IMMUNIZATION IS DEAD. LONG LIVE IMMUNIZATION.

(A Note on a Form of Return Enhancement in the South African Bond Market)

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Abstract
This note describes an approach that combines linear optimization with Taylor's theorem as used by Redington, but expanded to allow the inclusion of options in the investment selection process. The final product is a framework that provides excess relative returns by arbitraging what appear to be mispriced options.

There are three sections to the note, namely a description of the South African bond market, the theoretical development and two examples of how it can be used. The first section is included as general background and to give some insight as to why the approach developed as it did.

Everything in the note refers to bonds or options on bonds and the word "volatility" is used as in options terminology. For "derivative" read "partial derivative" where appropriate. Lastly, it may be helpful to re-read the first two pages of section three of Redington's 1952 paper before reading section two in this note.

Keywords
Optimization, Options, Fixed Interest

SECTION 1: THE SOUTH AFRICAN BOND MARKET

Until 1979 the secondary market in South Africa was for practical purposes non-existent. In 1979 the rules of the Johannesburg Stock Exchange (JSE) were changed to permit dual capacity trading and effectively, negotiated commissions (a relatively low maximum commission was introduced and brokers were allowed to charge less at their discretion).

Turnover grew rapidly (see table below) and the banks, who had hitherto shown little interest in a secondary market, became active players, and have advertised prices via Reuters since 1982. Today it is thought that more turnover takes place outside of the JSE than on the JSE but different counting conventions prevent a true comparison. The table represents JSE turnover only.

Calendar year	Turnover (R million nominal)
1978	625
1979	2 916
1980	4 219
1981	5 100
1982	16 640
1983	23 071
1984	26 573
1985	51 220
1986	106 939
1987	165 192
1988	175 763
1989	195 371
1990	238 409
1991	249 115
1992	551 234
1993	746 000
1994	967 000

The major issuers are the central government, Eskom (electricity supplier), Transnet (railways, national airline) and Telkom (telecommunications utility). From the amounts in issue as at 1 June 1993 (given below) seen in relation to the turnover figures given above it can be seen that it is a surprisingly liquid market, with annual turnover as a percentage of the amount in issue running at around 300%.

Issuer	**Nominal amount in issue (R million)**
RSA	136 410
ESKOM	22 548
TRANSNET	9 350
TELKOM	7 000
OTHER	6 929
Total	182 237

There are three characteristics of the South African market that may seem a little strange to participants in other markets. Firstly, trade takes place at a "yield to maturity" which is converted to a price for settlement purposes. Secondly, although there is a fairly well-developed option market, futures turnover is almost non-existent. Finally, settlement takes place on the second Thursday following the day upon which a deal is struck. This introduces an element of forward trading which may partially explain why the futures market has not developed.

Strike prices for options are also expressed in terms of "yield to maturity" (ytm for brevity) and strike dates are standardised to the first Thursday of February, May, August and November.

The option pricing model most commonly used is Black-Scholes with a fairly crude adjustment for coupon payments. Perhaps a handful of participants use the Binomial model.

SECTION 2: THEORETICAL DEVELOPMENT

The concept "ytm" fell into disfavour with theoreticians towards the end of the seventies and early eighties to be replaced by the concept of "spot rates".

In South Africa, however, the practice of trading at a ytm more or less perpetuated the concept and any theoretical objections were shrugged aside by the pragmatic attitude that it is cheaper to buy a particular stock at say 15.00% than at 14.80%.

It is probably appropriate at this point to examine the theoretical objections to using a ytm. The first objection is that in order to actually earn the ytm one would have to re-invest coupons on average at the ytm. Since for each price there is only one ytm and for each ytm there is only one price, price and ytm are really just different ways of looking at the same thing. This objection is therefore clearly nonsense.

The second objection is best illustrated by an example. Consider two stocks, a 15% 15.05.1997 and a 15% 15.05.2007 with ytm's of 10% p.a. and 12% p.a., respectively. The reasoning goes that it is illogical to discount two identical streams of income (7.5 per half-annum from now until 15.05.1997) at two different rates of interest.

This led to the development of the "spot rate" concept, with income receivable at time t being associated with an interest rate i_t and most theoretical developments in the 1980's and 1990's have been based on this framework.

However, the framework that one chooses should surely be a function of the particular application at hand, and the answer to the question "*At what pace will my portfolio grow over time if nothing else changes?*" is far more easily answered in a ytm framework than in a spot-rate framework.

In the earlier work, all that was done was to optimise ytm subject to the portfolio having what the Americans call the same "duration" as that of the South African All-Bond Index. The British call it "discounted mean term" and the writer prefers "first partial derivative of value with respect to force of interest, per unit of value". A little pedantic, perhaps, but necessary for further theoretical development. The results obtained were only mildly encouraging, with typical yield pick-ups of between 30 and 50 points.

It was then decided to attempt to introduce options into the process and it was reasoned that the key to this lay in adopting measures of risk and reward that apply equally to both stock and options.

Although the first derivative of value with respect to time per unit of value is a satisfying measure of reward for stock, the acceleration in the so-called "time decay" of options makes it inappropriate for the latter.

It therefore became necessary to introduce a time horizon, and the measure of reward finally adopted was:

"Value added over the time horizon chosen, per unit of value at the start, should interest rates and volatility remain unchanged."

For measures of risk, there was no need to look any further than Taylor's theorem as used by Redington but replacing V_L with V_B where V_B is the value of some benchmark that one

is attempting to outperform.

To wit:

Let V_B = Value of benchmark at force of interest δ

V_P = Value of portfolio at force of interest δ.

Then, if $V_B = V_P$ and the force of interest changes from δ to $\delta+\epsilon$ ($\epsilon < 0$ or $\epsilon > 0$) and V'_B and V'_P denote the new value of V_B and V_P respectively,

$$V'_P - V'_B = (V_P - V_B) + \varepsilon\frac{d}{d\delta}(V_P - V_B) + \frac{\varepsilon^2}{2!}\frac{d^2}{d\delta^2}(V_P - V_B) + \frac{\varepsilon^3}{3!}\frac{d^3}{d\delta^3}(V_P - V_B) + \frac{\varepsilon^4}{4!}\frac{d^4}{d\delta^4}(V_P - V_B) + \ldots\ldots\ldots^{\infty}$$

The risk measures are thus the various orders of derivatives of value with respect to force of interest, but expressed per unit of value in order to facilitate the selection process.

Early tests showed that it was necessary to go beyond second order derivatives with respect to force of interest in the case of options and later tests showed that it was adequate to stop at fourth order derivatives. This is a function of interest rate levels, however, and findings in other countries may differ.

Also referring to options, first order derivatives with respect to volatility and the risk-free rate of interest have proved sufficient. This too is a function of interest rates levels.

In order to permit the writing or granting of options it was necessary to allow the x's in the general linear optimisation problem to be conceptually negative.

The approach can now finally be expressed as:

"Optimise the value added to the portfolio over the time horizon chosen, should interest rates and volatility remain unchanged, subject to the risk measures on the horizon date being such that a uniform interest rate change across the yield curve will not cause underperformance and subject to volatility and risk-free interest rate changes not causing intolerable underperformance."

The best results seem to be achieved by choosing the time horizon to fall one week before the option close-out date, which in South Africa implies four optimisations per year. The choice is partly due to the South African practice of Thursday settlements, and the idea was

to give the dealer a week in which to execute the transactions required in order to restructure the portfolio according to the latest optimisation.

SECTION 3: EXAMPLES

A little reflection leads one to realise that the approach allows one to simulate or synthesize the behaviour of any fixed interest instrument that one chooses, at terms that are often more favourable than those directly available in the market. The instrument chosen could be a particular stock, and index, liabilities of a fixed interest nature, a three-month treasury bill or even cash. In the last case the time horizon would be changed appropriately.

A point worth noting is that once a portfolio has been put in place, it is not necessary to change anything until the next re-optimisation date. This distinguishes the process from the typical delta-neutral curve trading techniques, which require continual changes in stock holdings as interest rates change.

The two examples given are the South African All-Bond Index and a three month zero coupon bond.

EXAMPLE 1

ALL-BOND INDEX VALUATION AT CLOSE ON 28.10.1993 FOR SETTLEMENT 11.11.1993									
Code	**Stock description**	**Duration**	**Market yield**	**Nominal (R million)**	**Market value (R million)**	**% of portfolio**	**Running yield**	**All-in price**	**Convexity**
R156	12.00% 15.07..1994	0.61	11.64	3783	3936.949	2.34	11.98	104.07	0.70
E158	9.25% 01.09..1994	0.74	11.20	1534	1538.969	0.92	9.39	100.32	0.90
T013	15.00% 01.10..1995	1.60	11.67	3550	3803.878	2.26	14.22	107.15	3.52
R004	12.50% 30.11.1995	1.78	11.58	5612	5668.840	3.37	12.30	101.01	4.13
E167	12.00% 01.05.1996	2.08	11.66	2010	2030.808	1.21	11.92	101.04	5.62
R145	12.50% 30.05.1996	2.15	11.62	8139	8242.427	4.91	12.27	101.27	5.95
TK07	13.00% 30.06.1996	2.10	11.76	2000	2148.782	1.28	12.67	107.44	5.91
R146	12.50% 30.11.1996	2.50	11.69	6329	6417.903	3.82	12.25	101.40	7.96
R119	14.00% 15.08.1997	2.81	12.02	9057	9885.699	5.89	13.24	109.15	10.49
TK05	12.00% 31.03.1998	3.28	12.20	2000	2013.785	1.20	12.09	100.69	13.95
E169	15.00% 01.10.1998	3.44	12.16	2641	2955.007	1.76	13.62	111.89	15.71
R160	10.75% 15.12.1998	3.62	12.18	1000	990.411	0.59	11.36	99.04	17.54
R147	11.50% 30.05.2000	4.48	12.51	17336	16468.890	9.80	12.04	95.00	26.29
T001	12.50% 01.04.2002	5.02	12.70	500	501.802	0.30	12.64	100.36	35.32
R150	12.00% 28.02.2005	5.80	12.91	35773	34702.474	20.66	12.69	97.01	50.80
UG55	15.00% 30.09.2005	5.65	13.44	1400	1551.497	0.92	13.76	110.82	49.27
TK01	10.00% 31.03.2008	6.65	13.10	3000	2438.277	1.45	12.49	81.28	69.49
T004	7.50% 01.04.2008	7.13	12.90	4300	2833.415	1.69	11.54	65.89	77.78
E168	11.00% 01.06.2008	6.73	12.90	13970	12164.861	7.24	12.55	87.08	70.25
R153	13.00% 31.08.2010	6.57	13.10	27160	27666.137	16.47	13.10	101.86	72.47
R157	13.50% 15.09.2015	6.97	13.15	18230	19064.257	11.35	13.18	104.58	86.90
E170	13.50% 01.08.2020	7.10	13.11	893	951.614	0.57	13.14	106.56	95.65
				170217	167976.681	100.00			

From the above table, the following can be computed:

- Index duration 4.97
- Index convexity 45.53
- Index running yield 12.68
- Index yield to redemption 12.60
- Clean market value of index (R million) 165472.909
- All-in market value of index (R million) 167976.681.

The important things to note are that the index has a first partial derivative of value with respect to force of interest of 4.97 (the "duration") and a second partial derivative of the value with respect to force of interest of 45.53 (the "convexity").

SYNTHESISED ALL BOND INDEX

The universe of assets from which the synthesised All-Bond Index was constructed was:

(a) all of the stocks that make up the All-Bond Index; plus
(b) call and put options on R150's and E168's, all with a strike date of 3 February 1994 and with strike yields as follows:

		Strike Yields (%)
R150	Calls	13.00,12.75 and 12.50
R150	Puts	12.75, 13.00 and 13.25
E168	Calls	13.00, 12.75 and 12.50
E168	Puts	12.75,13.00 and 13.25

The idea was to allow the process to select from both "in the money" and "out of the money" options.

The process was to optimise the value added by 27 January 1994 (the horison date) subject to

(a) 1st and 3rd order derivatives of value with respect to force of interest being equal to those of the index;
(b) 2nd and 4th order derivatives with respect to force of interest being greater than or equal to those of the index;
(c) No more than 20% of the total market value of the portfolio being in any one stock; and
(d) No more than 0.5% of the total market value of the portfolio being in any one option (either long or short).

The outcome was a selection of six stocks, long positions in five options and short positions in five options, as set out below.

Stocks selected:

Code	Stock	Dura-tion	Con-vexity	Market yield	Nomi-nal (R m)	All-in price	% of Market value
UG55	15.00% 30.09.2005	5.70	49.91	13.44	18	109.99	19.8
TK01	10.00% 31.03.2008	6.70	70.21	13.10	25	80.68	20.2
E168	11.00% 01.06.2008	6.39	66.83	12.90	1	91.88	0.9
R153	13.00% 31.08.2010	6.62	73.23	13.10	20	101.12	20.2
R157	13.50% 15.09.2015	7.02	87.69	13.15	19	103.81	19.7
E170	13.50% 01.08.2020	7.15	96.44	13.11	19	105.79	20.1
Total market value R100 935 000							

Options selected

Position	Nominal (R m)	Stock code	Type	Strike yield %	Strike date	Value (R)
Long	48	E168	Call	12.50	03.02.1994	501 206
Long	38	E168	Put	13.25	03.02.1994	499 269
Long	26	E168	Put	13.00	03.02.1994	497 123
Long	2	E168	Put	12.75	03.02.1994	53 523
Long	16	R150	Put	13.25	03.02.1994	237 320
Short	18	R150	Call	13.00	03.02.1994	398 824
Short	31	R150	Call	12.75	03.02.1994	500 286
Short	35	R150	Call	12.50	03.02.1994	395 364
Short	24	R150	Put	13.00	03.02.1994	498 562
Short	73	R150	Put	13.75	03.02.1994	496 826
Net market value of options: - R501 421 (i.e. net "written")						
Total market value of stock plus options R100 433 579						

Although not obvious, the stock portfolio above, when combined with the option portfolio, behaves similarly to the index under uniform interest rate change.

BEHAVIOUR OF SYNTHESIZED ALL-BOND INDEX UNDER INTEREST RATE CHANGE

The table below compares the performance of the optimal portfolio with that of the index under uniform interest rate changes on the horizon date. Column 5 (the portfolio value excluding interest receipts) equals column 2 plus column 3 less column 4. The starting value was R100 433 579. The excess performance (column on extreme right) is expressed as an annualized return and ranges from 1.52% per annum to 2.19% per annum. The

variance of this column is not fully understood by the writer but the level of excess performance is quite encouraging.

Of practical concern is the fairly rapid deterioration in the excess performance as yields move from +95 points to +100 points. Here a little "hand polishing" is probably in order. One could start by repeating the process but writing 72 instead of 73 of the 13.75% February puts in R150's and so on until one feels comfortable.

BEHAVIOUR OF SYNTHESIZED ALL-BOND INDEX UNDER UNIFORM INTEREST RATE CHANGE

Points change	Stock value	Long options value	Short options value	Portfolio value excluding interest	Index value including interest	Interest on portfolio	Portfolio value including interest	Return on index % p.a.	Return on portfolio % p.a.	Excess performance % p.a.
1	2	3	4	5	6	7	8	9	10	11
-100	109 444 623	1 808 616	3 888 432	107 364 807	109 645 100		110 133 821	30.45	32.11	1.66
-95	109 071 771	1 633 519	3 633 644	107 091 464	109 366 200		109 853 615	29.5	31.16	1.66
-90	108 701 076	1 499 234	3 379 973	106 820 337	109 088 500		109 575 309	28.56	30.21	1.65
-85	108 332 524	1 345 763	3 127 405	106 550 882	108 812 100		109 298 904	27.63	29.28	1.65
-80	107 966 097	1 193 097	2 880 658	106 278 536	108 537 100		109 019 534	26.7	28.33	1.63
-75	107 601 780	1 048 477	2 637 6.7	106 012 650	108 263 300		108 746 791	25.78	27.41	1.63
-70	107 239 558	908 541	2 398 830	105 749 268	107 990 700		108 476 617	24.86	26.5	1.64
-65	106 879 415	774 741	2 165 266	105 488 889	107 719 500		108 209 522	23.95	25.6	1.65
-60	106 521 337	448 793	1 937 947	105 232 183	107 449 400		107 946 195	23.04	24.71	1.67
-55	106 165 307	532 519	1 721 209	104 976 617	107 180 700		107 684 038	22.14	23.83	1.69
-50	105 811 311	427 642	1 515 860	104 723 094	106 913 200		107 423 976	21.25	22.96	1.71
-45	105 459 335	335 627	1 322 245	104 472 716	106 646 900		107 167 142	20.36	22.1	1.74
-40	105 109 363	257 512	1 141 737	104 225 139	106 381 800		106 913 178	19.47	21.25	1.78
-35	104 761 381	193 815	975 464	103 979 732	106 117 900		106 661 443	18.6	20.41	1.81
-30	104 415 375	144 553	825 747	103 734 181	105 855 300		106 409 558	17.72	19.57	1.85
-25	104 071 329	109 331	694 046	103 486 615	105 593 800		106 155 607	16.85	18.72	1.87
-20	103 729 231	87 535	579 843	103 236 923	105 333 600		105 899 476	15.99	17.87	1.88
-15	103 389 065	78 540	483 525	102 984 079	105 074 500		105 640 111	15.13	17.01	1.88
-10	103 050 818	81 910	405 198	102 727 529	104 816 700		105 376 945	14.28	16.14	1.86
-5	102 714 475	97 545	344 689	102 467 331	104 560 000		105 110 036	13.44	15.25	1.81
0	102 380 024	125 733	301 540	102 204 217	104 304 400	2 635 919	104 840 136	12.59	14.36	1.77

BEHAVIOUR OF SYNTHESIZED ALL-BOND INDEX UNDER UNIFORM INTEREST RATE CHANGE

Points change	Stock value	Long options value	Short options value	Portfolio value excluding interest	Index value including interest	Interest on portfolio	Portfolio value including interest	Return on index % p.a.	Return on portfolio % p.a.	Excess performance % p.a.
1	2	3	4	5	6	7	8	9	10	11
0	102 380 024	125 733	301 540	102 204 217	104 304 400	2 635 919	104 840 136	12.59	14.36	1.77
5	102 047 451	167 126	275 003	101 939 573	104 050 000		104 568 667	11.76	13.46	1.70
10	101 716 741	222 625	264 074	101 675 293	103 796 800		104 297 570	10.93	12.57	1.64
15	101 387 882	293 220	267 518	101 413 584	103 544 700		104 029 112	10.1	11.69	1.59
20	101 060 861	379 803	283 955	101 156 709	103 293 800		103 765 612	9.28	10.82	1.54
25	100 735 663	482 982	311 953	100 906 693	103 044 000		103 509 148	8.46	9.98	1.52
30	100 412 277	602 956	350 154	100 665 080	102 795 300		103 261 303	7.65	9.17	1.52
35	100 090 698	739 400	397 405	100 432 684	102 547 700		103 022 914	6.84	8.39	1.55
40	99 770 886	891 495	452 884	100 209 497	102 301 200		102 793 971	6.04	7.64	1.60
45	99 452 856	1 057 848	516 189	99 994 515	102 055 900		102 573 444	5.24	6.92	1.68
50	99 136 585	1 236 688	587 385	99 785 888	101 811 600		102 359 437	4.45	6.23	1.78
55	98 822 062	1 426 026	667 013	99 581 075	101 568 400		102 149 341	3.66	5.54	1.88
60	98 509 275	1 623 791	755 993	99 377 072	101 326 300		101 940 077	2.88	4.87	1.99
65	98 198 209	1 827 980	855 515	99 170 674	101 085 300		101 728 355	2.1	4.18	2.08
70	97 888 854	2 037 194	966 862	98 959 187	100 845 400		101 511 414	1.32	3.48	2.16
75	97 581 198	2 249 613	1 091 388	98 739 423	100 606 500		101 285 982	0.55	2.75	2.20
80	97 275 228	2 463 741	1 230 104	98 508 865	100 368 700		101 049 478	-0.21	1.98	2.19
85	96 970 932	2 678 696	1 383 085	98 266 543	100 131 900		100 800 906	-0.97	1.18	2.15
90	96 668 299	2 893 796	1 550 288	98 011 807	99 896 190		100 539 600	-1.73	0.34	2.07
95	96 367 317	3 108 596	1 731 133	97 744 780	99 661 500		100 265 687	-2.48	-0.54	1.94
100	96 067 974	3 322 800	1 924 509	97 466 265	99 427 860		99 979 989	-3.23	-1.46	1.77

EXAMPLE 2: SYNTHESISED 91-DAY TREASURY BILL

Stocks selected:

Code	Stock	Duration	Convexity	Market yield %	Nominal (R m)	All-in price	Market value (R m)
R156 12.0%	15.07.1994	0.61	0.70	11.64	19	104.07	19.8
T013 15.0%	01.10.1995	1.60	3.52	11.67	14	107.15	15.0
UG55 15.0%	30.09.2005	5.65	49.27	13.44	18	110.82	19.9
R153 13.0%	31.08.2010	6.57	72.47	13.10	6	101.86	19.9
R157 13.5%	15.09.2015	6.97	86.90	13.15	19	104.58	20.0
E170 13.5%	01.08.2020	7.10	95.65	13.11	19	106.56	20.2
Market value of stocks: R100 950 100							

Options selected

Position	Nominal (R m)	Stock code	Type	Strike yield %	Strike date	Value (R)
Long	55	E168	Call	12.50	03.02.1994	498 720
Long	44	E168	Put	13.25	03.02.1994	497 101
Long	29	E168	Put	13.00	03.02.1994	496 911
Long	20	E168	Put	12.75	03.02.1994	495 628
Long	27	R150	Put	13.25	03.02.1994	346 756
Short	24	E168	Call	13.00	03.02.1994	492 431
Short	19	E168	Call	12.75	03.02.1994	266 861
Short	24	R150	Call	13.00	03.02.1994	497 909
Short	34	R150	Call	12.75	03.02.1994	498 160
Short	39	R150	Call	12.50	03.02.1994	384 734
Short	94	R150	Put	13.75	03.02.1994	502 651
Net market value of options - R307 630 (i.e. net "written")						
Total net market value R100 642 470						

Here it is worth noting the similarity between the stocks selected to simulate the All-Bond Index and the much shorter three-month zero coupon bond. It is the options that "shorten" the portfolio to the appropriate interest rate sensitivity level.

BEHAVIOUR OF SYNTHESIZED 91-DAY TREASURY BILL UNDER INTEREST RATE CHANGE							
Points	Stock value	Long options value	Short options value	Portfolio value excluding interest	Interest on port-folio	Portfolio value	Return on port-folio %
-100	106 200 207	2 072 376	7 172 575	101 100 008		103 922 934	13.14
-95	105 942 397	1 894 665	6 739 413	101 097 649		103 920 510	13.13
-90	105 686 059	1 717 893	6 308 268	101 095 683		103 918 489	13.12
-85	105 431 181	1 542 067	5 879 120	101 094 128		103 916 891	13.11
-80	105 177 753	1 367 199	5 457 218	101 087 734		103 910 317	13.09
-75	104 925 762	1 201 616	5 040 183	101 087 195		103 909 764	13.08
-70	104 675 199	1 041 525	4 628 647	101 088 077		103 910 671	13.09
-65	104 426 051	888 694	1 223 638	101 091 107		103 913 786	13.1
-60	104 178 309	745 247	3 826 259	101 097 297		103 920 148	13.13
-55	103 931 960	613 510	3 441 236	101 104 234		103 927 279	13.15
-50	103 686 995	495 793	3 071 798	101 110 990		103 934 223	13.18
-45	103 443 402	394 211	2 717 755	101 119 859		103 943 340	13.22
-40	103 201 172	310 491	2 380 491	101 131 172		103 954 968	13.27
-35	103 960 293	245 828	2 061 595	101 144 527		103 968 696	13.32
-30	102 720 756	200 871	1 764 220	101 157 407		103 981 936	13.38
-25	102 482 550	175 761	1 492 817	101 165 494		103 990 250	13.41
-20	102 245 665	170 279	1 247 049	101 168 895		103 993 746	13.42
-15	102 010 092	184 037	1 026 576	101 167 553		103 992 366	13.42
-10	101 775 819	216 670	831 739	101 160 750		103 985 372	13.39
-5	101 542 837	267 998	662 388	101 148 448		103 972 727	13.34
0	101 311 137	338 119	517 836	101 131 419	2 823 804	103 955 223	13.27
5	101 080 708	427 432	396 896	101 111 244		103 934 484	13.18
10	100 851 541	536 567	297 954	101 090 154		103 912 806	13.1
15	100 623 627	666 256	219 038	101 070 845		103 892 957	13.01
20	100 396 956	817 162	157 981	101 056 137		103 877 839	12.95
25	100 171 519	989 681	112 572	101 048 628		103 870 120	12.92
30	99 947 306	1 183 798	80 753	101 050 351		103 871 892	12.93
35	99 724 308	1 398 934	60 802	101 062 440		103 884 318	12.98
40	99 502 517	1 634 399	51 501	101 085 415		103 907 934	13.08
45	99 281 923	1 888 010	52 250	101 117 682		103 941 103	13.21
50	99 062 517	2 157 332	63 118	101 156 730		103 981 240	13.37
55	98 844 290	2 439 904	84 819	101 199 375		104 025 076	13.55
60	98 627 234	2 733 133	118 595	101 241 772		104 068 657	13.73
65	98 411 339	3 034 490	166 039	101 279 791		104 107 737	13.89
70	98 196 598	3 342 135	228 865	101 309 868		104 138 654	14.02
75	97 983 002	3 653 774	308 639	101 328 137		104 157 434	14.09
80	97 770 541	3 967 526	406 550	101 331 517		104 160 980	14.11
85	97 559 209	4 282 239	523 224	101 318 223		104 147 243	14.05
90	97 348 996	4 597 017	658 583	101 287 430		104 115 590	13.92
95	97 139 894	4 911 272	811 856	101 239 310		104 066 126	13.72
100	96 931 895	5 224 612	981 612	101 174 895		103 999 912	13.45
91-day Treasury bill at time of simulation, converted to a return: 12.25%.							

CONCLUSION

The use of formulae has been limited in the hope that this would lead to a wider readership. In addition, certain details have deliberately been left out in the hope that this may stimulate discussion. If the impression has been created that the project was first of all thought through in an orderly fashion and then tested calmly in practice with immediate success, it is a false impression. The old proverb "if you don't know where you're going, it doesn't matter how you get there," gives the writer great comfort.

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