MODELING JAPANESE FINANCIAL MARKETS FOR PENSION ALM SIMULATIONS

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Abstract

SIMULATION ALM, one of several pension ALM techniques, uses Monte Carlo simulations run under different scenarios to make financial forecasts of pension funding systems. Since these forecasts are not based simply on one set of assumptions, they help to understand outcomes that might occur from a range of possibilities.

The financial markets model used in the simulation requires not only that the model conforms adequately to the real market, but that it also be simple enough to make simulations practical in terms of time and cost.

Building on research done by Ibbotson and Sinquefield (1976) of the U.S. market and Wilkie (1986) of the U.K. market, this paper introduces a method for modeling Japanese financial markets for long-term simulations, and examines features for describing these long-term movements. Specifically:

For the three assets of stocks, bonds, and cash, after checking for and determining the stationary distribution of rates of return, we estimated the parameters for a time series model.

When constructing a long-term asset allocation, using a mean-variance model that ignores this time series structure can lead to erroneous results.

Keywords

Monte Carlo Simulation, ADF test, ARMA model, VAR model

1. Definition of the Model

1.1 Outline of the Ibbotson-Sinquefield (IS) model

Ibbotson-Sinquefield (1976) define their model for the following five asset returns.

$\tilde{R}_m$: Stock (S&P 500, including dividends)
$\tilde{R}_g$: Long-term Treasury bonds (20-year Treasury bonds)
$\tilde{R}_c$: Long-term corporate bonds (High-grade 20-year straight bonds)
$\tilde{R}_f$: Risk-free rate (30-day Treasury notes)
$\tilde{R}_I$: Inflation rate (% change in CPI)

These returns are then broken down into premiums and real returns using the following equations.

$\tilde{R}_p = \left(1+\tilde{R}_m\right)/\left(1+\tilde{R}_f\right)-1$: Stock risk premium
$\tilde{R}_L = \left(1+\tilde{R}_g\right)/\left(1+\tilde{R}_f\right)-1$: Bond maturity premium
$\tilde{R}_d = \left(1+\tilde{R}_c\right)/\left(1+\tilde{R}_g\right)-1$: Bond default premium
$\tilde{R}_r = \left(1+\tilde{R}_f\right)/\left(1+\tilde{R}_I\right)-1$: Real interest rate

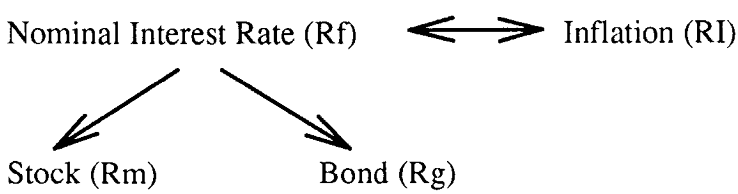
The Rp, RL, Rd, Rr and RI obtained by these equations, the model is estimated by applying time series and cross sectional analysis. By working backward from the decomposing process, the predicted returns for each asset are obtained.

1.2 Japanese Financial Markets Model

On the other hand, the financial markets model for Japan aims to build a model for four assets. Corporate bonds are excluded because there is no corporate bond data over a long period.

Rather than assume a multi variate structure wherein variables effect mutually, we assume a cascade structure in which variables are determined in sequence as shown in the figure below.

Figure 1 Structural Relationship of Variables



1.2.1 On Rf, Rr, and RI (Nominal Interest Rate, Real Interest Rate, and Inflation Rate)

The IS model first estimates Rr and RI, and obtains Rf from these two. This procedure suggests the use of the Fisher hypothesis. In Japan, on the other hand, while Asako and Mura (1991) find that over the past 100 years the Fisher hypothesis does not hold, the hypothesis is undeniable for the post war era and particularly for recent data. Assuming that the Fisher hypothesis does hold true, the method of estimating expected inflation becomes an important issue. Using actual CPI values as a proxy for expected inflation, the explanatory power of the time series model is diminished because of large measurement errors in the CPI. As a result, the nominal interest rate obtained by adding inflation and real interest rates tends to be more volatile compared to forecasting with a time series model of nominal interest rates.

In this paper, we estimate the time series structure of the nominal interest rate directly rather than from adding inflation and real interest rates. To obtain expected inflation, we use a moving average of the CPI with coefficients adjusted to maximize explanatory power from the viewpoint of rational expectations hypothesis.

1.2.2 On Rg (Return on Bonds)

When the return on bonds is divided into income gains and capital gains, capital gains can be obtained by multiplying the fluctuation range of market interest rates and the duration of the bond portfolio. If we assume that the interest rate yield curve is flat, and that fluctuations occur only in small parallel shifts, the bond risk premium can be described as in the following equation. Note that the bond default premium is not evaluated separately but included in μ_L.

$$R_{Lt} = \left(1 + R_{gt}\right) / \left(1 + R_{ft}\right) - 1 = -D\left(R_{ft} - R_{ft-1}\right) + \mu_L + \sigma_L u_{Lt}$$

R_{Lt} : Bond risk premium in period t

R_{ft} : Nominal interest rate (risk-free rate) in period t

R_{gt} : Bond return in period t

D: Duration

μ_L: Bond risk premium

σ_L: Residual risk

u_{Lt} : White noise in period t

1.2.3 On Rm (Return on stocks)

Learning from the IS model, we broke down the return on stocks into risk-free and premium components, and estimate the premium model.

$$R_{pt} = (1 + R_{mt}) / (1 + R_{ft}) - 1 = \mu_p + \sigma_p u_{pt}$$

R_{pt}: Risk premium of stocks in period t

R_{mt}: Return on stocks in period t

μ_p: Expected value of stock risk premium

σ_p: Residual risk

u_{pt}: White noise in period t

2. Data and Analytical Method

2.1 Data

2.1.1 Basic Data

For the analysis, we used data covering 77 quarters from March 1975 to March 1994. Table 1 shows the specific data series used; rates of change are used except for the nominal interest rate. Our reasons for choosing 1975 as the starting point are:

(1)Structural changes in Japan's economy--the shift from fast growth to stable growth, and from fixed to flexible exchange rate--occurred in the mid 1970s.

(2)Inclusion of the oil shocks of the early 1970s would adversely affect the estimation of the future market structure.

Table 1 Data Used in Analysis

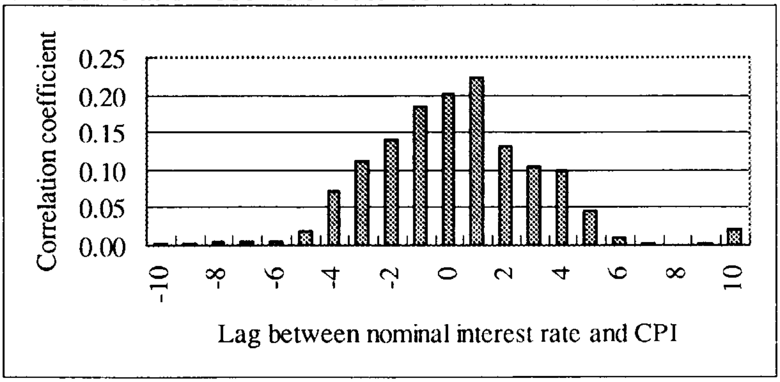
Inflation	: Consumer price index (overall)
Nominal interest rate	: 1-month Gensaki
Bonds	: NRI bond performance index composite
Stocks	: Topix return data (including dividends)

Note: The analysis used raw quarterly data (not annualized).

2.1.2 Expected Inflation Data

According to the Fisher hypothesis, expected inflation has the effect of raising the nominal interest rate. There are numerous ways of estimating expected inflation, including advanced quantitative models, time series models and state-space models, as well as the Carson Perkin method, which quantifies survey results that directly reflect expectations. Keeping in mind that we are conducting a simulation, we chose the following simple method. As Figure 2 shows, the correlation coefficient between nominal interest rates and the CPI is highest in the four quarters preceding and following the CPI value. We thus devised an index for expected inflation from the weighted average of the CPI over this period, using covariance ratios as weights.

Figure 2 Correlation Coefficient between Nominal Interest Rate and CPI



2.2 Analytical Method

The analytical procedure consists of the following steps.

(1)Estimate the single variable time series model with the Box-Jenkins method.

(2)Estimate the VAR model and compare results.

(3)Formulate the risks and returns for each asset with the estimated model.

(4)Compare the portfolio made based on the risk/return results obtained by the time series model with the portfolio made from the ordinary stochastic model.

3. Results

3.1 Test for Stationarity

Prior to the analysis, we conducted an ADF (Augmented Dickey Fuller) test for stationarity on the data. As Table 2 shows, the stationarity for each data is :

(1)Rp and RL can be verified stationarity.

(2)Rf can be treated as stationary if we assume a relatively low dimension of AR process.

(3)RI can be treated as stationary if we assume a relatively high dimension of AR process.

Table 2 ADF Test Results

AR dimension	Rp	RL	Rf	RI
0	-82.6**	-78.1**	-9.1	-5.4
1	-75.7**	-83.2**	-30.4**	-10.3
2	-58.1**	-26.7*	-23.8*	-10.2
3	-60.4**	-18.3	-26.6*	-30.2**
4	-74.0**	-21.7*	-41.7**	-31.0**

Notes: * Rejects the unit root process at the 5% significance level.

** Rejects the unit root process at the 1% significance level.

In this test, the distribution that should converge is special, and a test based on normal stochastic process cannot be applied. Here our decision is based on critical values obtained empirically from Monte Carlo simulation. Thus, we refereed the significance probability from Fuller(1976) study printed in the "Time Series Analysis in Economics (by Taku Yamamoto)".

All calculations are completed by RATS 3.0.

3.2 Model Estimation with the Box-Jenkins Method

3.2.1 Auto-correlation and Partial Auto-correlation Analysis

We estimated ARMA(p, q) using the Box-Jenkins method. First we obtained the auto-correlations and partial auto-correlations for the data, and then determined approximate p and q values from those configurations (results omitted).

3.2.2 AIC Test

For Rf and RI, which are assumed to have a time series structure, we observed AIC while allowing the degree p of AR to fluctuate. As results:

(1)Rf uses ARMA(2, 0), which has the lowest AIC.

(2)RI uses ARMA(4, 0), which indicates a critical improvement.

Table 3 Identification of ARMA(p, q) with AIC

p	q	Rf	RI
1	0	-14.2	-85.0
2	0	<u>-37.3</u>	-100.9
3	0	-35.7	-99.2
4	0	-33.8	<u>-107.4</u>
5	0	-31.9	-105.4
6	0	-29.9	-104.8

3.2.3 Estimation of Parameters

We estimated the parameters for the return components.

$$R_{pt} = 1.512 + 9.427 u_{pt}$$

(1.35)

$$R_{Lt} = 0.295 - 5.108 \left(R_{ft} - R_{ft-1} \right) + 2.192 u_{Lt}$$

(1.12) (-3.34)

$$R_{ft} = 0.121 + 1.485R_{ft-1} - 0.560R_{ft-2} + 0.183u_{ft}$$
$$\quad (6.27) \quad (15.62) \qquad (-5.86)$$

$$R_{lt} = 0.032 + 1.418R_{lt-1} - 0.549R_{lt-2}$$
$$\quad (2.86) \quad (12.60) \qquad (-2.72)$$
$$+0.486R_{lt-3} - 0.392R_{lt-4} + 0.101u_{lt}$$
$$(2.43) \qquad (-3.47)$$

Note:( ) means t-value.

3.2.4 Cross-Section Analysis

We analyzed the cross-sectional correlations between residuals of the return components.

Table 4 Correlation Coefficients between residuals

	u_p	u_L	u_f
u_p	1.000		
u_L	0.071	1.000	
u_f	-0.031	0.036	1.000

3.3 Model Estimation with VAR Model

3.3.1 Estimation of Parameters

The VAR (vector autoregressive) model which assumes a bi-directional relationship among variables is often used for estimating not only the structure but the relation of cause and effect around economics. In this section, we estimated a VAR model for comparison. The model was estimated with the ordinary least square method (the degree of lag was determined base on AIC).

$$\begin{pmatrix} R_{ft} \\ R_{lt} \\ R_{pt} \\ R_{Lt} \end{pmatrix} = \begin{pmatrix} 1.312 & 0.027 & -0.000 & 0.001 \\ 0.543 & 0.253 & 0.004 & -0.025 \\ 5.846 & -0.374 & -0.163 & 1.196 \\ 0.359 & 0.023 & -0.060 & 0.015 \end{pmatrix} \begin{pmatrix} R_{ft-1} \\ R_{lt-1} \\ R_{pt-1} \\ R_{Lt-1} \end{pmatrix}$$

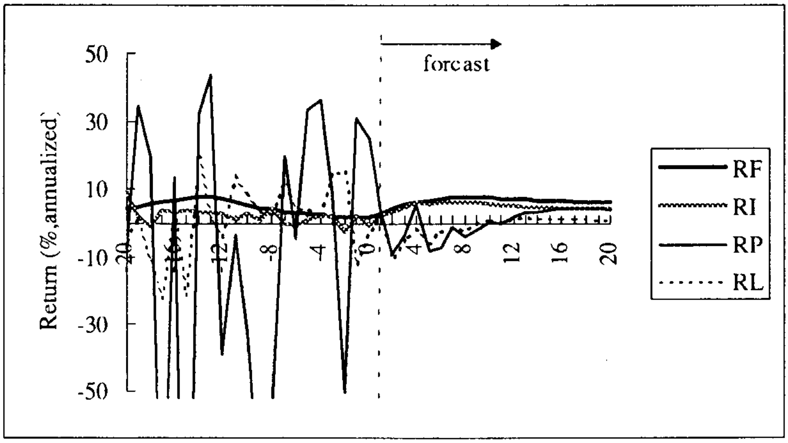
$$+ \begin{pmatrix} -0.516 & 0.018 & 0.003 & -0.011 \\ 0.062 & -0.123 & 0.002 & -0.017 \\ -0.845 & -1.563 & 0.115 & 0.164 \\ 0.302 & 0.188 & -0.010 & 0.037 \end{pmatrix} \begin{pmatrix} R_{ft-2} \\ R_{lt-2} \\ R_{pt-2} \\ R_{Lt-2} \end{pmatrix}$$

$$+ \begin{pmatrix} 0.060 & -0.010 & 0.001 & -0.025 \\ -0.276 & 0.322 & -0.012 & 0.026 \\ -3.351 & -0.177 & 0.036 & 0.599 \\ 0.081 & -0.181 & 0.001 & 0.375 \end{pmatrix} \begin{pmatrix} R_{ft-3} \\ R_{lt-3} \\ R_{pt-3} \\ R_{Lt-3} \end{pmatrix} + \begin{pmatrix} 0.191 \\ -0.073 \\ -0.640 \\ -0.883 \end{pmatrix}$$

3.3.2 Estimation of Scenarios with the VAR Model

As Figure 3 shows, the fluctuation forecasted scenarios about Rp or RL is rather volatile than the others because of the so-called overfitting caused by tight restrictions for cross sectional and time serial relations. Consequently, we don't consider the VAR model as suitable for the ALM simulation model.

Figure 3 Forecast of Scenarios



3.4 Formulation of Risk/Return for Various Assets

3.4.1 Rp (Stock Risk Premium)

When a random walk process is expressed as a probability model, asset price P1 at time 1 can be expressed as follows:

$$P_1 = P_0 \exp(\mu + \sigma z_1)$$

z_t: random drawing from a standardized normal distribution in period t

Price Pn at time n in the future is,

$$P_n = P_0 \exp\{n\mu + \sigma(z_0 + z_1 + \cdots + z_n)\}$$

Thus cum.R_{T+n}, the cumulative return from time T to T+n is,

$$cum.R_{T+n} = n\mu + \sigma(z_1 + z_2 + \cdots + z_n)$$

The stochastic process of stock risk premium Rp is considered to be consistent with those of random walk model. Thus, the expected value and variance of cumulative return of Rp is,

$$E\left(cum.R_{pT+n}\right) = n\mu_p$$

$$Var\left(cum.R_{pT+n}\right) = n\sigma_p^{\ 2}$$

3.4.2 Rf (Risk-free Rate)

Since Rf has a time series structure (AR(2)), it can be expressed as follows :

$$R_{fT+1} = \mu_f + \phi_1 R_{fT} + \phi_2 R_{fT-1} + u_{fT+1}$$

For any arbitrary time in the future, R_{T+n} can be calculated consecutively. E(cum.Rf_{T+n}), the expected cumulative return from period T to T+n, produces the same results calculated consecutively when the all error terms are set at zero, and is easy to obtain from fixed information.

$$E\left(cum.R_{fT+n}\right) = f\left(\mu_f, \phi_1, \phi_2, R_{fT}, R_{fT-1}, n\right)$$

On the other hand, the standard deviation of the residual risk for Rf_{T+n} is as follows:

$$\tilde{e}_{fT+n} = u_{fT+n} - \psi_1 u_{fT+n-1} - \cdots - \psi_{n-1} u_{fT+1}$$

However, $\psi_1 = -\phi_1, \psi_2 = -\left(\phi_1^2 + \phi_2\right), \psi_j = \phi_1 \psi_{j-1} + \phi_2 \psi_{j-2}$

Thus the risk for cumulative return cum.Rf_{T+n} can be obtained as the sum of the residuals .

$$\sum_{i=1}^{n} \tilde{e}_{fT+i} = u_{fT+1} + \{(1-\psi_1)u_{fT+1} + u_{fT+2}\}$$
$$+\{(1-\psi_1-\psi_2)u_{fT+1} + (1-\psi_1)u_{fT+2} + u_{fT+3}\}$$
$$+\{\cdots\}$$
$$+\{\ (1-\psi_1-\psi_2-\cdots-\psi_{n-1})u_{fT+1}+\cdots$$
$$+(1-\psi_1)u_{fT+n-1} + u_{fT+n}\ \}$$
$$= \sum_{i=1}^{n}\left(1-\sum_{j=1}^{n-i}\psi_j\right)u_{fT+i}$$

$$\therefore Var(cum.R_{fT+n}) = Var\left(\sum_{i=1}^{n}\tilde{e}_{fT+1}\right) = \sigma_f{}^2\left\{\sum_{j=1}^{n-i}(1-\psi_j)^2\right\}$$

3.4.3 RL (Bond Premium)

Bond premium RL was modeled as follows:

$$R_{Lt+1} = \mu_L - D(R_{ft+1} - R_{ft}) + \sigma_L u_{Lt+1}$$

Cumulative return cum.R_{T+n} becomes,

$$cum.R_{LT+n} = n\mu_L - D(R_{fT+n} - R_{fT}) + \sigma_L(z_1 + z_2 + \cdots + z_n)$$

Thus the expected value and variance of cumulative return cum.R_{T+n} are,

$$E(cum.R_{LT+n}) = n\mu_L - D\{E(R_{fT+n}) - R_{fT}\}$$
$$Var(cum.R_{LT+n}) = D^2Var(R_{fT+n}) + n\sigma_L^2 = D^2Var(\tilde{e}_{fT+n}) + n\sigma_L^2$$

Remembering that

$$\tilde{e}_{fT+n} = u_{fT+n} - \psi_1 u_{fT+n-1} - \cdots - \psi_{n-1}u_{fT+1}$$

We obtain

$$Var\left(cum.R_{LT+n}\right)=D^2\sigma_f^2\left(1-\psi_1^2-\psi_2^2-\cdots-\psi_{n-1}^2\right)+n\sigma_L^2$$

However, $\psi_1=-\phi_1, \psi_2=-\left(\phi_1^2+\phi_2\right), \psi_j=\phi_1\psi_{j-1}+\phi_2\psi_{j-2}$

3.4.4 Rm, Rg, and Rf (Return on Stocks, Bonds, and Cash)

Having previously estimated the risk premiums Rp, Rf and RL, we now estimate the expected values and variances of returns Rm and Rg, and of the risk-free rate Rf. Since the expected values can be obtained with a simple addition, we look only at the variances.

Let us first look at the variance of Rm:

$$\begin{aligned} Var\left(\tilde{R}_m\right) &= Var\left\{\left(1+\tilde{R}_p\right)\left(1+\tilde{R}_f\right)-1\right\} \\ &\cong Var\left(\tilde{R}_p\right)+Var\left(\tilde{R}_f\right)+Cov\left(\tilde{R}_p,\tilde{R}_f\right) \end{aligned}$$

Here,

$$\begin{aligned} R_{pt} &= \mu_p+\sigma_p u_{pt} \\ R_{ft} &= \mu_f'+\phi_1 R_{ft-1}+\phi_2 R_{ft-2}+\sigma_f u_{ft}' \\ &= \mu_f'+\sigma_f\left(u_{ft}'-\psi_1 u_{ft-1}'-\psi_2 u_{ft-2}'-\psi_3 u_{ft-3}'-\cdots\right) \end{aligned}$$

($MA(\infty)$ expression)

Thus,

$$\begin{aligned} Cov(\tilde{R}_p, \tilde{R}_f) &= \sum_t P_t\{(R_{pt} - \mu_p)(R_{ft} - \mu_f)\} \\ &= \sum_t P_t\left[\sigma_p u_{pt}\{\mu_f + \sigma_f(u'_{ft} - \psi_1 u'_{ft-1} - \psi_2 u'_{ft-2} - \psi_3 u'_{ft-3} - \cdots)\}\right] \\ &= \left\{\sigma_p u_{pt}(\mu'_f - \mu_f)\sum_t P_t u_{pt}\right\} \\ &\qquad + \left\{\sigma_p \sigma_f \sum_t P_t u_{pt}(u'_{ft} - \psi_1 u'_{ft-1} - \psi_2 u'_{ft-2} - \psi_3 u'_{ft-3} - \cdots)\right\} \end{aligned}$$

Since u_{pt}(residual of stock return) and u'_{ft}(residual risk of risk free rate) are white noise, the expected value is zero. If u_{pt} and u'_{ft} are independent, their value when multiplied together will still be zero. As Table 4 shows, the correlation between u_{pt} and u'_{ft} is -0.031, or almost zero. Moreover, while we did not measure the covariances of lags such as $Cov(u_{pt}, u_{ft-n})$, if we assume them to be zero, then

$$Cov(\tilde{R}_p, \tilde{R}_f) = \rho\sigma_p\sigma_f \cong 0$$

Similar to Rm (stocks), we assume Rg (bonds) to have a covariance of zero with Rf (risk-free rate).

3.5 Optimal Portfolio Selection Simulation

3.5.1 Risk/Return Estimation

Based on these formulae, we obtained the risks and returns for any future time periods. For example, we estimate over 12 quarters cumulative risk/return as Table 5.

Table 5 Risk/Return (over 12 quarters) Estimates of the two models

	Time Series Model		Old Model	
	Return	Risk	Return	Risk
Stock	37.47	33.13	43.64	33.03
Bond	17.30	9.93	27.72	8.66
Cash	14.82	5.61	20.04	1.97
	Correlation		Correlation	
Stock-Bond	0.096		0.273	
Bond-Cash	0.565		0.235	
Cash-Stock	0.169		-0.028	

Note: The old model parameters are estimated from the monthly data whose stochastic process would be according to random walk.

3.5.2 Constructing Efficient Frontiers

We built and compared optimal portfolios using the predicted risks and returns (Table 5) of the time series model and the old model over the next three years.

Figure 4 Efficient Frontier

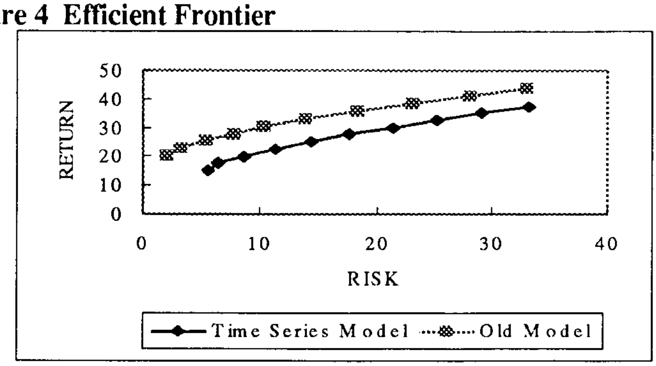


Figure 5 shows a comparison of asset composition ratios obtained along the efficient frontier of each model.

Figure 5.1 Time Series Model

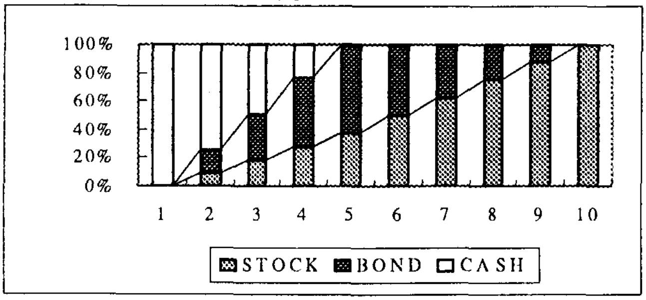
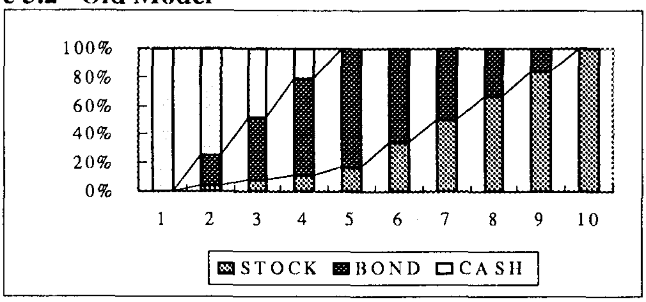


Figure 5.2 Old Model



4. Conclusion

In this paper, for the purpose of use in pension ALM simulation, we estimated an asset return variable model for Japan's stock and bond markets, nominal interest rate, and inflation rate using the Box-Jenkins method. Furthermore, based on the time series model we estimated, we estimated relatively long-term returns and risks, and built an optimal portfolio to compare our results with those obtained by conventional simple mean/variance based on the random walk stochastic process assumption.

To use the single variable time series model related to the asset returns estimated here in an actual pension ALM simulation, the cross-sectional correlation coefficients between the model's residuals are designated as constraints. This is how we express the fluctuating structure that considers the effect of variables each other. In the VAR model, the bi-directional relationship structure among variables can be directly estimated. But when we look at the results of applying VAR, the relationship among the variables tighter than when constrained simply by the covariance of residuals. Thus misunderstanding of the simulation results can be caused by the fluctuate scenarios forecasted with the influence of overfitting. When using VAR, it is better to use a structured VAR model in which any parameter is set to zero beforehand, or the Baysian VAR model, which defines coefficient parameters as probability variables.

The group of single variable time series models estimated here describe the average image of the past, they are not reliable in forecasting the future. The applicability will be to use them considering macroeconomics forecasting, make additional revisions to the model parameters, and use them while keeping their limitations in mind.

The optimal portfolio built using the time series model is quite different from one built using risks and returns obtained by conventional simple method. Leaving aside the question of which estimation method is more accurate, the conventional method was found to invite the possibility of producing erroneous results. The reasons for this are as follows. In other words, the return on short-term financing assets (short-term interest rate) and bond returns, because they express volatility that indicate a time series structure, there is a high possibility that they estimate risks and returns more accurately. Moreover, pension asset management, which assumes an extremely long time horizon, consideration of this type of time series structure is an important issue.

References

Japanese

1. Tanaka, Shuji, and Inui, Koji, "Prospects for Pension ALM," Chosa Geppo, NLI Research Institute, March 1994.
2. Inui, Koji, "Building Financial Markets Models for Pension ALM Simulation," Second Conference of the Japan Finance Society, May 1994.
3. Asako, Kazumi, and Mura, Tatsuo, Interest Rates and Inflation: On Japan's Gibson Paradox, MOF Fiscal and Monetary Research Center, March 1991.
4. Asano, Yukihiro, Pension Obligations and Asset Management," Sumitomo Trust Bank Investment Research Department, Working Paper No. 23.
5. Adachi, Seiji, On Interest Rate Movements, Daiwa Investment Report, March 1994.
6. Iwabuchi, Junichi, "On the Effect of Monetary Variables on Real Variables," BOJ, Monetary Research, October 1990.
7. Shimizu, Hiroshi, "On the Formulation Process of the Wilke Model," Kyushu University.
8. Yamamoto, Taku, Economic Time Series Analysis, Sobunsha.
9. Yoshida, Tomoo, "On the Stability of Currency Supply and Demand Function, BOJ, Monetary Research, October 1989.

English

1. Hamao, Yasushi, "A Standard Dababase for the Analysis of Japanese Security Markets," Journal of Business, 1991, vol. 64, no. 1.
2. Wilkie, A.D., "A Stochastic Investment Model for Actuarial Use," T.F.A. 39, 1986.
3. Roger G. Ibbotson, and Rex A. Sinquefield, "Stocks, Bonds, Bills and Inflation: Simulations of the Future (1976-2000)," Journal of Business, July 1976.
4. Daykin, C.D., T. Pentikäinen, and M. Pesonen, "Practical Risk Theory for Actuaries," Chapman & Hall.