PRICING INTEREST RATE INSURANCE

CARLO MOTTURA
UNIVERSITY OF ROME "LA SAPIENZA"
DEPARTMENT OF ACTUARIAL AND FINANCIAL SCIENCES
VIA NOMENTANA, 41
00161 ROME - ITALY
TEL : (0039.6) 44.250.297
FAX : 44.250.289
E-MAIL : ECOFIN@ITCASPUR.BITNET

Abstract :

An interest rate insurance written on a stream of stochastic cash flow is a policy that guarantees an assigned holding period return on the stream, against random shift of the term structure of interest rates. The problem has already been considered for deterministic cash flows. In the work, interest rate insurance has been characterized in terms of interest rate options either with deterministic and stochastic exercise price. The insurance premium has been analyzed in continuous time framework and closed form solutions have been derived in the Cox, Ingersoll & Ross model.

Introduction

Let us consider, at time t (valuation time), a stream of stochastic cash flows X_k to be paid at time t_k, represented by a random vector $\mathrm{X} = \{X_k\}$, $(k = 1, 2, \ldots, m)$ and a "target" $L > 0$ with maturity H $(t \leq H)$. An interest rate insurance on cash flow X is a policy that guarantees the holding period return corresponding to L, against random shift of the term structure of interest rates.

The economic agent, to cover interest rate risk on cash flow X until time H, will pay a premium, as in every insurance policy. To determine this cost, let us consider the payoffs to the various claims on the maturity date. If the income $R(H, \mathrm{X})$ produced by X exceed the promised payment L, then the agent will receive $R(H, \mathrm{X})$; if the value is less than the promised payment, then the guarantor has a net payout or loss of $[L - R(H, \mathrm{X})]$. Hence, if $G(r(H), H, \mathrm{X}, L)$ is the value of the insurance premium at maturity, it results:

$$G(r(H), H, \mathrm{X}, L) = max\{0, L - R(H, \mathrm{X})\}\,, \tag{1}$$

that is identical to the payoff structure of a put option on the stream X, with exercise price L and maturity H. In this sense, pricing interest rate insurance can be characterized as an application of bond option pricing theory.

This methodolgy was originally used by Merton with reference to the problem of deposit insurance pricing model that "has as its foundation the isomorphic relantionship between deposit insurance and common stock put options" (1) .

In the paper, interest rate insurance has been characterized in terms of interest rate options either with deterministic and stochastic exercise price. In section 2, the continuous market model is defined; the general setting of the pricing model for the insurance premium is described in

(1) In particular, Merton priced the premium of deposit insurance by the classical Black-Scholes model (see [7], pp.13-14, [5], p.4 and [6]). Recently, Boyle ([1]) considered the same problem referring to pricing models with barriers.

section 3, considering the case of a policy written on deterministic cash flows (subsection 3.1) and the case of stochastic cash flows (subsection 3.2). In section 4, closed form solutions have been derived in the one factor Cox, Ingersoll e Ross term structure model. An application to interest rate sensitive bond portfolios has been considered in section 5. The insurance premium appears a quite relevant risk measure, even with respect to the results of financial immunization theory, as it allows to characterized a "price" for interest rate risk.

2. The market model

The value (price) of X at time t, by the no arbitrage principle, is assumed to be a continuous linear functional, given by the expression:

$$P(t, \mathrm{X}) = \sum_{k=1}^{m} \widetilde{\mathrm{E}}_t \left[X_k \, \varphi(t, t_k) \right] \quad , \tag{2}$$

where $\varphi(t, t_k)$ is the stochastic discount factor from time t_k to time t and $\widetilde{\mathrm{E}}$ represents the expectation operator, given the information at time t, under the relevant probability distribution ("risk neutral" probability measure).

Referring to continuous-time models, the model price (2) can be written as:

$$P(t, \mathrm{X}) = \sum_{k=1}^{m} \widetilde{\mathrm{E}}_t \left[X_k \, e^{-\int_t^{t_k} r(u) du} \right], \tag{3}$$

where r is the (nominal) spot rate. In the case of interest rate sensitive (IRS) contracts, the price $P(t, \mathrm{X})$ depends, by definition, on the spot rate process $\{r(t)\}$. If $\mathbf{1}_k$ is a unitary zero coupon bond with maturity t_k, paying one unit of money at time t_k (with certainty), then: $P(t, \mathbf{1}_k) = \widetilde{\mathrm{E}}_t \left[e^{-\int_t^{t_k} r(u) du} \right] = v(t, t_k)$, is the discount factor from time t_k to time t.

In general, in continuous-time markov models, the random evolution of the spot rate, $r(t)$, the state variable, can be described by the Ito stochastic differential equation: $dr(t) = f(r(t),t)dt + g(r(t),t)dZ(t)$, where f and g^2 are the infinitesimal parameters of the diffusion process $\{r(t)\}$, $\{Z(t)\}$ is the standard Brownian motion; under the usual market assumption, by Ito's lemma and the no arbitrage argument, the model price is the solution of the general valuation equation $P_t + \widetilde{f} P_r + \frac{1}{2} g^2 P_{rr} - rP = 0$, with the proper boundary condition [(2)] , being $\widetilde{f}$ the risk-adjusted drift.

The income $R(H, \mathrm{X})$ produced by X at time H $(t \leq H \leq t_m)$ for the holding period $H - t$, can be expressed in the form:

$$R(H,\mathrm{X}) = R'(H,\mathrm{X}) + R''(H,\mathrm{X})$$

$$= \sum_{t_k \leq H} \frac{X_k}{v(t_k, H)} + \sum_{t_k > H} \widetilde{\mathrm{E}}_H \left[X_k \varphi(H, t_k)\right], \tag{4}$$

where $R'(H,\mathrm{X})$ is the sum of the payments in the time interval $[t, H]$ and of their reinvestment until time H (at the market accumulation factors, $v(t_k, H)^{-1}$, known at time H), and $R''(H,\mathrm{X})$ is the price, at time H, of the residual cash flows after H.

(2) As opposed to IRS contracts, we can consider random variables X_k independent on the process $\{r(t)\}$, in order to model, for example, derivatives on equities. It results: $P(t,\mathrm{X}) = \sum_{k=1}^{m} \widetilde{\mathrm{E}}_t \left[X_k\right] \widetilde{\mathrm{E}}_t \left[e^{-\int_t^{t_k} r(u)du}\right]$.

3. The general valuation framework

3.1 Interest rate insurance on deterministic cash flows

If X is a vector of deterministic cash flows, expression (3) and (4) can be rewritten in the form:

$$P(t,\mathbf{X}) = \sum_{k=1}^{m} X_k \,\widetilde{\mathbf{E}}\,[e^{-\int_t^{t_k} r(u)du}] = \sum_{k=1}^{m} X_k\, v(t,t_k) \quad ,$$

and:

$$R(H,\mathbf{X}) = \sum_{t_k \le H} \frac{X_k}{v(t_k,H)} + \sum_{t_k > H} X_k v(H,t_k) \quad ,$$

where $v(t,t_k)$ describes the price structure of the market at time t. The boundary condition (1) is path-dependent and this determines relevant problems for pricing the premium if a continuous evolution of the spot rate $r(t)$ is assumed. To avoid these problems, we will consider the following assumption:

$$\sum_{t_k \le H} \frac{X_k}{v(t_k,H)} = \sum_{t_k \le H} X_k \mathbf{E}_t \left[\frac{1}{v(t_k,H)}\right] , \tag{5}$$

considering, at time t, the reinvestment component produced by X at time H as an exogenous variable, in terms of its expected value calculated under the natural probability measure at time t [(3)] . Expression

(3) We could also assume:

$$\sum_{t_k \le H} \frac{X_k}{v(t_k,H)} = \sum_{t_k \le H} \frac{X_k}{v(t,t_k,H)} = \sum_{t_k \le H} X_k \frac{v(t,t_k)}{v(t,H)} ,$$

considering, at time t, the value of the reinvestment component at time H, as its forward value calculated by the forward rates, $v(t,t_k,H)$, derived from the term structure of spot rates at time t. This assumption is operationally consistent being Forward Rate Agreement (FRA) daily quoted on international capital markets. The reinvestment rates, otherwise, could be strategically fixed.

(1) can be written again in the form:

$$G(r(H), H, \mathrm{X}, L') = max\{0, L' - R''(H, \mathrm{X})\}\,, \tag{6}$$

that is identical to the payoff structure of a put option with maturity H and exercise price $L' = L - \sum_{t_k \leq H} X_k \mathrm{E}_t[1/v(t_k, H)]$, written on the cash flows $\{X_k\}$ with maturities after H.

The premium at time t [4] can be derived by solving the general valuation equation with the boundary condition (6). The integral solution is given by:

$$G(r(t), H, \mathrm{x}, L') = \widetilde{\mathrm{E}}_t\left[max\left\{0, L' - R''(H, \mathrm{X})\right\} e^{-\int_t^H r(u)du}\right];$$

it can be also written in the form:

$$G(r(t), H, \mathrm{x}, L') = v(t, H)\, \mathrm{E}_t^*\left[max\left\{0, L' - R''(H, \mathrm{X})\right\}\right]. \tag{7}$$

where E_t^* is the expectation operator, given the information at time t, under the "forward adjusted" probability measure derived by Jamshidian [5].

It can be relevant to determine, at time t, the value of the premium if an insurance horizon $T - t$ is considered, being T a time before the maturity H of target L ($t \leq T \leq H$) [6]. The value of the insurance

[4] Referring to financial immunization, the insurance premium can be interpreted as the "price" of interest rate risk by considering the vector X as the cash flow of an immunized portfolio selected to cover a target: $L = P(t, \mathrm{X})/v(t, H)$ with maturity H (duration of the portfolio).

[5] See [4].

[6] Referring to financial immunization, the time interval $H - t$ is the duration (semideterministic or stochastic) of portfolio X; $T - t$ is the strategic rebalancing horizon. It appears relevant, in the stochastic immunization strategies that include expectation variables at time T in the objectve function (see [2], pp.144-150), to analyze the strategic time variable T through the premium on X at time T. In general, the premium at time T can be used as a price index to describe the interest rate risk evolution of the outstanding situation of the agent, for strategic time intervals.

premium at time T can be expressed in the form: $G(r(T), T, \mathrm{X}, L) = max\{0, Lv(T,H) - R(T,\mathrm{X})\}$; by assumption (5), it results:

$$G(r(T), T, \mathrm{X}, K(r(T))) = max\{0, K(r(T)) - R''(T,\mathrm{X})\},$$

that is identical to the payoff structure of a put option with maturity T and exercise price given by:
$K(r(T)) = L\,v(T,H) - \sum_{t_k \leq T} X_k\, \mathrm{E}_t\,[1/v(t_k,T)]$,
written on the residual stream of X with maturities after T. The insurance policy, in this case, can be therefore characterized in terms of put option with stochastic exercise price.

At the valuation time t, the integral form of the premium is given by the expression:

$$G(r(t), T, \mathrm{X}, K) = v(t,T)\,\mathrm{E}_t^*\,[max\,\{0, K(r(T)) - R''(T,\mathrm{X})\}]. \quad (8)$$

3.2 Interest rate insurance on stochastic cash flows

Let us consider the stochastic payment X_k, to be paid at time t_k, determined by the following indexation rule:

$$X_k = Cj(t_{k-1}, t_k) = C\left(\frac{1}{v(t_{k-1}, t_k)} - 1\right), \quad k = 2, 3, \ldots, m,$$

being $j(t_{k-1}, t_k)$ the reference index and C the nominal value of the contract ($t_0 = t$ and X_1 is deterministic). It can be demonstrated, by the no-arbitrage argument, that expression (2) is given by [7]: $P(t,\mathrm{X}) = (C + X_1)\,v(t,t_1)$. For every $H = t_j$, $j \in \{1, 2, \ldots, m\}$, expression (4) is: $R(H,\mathrm{X}) = R'(H,\mathrm{X}) + C$. The insurance policy, if assumption (5)

(7) See [2], pp.53-73.

is considered on the reinvestment component [8], is reduced to a deterministic contract. This is in consequence, otherwise, of the self-insuring nature of stochastic cash flow $\mathbf{X}$; uncertainty is in fact referred just to the reinvestment component and it cannot be analized by considering its expected value. For practical pourpouses, let us consider some particular values of H.

If $H = t_1$, it results: $R(t_1, \mathbf{X}) = X_1 + C$ and insurance policy is still a deterministic contract.

If $H = t_2$, it results:

$$R(t_2, \mathbf{X}) = \frac{X_1}{v(t_1, t_2)} + \left(\frac{1}{v(t_1, t_2)} - 1\right) C + C = \frac{X_1 + C}{v(t_1, t_2)}. \qquad (9)$$

The integral expression for the insurance premium, at time t, is given by:

$$G(r(t), t_2, \mathbf{X}, L) = v(t, t_1)\, \mathbf{E}_t^* \left[max \left\{0, L - \frac{X_1 + C}{v(t_1, t_2)}\right\} v(t_1, t_2)\right]; \quad (10)$$

the insurance policy is equivalent to a put option write on a security z that pays at time t_2 the (stochastic) amount defined by (9) and whose value at time t is: $P^z(t) = (X_1 + C)v(t, t_1)$, with exercise price L and exercise time t_2.

---

(8) It results:

$$\sum_{t_k \le H} \frac{X_k}{v(t_k, H)} = C\mathbf{E}_t \left\{ \sum_{t_1 < t_k \le H} \frac{1}{v(t_{k-1}, t_k) v(t_k, H)} - \frac{1}{v(t_k, H)} \right\}$$
$$+ X_1 \mathbf{E}_t \left\{ \frac{1}{v(t_1, H)} \right\}.$$

4. Specification of the stochastic model

In the one factor Cox, Ingersoll and Ross term structure model [3], the market is characterized by a diffusion process of the spot rate with parameters: $f(r,t) = \alpha(\gamma - r)$, $\alpha, \gamma > 0$, and $g(r,t) = \rho\sqrt{r}$, $\rho > 0$; the market price of risk is defined by the function: $q(r,t) = \pi\sqrt{r}/\rho$, π constant.

The premium (7) can be derived by the formulas for bond option and by the put-call parity. It results: (9):

$$G(t) = \sum_{t_k > H} X_k v(t,t_k)[\chi^2(y_k;\vartheta,\lambda_k) - 1] + L'v(t,H)[1 - \chi^2(y;\vartheta,\lambda)], \tag{11}$$

being $\chi^2(y;\vartheta,\lambda)$ the distribution function of the Cox, Ingersoll and Ross model, and the parameters ϑ, y, λ, y_k and λ_k as defined in [9], p.439 – 440.

The premium (8) is given by: (10) :

$$G(t) = \sum_{t_k > T} X_k v(t,t_k)[\chi^2(y_k;\vartheta,\lambda_k) - 1] + Lv(t,H)[1 - \chi^2(y;\vartheta,\lambda)]$$

$$- \sum_{t_k \leq T} X_k \mathrm{E}_t[1/v(t_k,T)]v(t,T)\,[1 - \chi^2(y';\vartheta,\lambda'')], \tag{12}$$

where ϑ, y, λ, y_k, λ_k, y' and λ'' are defined as in [9], p.443 – 444.

(9) See [2], pp.171-178. It must be: $0 < L' < \sum_{t_k > H} X_k A(H,t_k)$. If: $L' < 0$, the value of the insurance policy is always zero. If: $L' > \sum_{t_k > H} X_k A(H,t_k)$ the insurance policy is reduced to a deterministic contract, whose value is given by:

$$G(t) = L'v(t,H) - v(t,H)\mathrm{E}_t^*[R''(H,X)] = L'v(t,H) - P''(t,X),$$

being $P''(t,\mathrm{X})$ the price at time t of the payments with maturities after H. In the Cox, Ingersoll and Ross model a closed form solution for the "exercise probability" of the option can be derived (see [8], p.553).

(10) It must be: $0 < K(r(T)) < \sum_{t_k > T} X_k A(T,t_k)$. If $T = H$ expression (12) is obviously reduced to (11).

To derive the premium (10), in the case of stochastic cash flows, it results:

$$G(t) = v(t,t_1)\int_{r^*}^{+\infty} [Lv(t_1,t_2) - (X_1 + C)]\, dF^*$$

$$= L\, v(t,t_1)\left(\int_0^{+\infty} v(t_1,t_2)dF^* - \int_0^{r^*} v(t_1,t_2)dF^*\right)$$

$$-(X_1 + C)\, v(t,t_1)\left(\int_0^{+\infty} dF^* - \int_0^{r^*} dF^*\right),$$

being r^* the "strike" interest rate, i.e. the interest rate at which the exercise price of the option equals the value (at the exercise time) of the residual cash flows (11) . In the Cox, Ingersoll and Ross model the forward-adjusted process is characterized by the distribution function:

$$F^*(x|r(t)) = \chi^2(2a'x; \vartheta', \lambda' r),$$

being:

$$a' = \frac{q}{\rho^2(e^{d(t_1-t)} - 1)}, \qquad q = \frac{1}{(\alpha - \pi + d)(e^{d(t_1-t)} - 1) + 2d},$$

$$\vartheta' = \frac{2\alpha\gamma}{\rho^2}, \qquad \lambda' = \frac{4d^2 e^{d(t_1-t)} a}{q^2}, \qquad d = \{(\alpha - \pi)^2 + 2\rho^2\}^{1/2}.$$

(11) In this case:

$$r^* = \frac{1}{B(t_1,t_2)}\, ln\frac{A(t_1,t_2)}{X_1 + C}L;$$

it must be: $L > (X_1 + C)/A(t_1,t_2)$.

It results [12] , for $t_2 > t_1$:

$$v(t,t_1)\int_0^{r^*} v(t_1,t_2)\,dF^* = v(t,t_2)\chi^2(y;\vartheta,\lambda)$$

with:

$$y = 2r^*[a+b+B(t_1,t_2)]\,, \quad \vartheta = \frac{4\alpha\gamma}{\rho^2}\,, \quad \lambda = \frac{2a^2 r e^{d(t_1-t)}}{a+b+B(t_1,t_2)}\,,$$

being:

$$a = \frac{2d}{\rho^2[e^{d(t_1-t)}-1]}\,, \qquad b = \frac{a-\pi+d}{\rho^2}\,.$$

The insurance premium is given by:

$$G(t) = Lv(t,t_2)\,[1-\chi^2(y;\vartheta,\lambda)] - (X_1+C)v(t,t_1)\,[1-\chi^2(y'';\vartheta,\lambda''')]\,, \tag{13}$$

with:

$$y'' = 2r^*(a+b)\,, \quad , \quad \lambda''' = \frac{2a^2 r e^{d(t_1-t)}}{a+b}\,.$$

The expression (13) can be also written in the form:

$$G(t) = C(t) + Lv(t,t_2) - P^z(t)\,, \tag{14}$$

being $C(t) = P^z(t)\chi^2(y'';\vartheta,\lambda''') - Lv(t,t_2)\chi^2(y;\vartheta,\lambda)$ the value of a call option on the securities z with exercise price L and maturity t_2; expression (14) is the put-call parity for zero coupon bond (or for equity) and can also be derived by the no-arbitrage argument.

(12) See [4]. In other terms:

$$\int_0^{r^*} v(t_1,t_2)\,dF^* = v(t,t_1,t_2)\chi^2(y;\vartheta,\lambda)\,.$$

where $v(t_1,t_1,t_2) = v(t_1,t_2)$ is the future spot price and $v(t,t_1,t_2)$ is the forward price.

4. Empirical evidences

The valuation of insurance premium has been analized with reference to interest rate sensitive bond portfolios.

The estimation of the one factor Cox, Ingersoll and Ross model has been derived by the prices of the Italian Treasury bonds (*Buoni Ordinari del Tesoro, BTP*) and Italian Treasury bills (*Buoni Ordinari del Tesoro*) quoted on the screen-based market (*Mercato telematico titoli di Stato, MTS*) on january 29th 1991 and also by the *BOT's* time serie from january 1980 to january 1991, considering the cash flow net of witholding tax; a "three steps" estimation tecnique (see [2], pag.128) has been used. The model, in the quoted day, has been characterized by the parameter values: $\alpha = 0.21803\,, \gamma = 0.11274\,, \rho^2 = 0.0030133\,, \pi = 0.055569\,, r = 0.10387$. For the identification of the portfolios to be analized, the following securities (investment opportunities) has been considered, with payments defined on a semiannual grid for 10 years: 6 fixed rate bonds with face value of 100 *lire*, semiannual coupon of 5.5%, 5.25%, 4.5%, 5%, 3.94%, 5% and maturities, respectevely, of 5, 7, 8, 9, 10, and 4 years; 1 fixed rate bond with face value of 100 *lire*, annual coupon of 10% and maturity 6 years; 3 zero coupon bonds with face value of 50, 80 and 100 *lire* and maturities, respectevely, of 0.5, 1 and 1.5 years. Refererring to the interest rate sensitive portfolio selection strategies (as defined in [2], pp.13-39), the minimum risk and the minimum cost (maximum holding period return) programming problems has been solved (to identify the extreme points of the efficient portfolios frontier), considering an investment horizon (duration of the portfolios) of 3 years and a target (income) of 100 *lire*. The solutions of the problems are represented, respectevely, by the vectors $\mathbf{X}_r$ and $\mathbf{X}_c$ with components:

$\mathbf{X}_r = (X_k = 7.795, k = 0.5, 1, \ldots, 4, X_{4.5} = 1.99, X_5 = 38.185)$ and $\mathbf{X}_c = (X_{0.5} = 45.225, X_k = 1.606, k = 1, 1.5, \ldots, 9.5, X_{10} = 42.386)$. On the streams described by the vectors $\mathbf{X}_r$ and $\mathbf{X}_c$; the premium has been calculated varying the insurance horizon $H - t$, having assumed a target $L = 100$ *lire* and the reinvestments (of payments before H) at the "expected" term structure of interest rates (13) , as considered in (5); in table 1 and 2 the results for $\mathbf{X}_r$ and $\mathbf{X}_c$ are reported.

Table 1
Minimum Risk Immunized Portfolio

H-t *(years)*	G(t) *(lire)*	r* *(%)*	Holding Period *(%)*
2,5	4,48	4,76	14,54
3	0,52	11,79	11,98
3,5	0,00	26,21	10,18

Table 2
Minimum Cost Immunized Portfolio

H-t *(years)*	G(t) *(lire)*	r* *(%)*	Holding Period *(%)*
2,5	4,38	6,60	14,94
3	0,68	12,12	12,31
3,5	0,00	19,72	10,46

The premium decreases as $H - t$ increases as a combination of the effects of the decrease in the exercise price of the put and in the number of the residual payments.

The evolution of the premium variyng the target L is represented in table 3 for $\mathbf{X}_r$ and in table 4 for $\mathbf{X}_c$, having assumed an insurance horizon $H = 3$ years. To guarantee the target $L = 100$ in $H = 3$,

(13) In the Cox, Ingersoll and Ross the expected rates can be derived in a closed form solution (see [2], p.123).

i.e. the level at which both the portfolios are immunized, the insurance premium is significantly different from zero (0.516 forX_r and 0.681 for X_c) even if the exercise probability is quite low (0.3175 for X_r and

Table 3
Minimum Risk Immunized Portfolio

Target *(lire)*	G(t) *(lire)*	r* *(%)*	Holding Period (%)	Exercise Probability
94	0,00	22,04	9,69	0,00
95	0,00	20,25	10,08	0,00
96	0,01	18,47	10,47	0,00
97	0,03	16,74	10,85	0,01
98	0,08	15,05	11,23	0,04
99	0,23	13,40	11,61	0,13
100	0,52	11,79	11,98	0,32
101	0,98	10,23	12,35	0,58
102	1,58	8,69	12,72	0,82
103	2,26	7,20	13,09	0,95
104	2,97	5,74	13,45	0,99
105	3,68	4,31	13,82	0,99
106	4,39	2,91	14,18	1,00

Table 4
Minimum Cost Immunized Portfolio

Target *(lire)*	G(t) *(lire)*	r* *(%)*	Holding Period (%)	Exercise Probability
92	0,00	21,61	9,23	0,00
93	0,00	20,23	9,62	0,00
94	0,01	18,91	10,01	0,00
95	0,02	17,65	10,40	0,01
97	0,10	15,30	11,17	0,04
99	0,40	13,14	11,93	0,16
100	0,68	12,12	12,31	0,27
101	1,07	11,14	12,68	0,42
103	2,16	9,27	13,42	0,74
105	3,49	7,53	14,15	0,93
107	4,90	5,89	14,87	0,99
109	6,32	4,35	15,58	0,99
111	7,74	2,89	16,28	1,00

0.2718 for X_c).
The premium for the minimum risk portfolio is, in particular, lower than the premium of the maximum return portfolio, as it could be expected.

References

[1] P. H. Boyle, I. Lee, *Deposit insurance with changing volatility*, OFOR Paper, N. 93-02, 1993

[2] G. Castellani, M. De Felice, F. Moriconi, C. Mottura, *Un corso sul controllo del rischio di tasso di interesse*, Bologna, Il Mulino, 1993

[3] J. C. Cox, J. E. Ingersoll, S.A. Ross, *A theory of the term structure of interest rates*, Econometrica, 53, 1985.

[4] F. Jamshidian, *An exact bond option formula*, Journal of Finance, 44, 1989, 205-209

[5] R.C. Merton, *An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees: An Application of Modern Option Pricing Theory*, Journal of Banking and Finance, 1, June 1977, 3-11

[6] R.C. Merton, *On the Cost of Deposit Insurance When There Are Surveillance Costs*, Journal of Business, 51, July 1978, 439-52

[7] R.C. Merton, *Continuous-Time Finance*, Cambridge, Basil Blackwell, 1990

[8] C. Mottura, *Uno schema di "asset-liability management" per contratti finanziari con flusso di pagamenti aleatorio*, Atti del XVI Convegno A.M.A.S.E.S., Treviso, 1992

[9] C. Mottura, *Il prezzo della garanzia dal rischio di tasso di interesse*, Atti del XVIII Convegno A.M.A.S.E.S., Modena, 1994