.e. mortality. Though both factors are independent from each other (in the model as well as in reality) the behavior is influenced by both jointly. In a low interest environment the interest guarantee is more valuable compared to a high interest environment. In order to keep the value of the guarantee the rational policyholder is willing to accept some mortality losses, as long as the total value of both results is positive. The decision to pay the premium is made on the basis of the total value of the policy, not just the interest or mortality part.

Reduction in insured amount

The model allows for a reduction in the paid up insured amount when the policy is puped. Often this reduction reflects the part of the acquisition costs not yet earned back. In that case the reduction facto β_m declines when m (the number of premiums actually paid) increases. For a new policy the deferred acquisition costs tend to be large, this is partly recovered by applying a high β . Typical values for β are in the 2%-4% range. For reasons of simplicity all calculations in this article are performed using a flat $\beta_m=\beta$.

The factor β is important for the modeled behavior of the policyholder. If a number of premiums are paid, the paid up insured amount is relatively high compared to the premiums, so the policyholder will be more inclined to continue paying his premiums as the value of the reduction increases. In the initial technical reserves a higher β will result in lower technical reserves in both models. The factor β is a kind of terminal bonus when all the premiums are paid, it can be regarded as a very simplified way of including a specific form of profit-sharing in the model.

Fixed behavior

The initial value of the policy is influenced by both β and the estimated pup-probability s_x . In table 6 for different β and s_x the initial value of the technical reserves using the fixed behavior is presented. There are two "absurd" situations added: a β of 100% meaning that the policy is canceled when one premium is not paid. Another absurd situation is a bonus when the policy is puped ($\beta=-10\%$), so giving a serious stimulation not to pay the premium (especially the last premium).

Table 6: Reduction factor $\hat{\alpha}$ and $TR_{0,t}$ (fixed)

s_x	$\hat{\alpha}$	0%	2,5%	5%	100%	-10%
0%		319,92	319,92	319,92	319,92	319,92
2%		317,14	242,06	166,98	-2.686,08	617,46
5%		312,86	136,17	-40,53	-6.754,92	1.019,64
10%		305,48	-13,43	-332,34	-12.450,97	1.581,13

The table clearly indicates that the pup-probability and the reduction factor both influence the initial technical reserves (=initial loss to the company). The column with $\hat{\alpha}=0$ depicts a slightly decreasing initial value, that can only be attributed to less premiums received. As the tariff is initially loss making, fewer premiums lead to a lower initial loss. With higher reductions in the puped value this is clearly to the advantage of the company. The column with the 100% shows that the policy becomes pup supported: a higher pup-rate leads to a higher value for the company. The situation with $\hat{\alpha}=-10$ (i.e.: the policyholder receives a bonus when the policy is puped) shows that in this situation the company will be afraid of a higher pup-percentage.

Dynamic behavior

With \hat{a} it is possible that the technical reserves of the policy will be lower than the paid up value. Consider for instance the situation where at the second premium payment the zero rate is 6,80%, $\beta=2,5\%$, ${}_4p_{x+1}^{BE}=96,06\%$. The value of the premium now can be calculated in three subparts (there can be rounding differences in the calculations):

$$\begin{aligned}
 PP_{1,t+1} &= 21.138,86 * 96,06\% * e^{-4*6,80\%} \text{ -/ - } 16.705,72 = - 1.232,90 \\
 FV_{1,t+1} &= 764,71 \\
 RV_{1,t+1} &= 2,5\% * 22.420,01 * 96,06\% * e^{-4*6,80\%} = 410,19 \\
 VP_{1,t+1} &= \max(-1.232,90 + 764,71; -410,19) = - 410,19 \\
 RT_{1,t+1} &= 22.420,01 * 96,06\% * e^{-4*6,80\%} \text{ -/ - } 410,19 = 15.997,41
 \end{aligned}$$

In this example the loss of the pup-reduction \hat{a} is smaller for the policyholder then the loss that would be made by paying the premiums plus the right for the future FV. The model assumes thus that the policy will be puped.

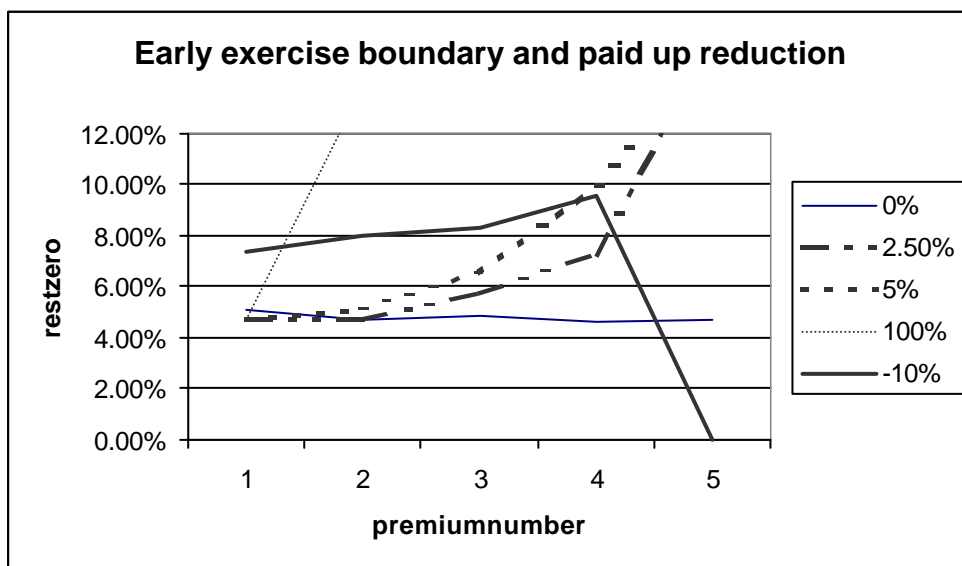
Table 7 shows the influence on the initial value of the policy for different β .

Table 7: Reduction factor β and value for the policyholder (dynamic)

β	$RT_{0,t}$
0%	780,48
2,5%	493,31
5%	372,71
100%	229,41
-10%	6 432,78

It is enlightening to draw the relation between the rational behavior at a certain point, the then prevailing interest rates and the value of β . In graph 2 the early exercise boundary for the five premium payments are drawn for different β .

Graph 2: Early exercise boundary and paid up reduction β



The early exercise boundary for the three situations with $\beta=0\%$, $2,5\%$ and 5% indicates that a higher β leads to a higher early exercise boundary, so decreasing the probability of policyholders leaving the policy. For $\beta=0\%$ the boundary does not differ much from the tariff-interest (5%), indicating that the loss of the right to pay in the future premiums is relatively low in that area.

With a very high β it is unlikely that the rational policyholder will pup his policy. In the case where the model indicates that he will never pup the policy the value of the policy equals the value of a policy without the right to pup; the option is immaterial. This constitutes an interesting check on the results. In the model that was built there remained some differences even when there was no rational moment to leave the policy. These differences are attributed to discretization errors that arise in the Hull-White model. In the graph the situation with $\beta=100\%$ demonstrates that the early exercise boundary starts at the lowest point from all the lines, this can be attributed to a low initial value of the policy. If the first payment is done then the policyholder will practically always continue with the policy, because the loss of all the paid up value will be larger than a loss due to a difference between the tariff and the actual interest rate.

The situation with a negative β gives an entirely different picture. The early exercise boundary is relatively high, indicating that the payment of the premiums is in general profitable for the policyholder. Only at the last premium the situation changes: now it is very profitable to pup the policy. This also influences previous premiums: the additional paid up insured amount from a premium payment will be increased by 10% if the policy is puped just before the last premium payment.

Policyholder motives

In the model with the fixed behavior the policyholder is assumed to be constant in his behavior. On the basis of past experience from the company it is possible to estimate the pup-parameter. It is possible that this value will change in the future, because due to a different environment policyholders can change. For instance, a more aggressive salesforce from other companies can lead to an increase in the number of clients shopping around. This will be more pronounced when competitors offer policies with a higher guaranteed interest, this will be the case when the interest increases. A general economic slowdown will lead to a smaller number of policyholders being able to pay the premiums.

Dependent on the circumstances, it can be possible that a higher pup-rate will lead to financial problems for the company. In cases when the technical reserves of the puped policy is higher than the technical reserves for the in force policy, an increase in pup-rate will lead to an increase in technical reserves that can get dangerous. The positive values of the future premiums that are expected to come do not materialize.

The model with the dynamic behavior assumes economic rational behavior from the policyholder. In general the policyholder will have other motives to pay his premiums, like the desire for insurance protection and fiscal stimuli. He also can have another perception of the state of the economy. The dynamic model does not include that kind of motives and does not try to model the actual behavior from the policyholder.

The dynamic model makes it impossible that actual behavior from the policyholder will have negative repercussions for the insurer, as the most disadvantageous behavior already is included.

If the policyholder pays his premiums when this is expected in the model no additional charge or profit arises. If in that situation the premium is not paid then a profit arises for the insurer. The opposite also applies: if the model does expect that the policy will be puped and the policyholder stops paying his premiums, then no profit or loss comes up. If the premium is paid, then the insurer makes a profit $RI_{m,t}$. This profit can be thought of as caused by irrational behavior from the policyholder. The value of the eventual result is at every node

$$RI_{m,t} = \max(PP_{m,t} + FV_{m,t}; -RV_{m,t}) - / - \min(PP_{m,t} + FV_{m,t}; -RV_{m,t})$$

Hedging the policy

Both models have a different hedge strategy. A hedge fully replicates the exposure of the liabilities with a portfolio of assets.

For the fixed behavior model the hedge can be constructed using long and short positions in zero bonds. The future premiums that are "certain" are replicated with short positions in zero bonds. All the future benefits to the policyholders are replicated with a long position in zero bonds, there is no difference between the replication of the already paid up future benefits and the benefits related to the future premiums. When a policy is puped, the replicating portfolio is changed, buying back the short position in zero bonds and selling the zero bonds that are related to the benefits related to future premiums. In general this liquidation will not be cost neutral because the future interest rates will be different from the interest rates implicated in the previous term structure of interest.

It is possible to define a replicating investment strategy for this policy when it is valued assuming dynamic behavior. It is split in two parts: the value of the paid up policy is invested in zero bonds, the value of the future premium can be replicated using a dynamic hedge. This position is self-financing: the value of the hedge portfolio automatically equals the value of the future premium. When a policy is puped when the model does not expect it, the unwinding of the hedge will lead to a cash flow to the company.

Final remarks

It is possible to use techniques from the financial economics to analyze situations that are typical for the life-insurance industry. These situations may include legal options for policyholders. So it is possible to better value these options and to integrate them in the risk management.

An investment policy based on the uncertainty of future premiums can both protect the paid up insured amounts and take into account the hedging of guarantees given on future premiums.

The valuation of future premiums is a technical actuarial problem but it can have big repercussions on the emergence of profit in a Fair Value accounting environment. The emergence of profit more closely follows the actual reasons of the profits rather than leading to a large one off profit. The price that must be paid lies in the reduction of transparency of the method.

In earlier times actuaries were often thought of as being overprudent, but in recent times the collapse of the Equitable in the UK and the problems for pension funds all over the world

gave that image of the actuary a blow. The proposed method using economic rationality diminishes the possibility for the actuary to be overoptimistic; it leads to inherently safe reserves and a method that does not allow for too much professional judgement. This can significantly increase the uniformity of the calculations.

References

- 1 Bouwknegt, P. and Pelsser, A. (2002) Market Value of Insurance Contracts with Profit Sharing *Journal of Risk Finance* **3**, 60-64.
- 2 Gerber, H. (1990) *Life Insurance Mathematics* Springer-Verlag Heidelberg
- 3 Grosen, A. and Jorgensen, P. (2000) Fair valuation of life insurance liabilities: The impact of interest rate guarantees, surrender options, and bonus policies *Insurance: Mathematics and Economics* **26**, 37-57
- 4 Gutterman, S. (2002) The Coming Revolution in Insurance Accounting *North American Actuarial Journal* **6**, 1-11
- 5 Hull, J. (2000) *Options, Futures, & Other Derivatives* Prentice-Hall International, New Jersey.
- 6 International Accounting Standards Board (2002) Draft Statements of Principles
- 7 Milevsky, M. and Salisbury, T. (2001) The Real Option to Lapse a Variable Annuity: Can Surrender Charges Complete the Market? *Proceedings of Xith AFIR Colloquium*, 537-563.
- 8 Pelsser, A. (2003) Pricing and Hedging Guaranteed Annuity Options via Static Option Replication *Working paper*, Erasmus University of Rotterdam
- 9 Smid C., and Wolthuis, H. (2001) *Bedrijfsanalyse en Embedded Value* Instituut voor Actuarie en Econometrie, Amsterdam
- 10 Wallace, M. (2002) Performance Reporting under Fair Value Accounting *North American Actuarial Journal* **6**, 28-61
- 11 Waters, H. Wilkie, A. and Yang, S. (2003) Reserving, Pricing and Hedging for Policies with Guaranteed Annuity Options *British Actuarial Journal*, forthcoming

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ⁱⁱ Also known as Continuation Option or Pay As You Go Option, www.risk.ifci.ch

ⁱⁱⁱ In classical actuarial accounting the right to pup a policy is often not seen as a danger, because the premium reserve just before and after the mutation remain the same (often even leaving the company with a small profit due to penalties). Management however often knows that they are loosing some future profitability due to the action. The classical accounting thus has difficulties to correctly assess the influence of the action, as it can show a profit even when from an economic perspective a loss would be more appropriate.

^{iv} It is often expected that under fair value accounting the issue of a policy will lead to an initial profit, in the subsequent years only followed by the expected release of the MVM, thus showing only a small run-off profit.

^v The general form of the Nelson-Siegel zerocurve is: $r(t) = \mathbf{g}_0 + (\mathbf{g}_1 + \mathbf{g}_2) \cdot \frac{(1 - e^{-t/\tau})}{t/\tau} - \mathbf{g}_2 e^{-t/\tau}$

^{vi} The tree is built on the basis of chapter 21.12 in Hull (2000)