FINANCIAL SAFETY INEQUALITIES BASED ON EXPECTED RISKS FOR CREDIT INSTITUTIONS

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Abstract
The stochastic mathematical model of the crediting process is examined. It is assumed that in unstable economical state the possible default of credit may be a reason of risk under the crediting. The fund value of the crediting is considered as some random variable that is changed step-wise at instants of the payments of a credits or at instants of their repayment. At the same time that process may be considered as a sum of the single impulse disturbances that appear at the random instants and are characterized by a set of parameters that may be random too. These parameters are determined by the conditions of crediting. Among them a value of loan, a term of loan, the interest rate of crediting, a payment tariff of overdue repayment, a time of repayment delay, the probability of loan repayment by borrower. It is considered that credit institution sets a boundary admissible level of the loss for security of expenses discharge and receipt of guaranteed gain. By assumption that different random variables are independent the expected risk of crediting for a single loan is found. On base of the expected risk formula the inequalities are derived to determine the values of above parameters that guarantee the increase of the expected value of crediting fund. Numerical results are brought.

Keywords : credit, expected risk, probability of loan repayment, boundary admissible level of loss.

1. INTRODUCTION

The money credits are most widespread means for the activity business support. At the same time the credit institutions may be subjected to financial impacts because of the risk of default. Provided that the legitimate economical interrelation are unsteady so far the default may occur from behind the bankruptcy of the borrower, the criminal cause or other reasons. Other particulars of the bad debts have been described in [1]. Therefore under the payment of the credit it is desirable to value the risk of the crediting taking into account the probability of the repayment of the loan.

Introduce the following notation :

t' - time of credit issue,
T - term of credit (measured in years),
S - value of given money loan,
t'' - time of credit repayment,
Z - value of repaid money sum,
J' - interest rate of crediting,
J'' - interest rate for overdue repayment,
p - probability that the loan will be repaid.

As it is doing by the belarussian banks,we will suppose that the value of the repaid money sum $Z(t'')$ is computed for short-term credit by the relation based on the principle of simple interest rate, i.e.

$$Z(t'') = S(1+JT)$$

in case when $t''-t' \leq T$. But if the loan return after finish of credit term $(t''-t' > T)$ this relation is converted in form

$$Z(t'') = S(1+JT)(1+J''(t''-t'-T))$$

It is convenient to introduce the quantity $X = (t''-t'-T)^+$ where it is notated $(x)^+ = 0$, if $x \leq 0$, $(x)^+ = x$, if $x \geq 0$. X has meaning of delay time of repayment of the loan. Then both relation may be written by the single formula

$$Z(t'') = S(1+JT)(1+J'X)$$

Note that T and X are measured in years.Let Y be the random variable of repayment of the loan : $Y = 1$ when the loan will be repaid and $Y = 0$ when the loan will not be repaid. Suppose that $E(Y) = p$. Then the repaid loan function may be written for any time t , $t \geq t'$,

$$Z(t) = 0 \qquad t' \leq t < t''$$

$$Z(t) = YS(1 + J'T)(1 + J''X) \qquad t'' \leq t$$

The quantities t', J', S, T, X have the nature of the random variables too. Let us consider them as independent variables.

We call the function of the changes of the money value due to the payment or the repayment of loan by the credit function $C(t)$. This function is defined by the following relations

$$C(t) = 0 \qquad t < t'$$

$$C(t) = Z(t) - S \qquad t' \leq t.$$

All preceding notation corresponded to the single credit. When the credit institute gave out certain number N of loans, it should be enumerated these credits in order of payments. Then the index k of each credit must be appropriated to all parameters that correspond to this credit. Sets of parameters for different credits are independent except parameter t' when for any k $t'_k < t'_{k+1}$. Thus if the initial money amount of the crediting was R_0, the working balance of cash $R(t)$ in time t is expressed by :

$$R(t) = R_0 + \sum_{k=1}^{N(t)} C_k(t),$$

where $N(t)$ - number of loans paid during time interval $(0,t)$.

The financial safety of the credit institution is connected with the properties of the working balance $R(t)$ that is the random process. $R(t)$ is changes step-wise at times of the payments or the repayments of the loans. The financial safety inequalities define the conditions that guarantee the sufficient rate of the increase of the expected value of the money amount $R(t)$. In present paper we will examine the problem of fulfillment of some necessary conditions of the financial safety that based on consideration of a single credit only.

2. INEQUALITIES OF FINANCIAL SAFETY

The loss function for a single short-term credit may be defined (similary [2]) as

$$L_k(t''_k) = S_k(1 + (t''_k - t'_k)i) - Y_k S_k(1 + J'_k T_k)(1 + J''_k X_k).$$

Here we use the principle of simple interest rate as for the short terms the accumulation with simple interest exceeds that with compound interest, if the term is less than one year [3]. The symbol i designates a basic rate of interest.

On base of that loss function one can formulate the different safety inequalities. Let ε be the boundary admissible level of the loss of the crediting, α be the probability significance level to make the decision. Then the probability safety inequality for single credit is

$$\mathrm{Prob}\{L_k > \varepsilon\} \leq \alpha$$

This probability is not dependent of index k if the parameter random values of all credit functions have the same probability distributions. ε may be defined as $\varepsilon = a + bS_k$. If $b = 0$ the inequality determines the safety condition on "one credit", and if $a = 0$ the inequality determines the safety condition on "one money unit". The probability safety inequality for N credits is

$$\mathrm{Prob}\{\sum_{k=1}^{N} L_k > \varepsilon\} \leq \alpha$$

This inequality may be prefered when N is great and it is possible to use the normal approximation. The probability principle of constructing the safety inequality gives the sufficiently cumbersome expressions to compute. The more simpler formulas are received from the expected value principle. The risk of crediting (on single credit) may be defined as the expected value of loss $R = E[L_k]$ and the safety inequality based on risk is $R < \varepsilon$. If the random variables that determine the credit function are simultaneously independent the risk expression may be present in explicit form by the expected values of the parameters of the credit function.

$$R = E[L] = E[S](1 + iE[t'' - t']) -$$
$$pE[S](1 + E[J']E[T] + E[J'']E[X] + E[J'J'']E[TX])$$

Derive that expression under some simplifying hypothesis. Let J' and J'' be functionally connected by equality $J'' = dJ'$, $d > 1$. In general $(t'' - t')$ and T are dependent too. It is convenient instead the variable $(t'' - t')$ to use relative variable $H = (t'' - t')/T$. Then $X = T\max\{0, H - 1\}$. And inequality for admissible risk gets following

$$E[S](1 + iE[HT]) - pE[S](1 + E[J']E[T] +$$
$$+ dE[J']E[T\max\{0, H - 1\}] + dE[J'^2]E[T^2\max\{0, H - 1\}]) < \varepsilon.$$

In this inequality the variables p, S, H, T are characteristics of the borrowers and the variables d, J' are defined by the credit institution. Therefore it is possibily to consider this inequality as means for the determination of the values d and J' on the base of the statistical data about the random values H, S, T, Y. In that case it should not be supposed that the variable J' is random. Let $J' = j$ for brevity. Then in inequality for risk instead of $E[J']$ and $E[J'^2]$ it is necessary to use j and j^2 respectively. May happen that the random variables H and T are mutually independent. If it is may be accepted the inequality for the determination of the rate of interest of the crediting is converting to the form

$$1 + jhE[T] - p(1 + jE[T](1 + dh_0) + j^2 dh_0 E[T^2]) < \varepsilon / E[S].$$

Here it is designed $h = E[H]$, $h_0 = E[\max\{0, H-1\}]$ for the compactness. This inequality may be resolved relatively the interest rate of crediting j. Let

$$U = dh_0 E[T^2]$$

$$V = (1 + dh_0)E[T]$$

$$W = -\varepsilon / pE[S] + (1-p)/p + ihE[T]/p.$$

Then inequality for j is

$$j > ((1 + 4UW/V^2)^{1/2} - 1)(V/2U).$$

Notice that $\varepsilon \le 0$ occur only therefore $W > 0$ always. When $4UW/V^2 << 1$ it is possibly approximately to write $j > W/V$. It sets the linear relation of the interest rates i and j.

3. NUMERICAL RESULTS

As the example of the application of the obtained inequalities we consider the data of one belarussian bank. The data include the information about $N = 150$ loans. The payments of the short-term credits have occured in the random intervals. The probability distribution of these intervals is very close to exponential one with the mean value $E[t'_{k+1} - t'_k] = 1.2$ (in days). The histogram of the occured terms of crediting T presented on Figure 1. The histogram of the occured loan values S (in millions of roubles) presented on Figure 2. The histogram of the ocurred interest rates of crediting J' presented on Figure 3. The interest rate for overdue repayment was set by formula $J'' = 3J'$ $(d = 3)$. The empirical means of the observed data are following :

$E[T]$ = 0.2788 (in years), $E[T^2]$ = 0.1238;

$E[S]$ = 1.9766 (in millions of roubles), $E[S^2]$ = 9.2391;

$E[J']$= 2.2142 (in precentage), $E[J'^2]$ = 7.7056.

The inequalities that are derived in preceding section determined the least values of the parameters of crediting j and d that garantee the admissible value of the risk under given probability characteristics of the credit random parameters H, S, T, Y. Evaluate the least value of the interest rate of crediting j for the case when the variables d, S, T have the properties shown above and the random parameter H with probability 0.8 is equal 1 and with probability 0.2 is uniformly distributed over interval (0.9, 1.1). In this case

$$h = E[H] = 1, \quad h_0 = E[\max\{0, H-1\}] = 0.005.$$

We take the safety loading coefficient $\varepsilon = 0$. Therefore

$$U = 0.0016, \quad V = 0.2829, \quad W = (1 - p + i0.2788)/p.$$

Here $p = E[Y]$ we consider as the parameter that varies in a reasonable way, i.e. takes a values greater than 0.95. Then under the reasonable values of the basic interest rate i (at most 0.1) $4UW/V^2 < 0.08$ and it is possible to use the approximate inequality for j

$$j > W/V = 3.5348(1 - p + i0.2788)/p.$$

In Figure 4 it is presented the least interest rate of crediting j for this case as a function of the basic interest rate i for a few values of the probability of repayment of loan p.

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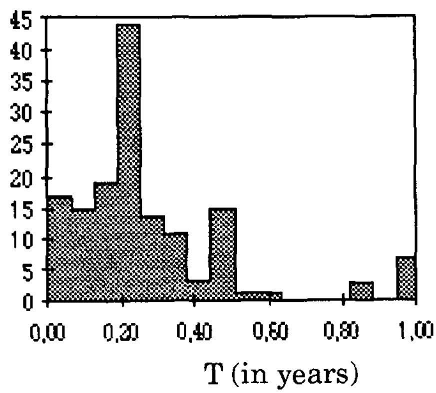

Figure 1.Histogram of terms of crediting for N=150 of credits

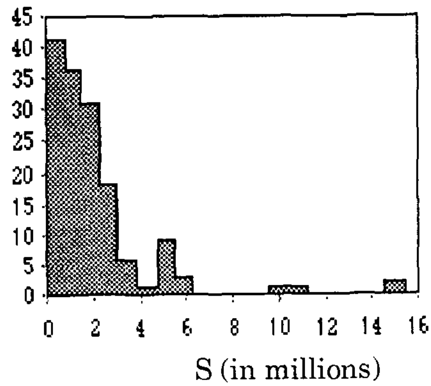

Figure 2. Histogram of loan values for N=150 of credits

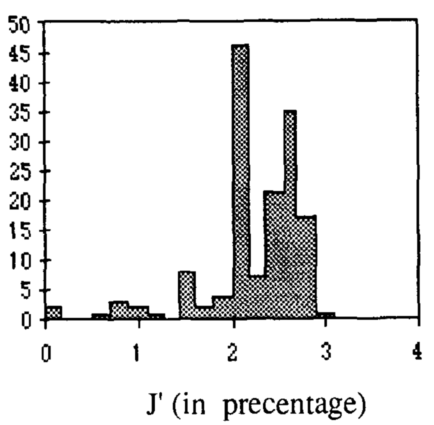

Figure 3. Histogram of interest rates of crediting for N=150 of credits.

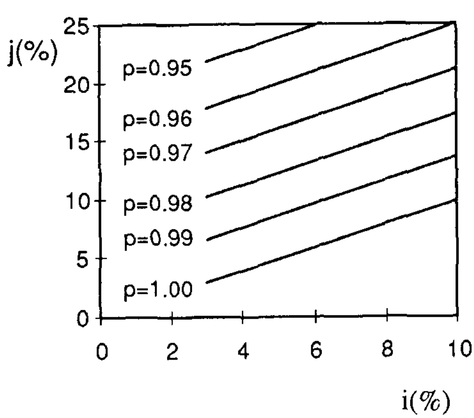

Figure 4. Least interest rate of crediting as a function of the basic interest rate i for different probabilities p .