PRICING PUTTABLE BONDS IN THE ITALIAN MARKET[1]

ADRIANA GNUDI
UNIVERSITA' DI MILANO
ISTITUTO DI SCIENZE STATISTICHE E MATEMATICHE "M. BOLDRINI"
6, VIA CONSERVATORIO
20122 MILANO - ITALY.
AND
MARIDA BERTOCCHI
UNIVERSITA' DI BERGAMO
DIPARTIMENTO DI MATEMATICA, STATISTICA, INFORMATICA E APPLICAZIONI
2, PIAZZA ROSATE
24129 BERGAMO - ITALY.

Abstract

This paper illustrates the results obtained from a new method to estimate both the term structure of interest rates in the Italian bond market and the equilibrium value of the Italian Treasury puttable bonds (Certificati del Tesoro con Opzione di Rimborso Anticipato - CTOs) using the Ho-Lee model.

Keywords. term structure, embedded options, Ho-Lee model.

[1] The work has been partly developed in the framework of Digital 1992, SIGE 1993 grants and CNR grant n.93.00754.CT10.

1 Introduction

The pricing of interest-rate sensitive securities is one of the topic widely discussed in financial literature (Ball and Torous [1], Barone e Cuoco [3], Brennan and Schwartz [6], Cox, Ingersoll and Ross [8],Cox, Ross and Rubinstein [9], Courtadon [10], Heath [12], Schaefer [17] and many others). Due to the fact that the value of the interest rate sensitive securities depends on the entire term structure of interest rates and on its stochastic movements as well, in order to evaluate any interest rate contingent claim, it is necessary to specify a model of the term structure.

Various models have been developed for the estimation of the term structure, ranging from the so-called *deductive approach* of Vasicek [18], Cox, Ingersoll and Ross [8], Courtadon [10] to the *inductive approach*, proposed by Ho and Lee [13] and already used in the Italian market by Giacometti and Nuzzo [11]. The former ones require estimation of the market price of risk, the latter one takes the current term structure as given and models its subsequent movements in such a way that no riskless arbitrage opportunities are possible. The model is tuned so that the equilibrium price of a chosen set of contingent claims is consistent with the initial yield curve which reflects the investors' market price of risk and their time preferences. Following this approach and its subsequent developments, we have estimated the term structure of interest rates on the Italian market using the information contained in the observed market data.

The paper is organised as follows: Section 2 briefly describes the features of the Italian puttable bonds (Certificati del tesoro con opzione - CTOs). Section 3 illustrates two methods to derive the term structure in the Italian market; Section 4 describes the Ho-Lee model and illustrates the methodology used to price interest contingent claims. Finally, Section 5 shows the results obtained applying the Ho-Lee model to price the CTOs.

2 Derivatives on Italian fixed income securities : Certificati del Tesoro con Opzione - CTO

Since 1988 the Italian Treasury has been issuing puttable bonds, (Certificati del Tesoro con Opzione di rimborso anticipato - CTOs). CTOs are coupon-bearing securities, which pay a semi-annual coupon and provide an option allowing their holder to obtain early redemption at a specified time and price (usually at par). We refer to this option as option embedded in bond, i.e. an option that is part of the underlying security and cannot be traded separately from it.

These bonds enable their owners to hedge now against a future fall in interest rates (since they are long-term fixed-income securities) and provide, at a later stage, protection, through the early redemption, from capital losses caused by a rise in interest rates.

As it is well known, CTOs can be decomposed in two parts: the underlying bond with maturity T and a put option that allows the holder to sell the bond at time τ ($\tau < T$) at a strike price equal to the face value of the bond (retractable bonds). Alternatively, a CTO can be viewed as a bond with maturity τ with a call option to buy at time τ another bond of maturity T-τ (extendible bonds).

Relationships between the equilibrium value of a CTO, the put and call options embedded in it and the price of a straight bond with the same maturity of the CTO can be found in Barone [4] and Giacometti [11]. The relationships show how the value of the option embedded in CTO crucially depends on the evolution of interest rates. Therefore, in order to determine the value of a retractable/extendible bond, as CTO, it is necessary to specify a model of the term structure.

3 Term structure estimation in the Italian market

As we will see in Section 4, the Ho Lee model implementation requires to estimate the discount function or, equivalently, the spot rate function at the initial date. Ho and Lee refer to the cubic-spline procedure, see Litzenberger and Rolfo [15], to compute the spot rate function while Giacometti and Nuzzo [11] prefers to work on the discount function approximation. In this section we present the method used by us to estimate the term structure in the Italian Treasury-bond market.

Estimating the term structure presents several problems on markets where no zero-coupons exist with maturities beyond the short term (Barone, Cuoco, Zautzik [2]). However, one can think of a coupon bond as a portfolio of pure discount bonds, thus the price of a coupon bond can be obtained as a linear combination of payoffs (interest and principal), with coefficients given by appropriate discount factors. We evaluate the discount factors starting from the spot rate function; in this way we avoid the situation where a small error in the discount factor value can produce a big error in the spot value.

The Italian secondary and Wire Treasury-bond market is sufficiently liquid to believe that market prices are significant and a spot rate function can be inferred.

After trying various functional forms, we came out with good fitting using the Bliss and Ronn [6] approach, in a slightly modified form:

$$r(T) = a + b \cdot \frac{1-\exp(-T/c)}{T/c} + d \cdot \exp\left(\frac{-T}{e}\right) \tag{3.1}$$

where $a, b, e \in \Re^+$ and $c, d \in \Re$,

and the following form chosen in order to catch the interest rate movements, but limiting the fluctuations

$$r(T) = a - \frac{b}{c \cdot T + d} + \frac{f - 0.02 \cdot \sin(g \cdot T + h)}{(j \cdot T + k)^3} \tag{3.2}$$

where $a, b, c, d, j, k \in \Re^+$ and $f, g, h \in \Re$.

The procedure starts, at a chosen time, with a number h of unknown parameters which are evaluated minimizing the sum, over all the coupon bonds priced on the market, of the difference between the actual price of each coupon bond and the price of the same bond evaluated as a portfolio of zero-coupon bonds.

The price of this portfolio is computed as a linear combination of payoffs with coefficients given by discount factors, $D(T)$, where:

$$D(T) = (1 + r(T))^{-T} \tag{3.3}$$

and T is the maturity time of each zero-coupon bonds.

The function to minimize is the sum of the squared errors that is

$$\min_p e'e = \min_p (b - Cd)'(b - Cd) \tag{3.4}$$

where:

d is the $(m\times1)$ vector of the discount factors computed by (3.3);

b is the $(n\times1)$ vector of the market coupon-bond prices;

C is the (n,m) matrix whose generic element $c(i,j)$ is the payment made by the i-th security at the j-th time.

p is the $(h\times1)$ vector of parameters used in function $r(T)$.

Conditions on the spot rate function are:

$$r(T) \text{ continuous and differentiable} \tag{3.5}$$

$$r(T) > 0 \qquad \text{for all } T \tag{3.6}$$

$$\lim_{T \to +\infty} r(T) = k^{-} \tag{3.7}$$

In particular (3.7) means that very long interest rates cannot diverge to infinity, on the contrary the term structure should behave asymptotically. The number h of parameters must be much less than n in order to avoid bad fluctuations of the function; the solution is found using a local optimisation method.

To solve (3.4) we use a routine (ZXSSQ routine), implemented in IMSL package, based on a modified Levenberg-Marquardt algorithm. The algorithm is iterative and needs an initial estimate of the solution, which has been chosen to be consistent with market data.

To estimate the term structure we use a sample of Italian Treasury coupon bonds (Buoni del Tesoro Poliennali - BTPs). We take Friday prices of BTPs traded on the Milan Stock Exchange and on the Wire market.

The sample period goes from January 1991 to May 1993. For the bonds traded on both markets, we use the Wire market quotations.

Since BTPs are issued by the Treasury, they are tipically default-free bonds. Hence there is no need to estimate the default-risk premium which otherwise would be embedded in their yields. Furthermore, being the Treasury bond market the most liquid of all Italian fixed-income securities, it can represent a benchmark for the pricing of other interest rate contingent claims.

Figures 3.1a and 3.1b show an example of the term structures obtained on April 19th 1991 and on May 28th 1993. Function described by (3.1) gives better fitting than (3.2) although using both curves for CTOs pricing give similar results.

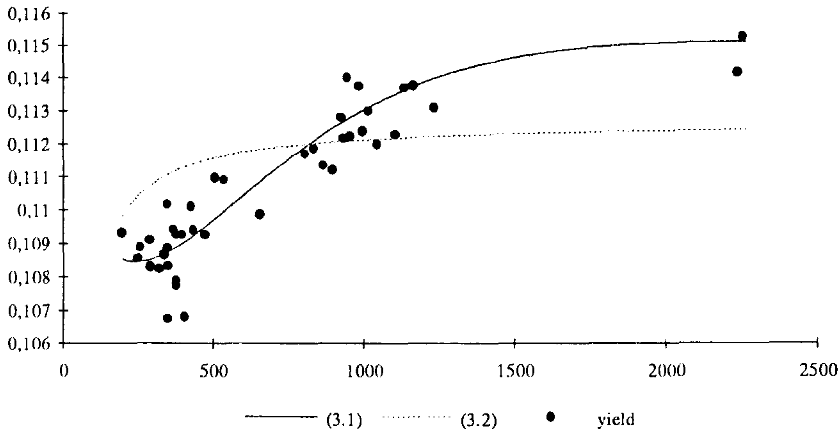

Figure 3.1a. Spot curves estimated on April 19th 1991.

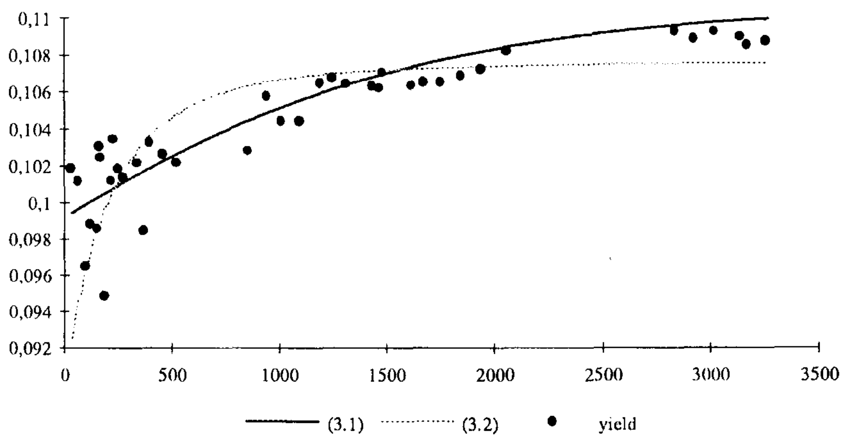

Figure 3.1b. Spot curve estimated on May 28th 1993.

4 The Ho-Lee model for pricing fixed income derivative securities

Ho and Lee [13] take the present data to estimate the initial discount function which is considered as given. They create the movements in the form of a binomial tree. The binomial process permits the entire term structure to move only up or down. Each node (t,i) of the tree, where t denotes time and i the state of the world, corresponds to a vector $D_{t,i}(\cdot)$ of values of discount bond prices. The subsequent discount functions are dependent only on the number of upward movements (path independent condition), i.e. an up-move followed by a down-move results in the same discount function as a down-move followed by an up-move.

In order to guarantee the absence of arbitrage opportunities, some constraints on the term structure movements have to be imposed.

Uncertainty is introduced by two perturbation functions $h(T)$ and $h^*(T)$ which specify the deviations of the discount from the forward functions in the upper and lower state, respectively, such that:

$$D_{t,i}(T) \begin{cases} \pi & D_{t+1,i+1}(T-1) = \dfrac{D_{t,i}(T)}{D_{t,i}(1)} h(T-1) \\ 1-\pi & D_{t+1,i}(T-1) = \dfrac{D_{t,i}(T)}{D_{t,i}(1)} h^*(T-1) \end{cases}$$

where:

$D_{t,i}(T)$ is the price, at time t and at state i, of a zero-coupon bond that pays 1 after T years.

Ho and Lee, under suitable postulates, found the following formula for the perturbation functions:

$$h(T) = \frac{1}{\pi + (1-\pi)\delta^T}, \qquad h^*(T) = \frac{\delta^T}{\pi + (1-\pi)\delta^T}$$

Given the initial discount function, the model can be completely specified by two constant parameters π and δ where π is the binomial or risk-neutral probability of a rise in interest rates and where $0 < \delta < 1$ is essentially a volatility parameter.

As it is well known, the Ho and Lee model has a serious drawback (see Ritcken and Boenawan [16] and Bliss and Ronn [6]), related to the fact that forward rates can become negative. Therefore, we introduced constraints on the discount function in order to obtain a monotically non increasing function bounded in the interval [0,1] at all nodes in the lattice. Once the binomial tree of interest rates has been built, any contingent claim can be priced using the model (Ho and Lee [12]).

Each security is priced by backwardation. According to this procedure, the price of a straight bond, at each node (t,i) is calculated as the weighted average of the bond prices at node $(t+1,i+1)$ and $(t+1,i)$, with the weights given by the probabilities π and $(1-\pi)$ of an upward and a downward movement of interest rates, discounted at the one-period rate prevailing at that node. Furthermore, in order to prevent arbitrage opportunities, at each node the expected holding period return must be the same for all bonds. Any other interest-rate-contingent claim can be priced similarly, provided that at each node the boundary conditions which define the domain of their possible values are satisfied.

The binomial lattice is uniquely determined given the initial discount function and once specified the two parameters π and δ. In the following sections we show how the two steps have been implemented using the Italian Treasury bond market data.

5 The pricing of CTOs using the Ho-Lee model

Once obtained the initial discount function, the second step to identify the binomial lattice is to estimate the parameters π and δ so that the theoretical prices of a chosen set of contingent claims result consistent with the market prices. This is equivalent to the assumption that the chosen contingent claims are well priced and we use them as a benchmark of the market behaviour. An optimization routine based on simulated annealing approach (Bertocchi [5]) for continuous global problems has been used to obtain the two values for π e δ. The minimum sum of the squared error between theoretical and observed prices is again used to judge the goodness of the fit.

As described in Section 4, the price of a CTO will be uniquely defined at each node (t,i) of the binomial lattice by backwardation. Let CTO be a retractable bond with fixed coupon C, maturing at time T.

We require that at maturity time T, the value of CTO, in all states i, is the value of the underlying straight bond.

This means that, if not redeemed at the exercise time τ, at T a CTO will have the same payoff of a BTP maturing at T, i.e. coupon plus principal. Hence, starting from the terminal condition, the CTO can be priced along the tree as follows:

$$\mathrm{CTO}_{t,i} = \left\{\pi\left[C + B_{t+1,i+1}\right] + (1-\pi)\left[C + B_{t+1,i}\right]\right\}D_{t,i}(1) \qquad (4.1)$$

where:

$D_{t,i}(1)$ is the one period discount bond price at node (t,i);

B is an Italian Treasury Bond with the same maturity T and coupon C as the CTO, but without embedded option, i.e. is a bond "equivalent" to the underlying straight cash flows of the CTO.

Furthermore, in the absence of arbitrage opportunities, at each node (t,i) with $t < \tau$, the price of a CTO with striking price equal to the par value must exceed both that of an equivalent straight bond with maturity τ and that with maturity T ($\tau < T$). At the exercise time τ, a CTO will be redeemed if the price of a BTP with coupon C and maturity T is lower than par. Therefore, valuing a CTO along the binomial tree, at the exercise time τ, the price of the CTO will be replaced with its exercise price if, according to the term structure movements, the CTO is likely to be early redeemed. We then continue with the rolling back procedure using (4.1) in all states until, after T steps, we reach the CTO's theoretical price at the current time.

In practice, the implementation of the binomial lattice through the CTOs market values requires the following steps:

1) the initial discount factors are estimated for each day of observation on the basis of the BTP market prices using the procedure illustrated in Section 3;

2) CTO prices are used to estimate π and δ. Simulated annealing is adopted to obtain the best fit estimate of π e δ related to the observed price.

At that stage, the estimated tree can be used to price simultaneously and consistently relative to each other all other derivatives securities on such bonds.

We organize the numerical tests along the following steps:

1) CTOs market prices were used to calibrate the parameters π and δ in a given date;

2) the same parameters values are used a week and three weeks later to evaluate CTO theoretical prices on the lattice;

3) the current market CTO prices are used to evaluate the new parameters and the current theoretical prices;

4) differences between step 2 and 3) are used to evaluate the validity of the Ho-Lee model along various time lags.

From numerical tests it comes out that parameter π does not influence the pricing process while parameter δ gives information about the volatility of short term interest rate. Changes in volatility depends heavily in the third decimal digits of δ. A value of δ equal to 0.996 gives a volatility of 11% p.a., while a value of δ equal to 0.993 gives a volatility of 15% p.a.

The following tables contain the results valuing CTOs, traded on the Milan Stock Exchange and the Wire Market, on 26th April 1991 (Table 1) with $\pi = .5$ and $\delta = .996$ based on the information of 10th April (one week). Table 3 shows the results valuing CTOs on 10th May 1991 based on the information of 10th April (three weeks). Table 5 contains the features of CTOs quoted on that day.

A.B.I.code	Theor. price	Market price	call price	put price
CTO13029	100.26	100.35	0.0000	5.5659
CTO13043	104.51	104.54	0.1675	1.3617
CTO13044	104.04	104.01	0.1613	1.3931
CTO13045	103.03	103.10	0.1593	1.3816
CTO13049	102.20	102.19	0.1524	1.4089
CTO13055	101.11	101.05	0.1502	1.3969
CTO13061	100.22	100.31	0.1463	1.4021
CTO13065	104.65	104.71	0.2124	1.4382
CTO13068	103.80	103.83	0.2030	1.4430
CTO13070	102.84	103.03	0.2018	1.4278
CTO13073	101.82	101.90	0.1974	1.4186
CTO13080	104.77	104.77	0.2443	1.4672
CTO13083	103.83	103.87	0.2379	1.4750
CTO13086	100.77	100.93	0.2396	1.3937

Table 1. CTOs theoretical, market prices, embedded call and put theoretical prices, (Friday 26th April 1991 on Milan stock exchange and wire market) obtained by (3.1) and a term structure with -1 week lag.

A.B.I.code	Theor. price	Market price	call price	put price
CTO13029	100.33	100.35	0.0000	6.9402
CTO13043	104.58	104.54	0.1481	1.5443
CTO13044	104.10	104.01	0.1413	1.5795
CTO13045	103.09	103.10	0.1384	1.5733
CTO13049	102.26	102.19	0.1306	1.6061
CTO13055	101.16	101.05	0.1275	1.5999
CTO13061	100.27	100.31	0.1228	1.6098
CTO13065	104.66	104.71	0.1488	1.6109
CTO13068	103.80	103.83	0.1378	1.6187
CTO13070	102.84	103.03	0.1350	1.6063
CTO13073	101.81	101.90	0.1292	1.5998
CTO13080	104.68	104.77	0.2036	1.6847
CTO13083	103.83	103.87	0.1966	1.6941
CTO13086	100.75	100.93	0.1973	1.6150

Table 2. CTOs theoretical, market prices, embedded call and put theoretical prices, (Friday 26th April 1991 on Milan stock exchange and wire market) obtained by (3.1) and the current term structure.

A.B.I.code	Theor. price	Market price	call price	put price
CTO13029	100.68	101.06	0.0000	6.6555
CTO13043	104.95	105.16	0.1706	1.3515
CTO13044	104.47	104.57	0.1641	1.3841
CTO13045	103.46	103.57	0.1619	1.3744
CTO13049	102.63	102.71	0.1547	1.4036
CTO13055	101.53	101.62	0.1522	1.3934
CTO13061	100.64	100.88	0.1481	1.4002
CTO13065	105.09	105.22	0.2156	1.4394
CTO13068	104.24	104.45	0.2057	1.4452
CTO13070	103.27	103.55	0.2040	1.4311
CTO13073	102.25	102.57	0.1991	1.4228
CTO13080	105.12	105.34	0.2453	1.4739
CTO13083	104.27	104.49	0.2386	1.4824
CTO13086	101.20	101.45	0.2399	1.4028

Table 3. CTOs theoretical, market prices, embedded call and put theoretical prices, (Friday 10th May 1991 on Milan stock exchange and wire market) obtained by (3.1) and a term structure with -3 weeks lag.

A.B.I.code	Theor. price	Market price	call price	put price
CTO13029	100.99	101.06	0.0000	7.0301
CTO13043	105.14	105.16	0.0817	1.6084
CTO13044	104.67	104.57	0.0734	1.6556
CTO13045	103.67	103.57	0.0692	1.6628
CTO13049	102.85	102.71	0.0609	1.7030
CTO13055	101.76	101.62	0.0581	1.6984
CTO13061	100.87	100.88	0.0544	1.7050
CTO13065	105.27	105.21	0.0721	1.6880
CTO13068	104.40	104.45	0.0702	1.6936
CTO13070	103.42	103.55	0.0701	1.6688
CTO13073	102.39	102.57	0.0696	1.6489
CTO13080	105.22	105.34	0.1192	1.6531
CTO13083	104.35	104.49	0.1166	1.6467
CTO13086	101.24	101.45	0.1320	1.5172

Table 4. CTOs theoretical, market prices, embedded call and put theoretical prices, (Friday 10th May 1991 on Milan stock exchange and wire market) obtained by (3.1) and the current term structure.

A.B.I. code	Coupon %	Exerc. data	Exe.price[2]	Matur. data
CTO13029	10.25	01/12/1992	99.3750	01/12/1996
CTO13043	12.50	01/06/1992	99.8250	01/06/1995
CTO13044	12.50	19/06/1992	99.9125	19/06/1995
CTO13045	12.50	18/07/1992	99.7750	18/07/1995
CTO13049	12.50	16/08/1992	99.8435	16/08/1995
CTO13055	12.50	20/09/1992	99.7875	20/09/1995
CTO13061	12.50	19/10/1992	99.7750	19/10/1995
CTO13065	12.50	20/11/1992	99.6435	20/11/1995
CTO13068	12.50	18/12/1992	99.6810	18/12/1995
CTO13070	12.50	17/01/1993	99.6685	17/01/1996
CTO13073	12.50	19/02/1993	99.6435	19/02/1996
CTO13080	12.50	16/05/1993	99.6438	16/05/1996
CTO13083	12.50	15/06/1993	99.7813	15/06/1996
CTO13086	12.50	19/09/1993	99.6813	19/09/1996

Table 5. Characteristics of CTOs, (Friday 26th April 1991), Milan stock exchange and wire market prices.

[2] Net of 12,5% withholding tax.

Comparing the theoretical price in Table 1 and 2, it comes out the mean average difference is of 0.022, while in Table 3 and 4 is of 0.174. It means it is reasonable not to recompute the term structure within two weeks time, while it seems to be necessary recomputation over two weeks.

Looking at Table 1 and considering the difference between theoretical and market prices, we can accept the null hypothesis H_0: average mean = 0 at the level of significance 0.01.

During the whole period the average error between theoretical and observed prices in CTOs is 0.0229 with a standard deviation of 0.21.

Figure 5.1 shows in the case of CTO 13068 that the put-call parity condition was always met during the whole sample period.

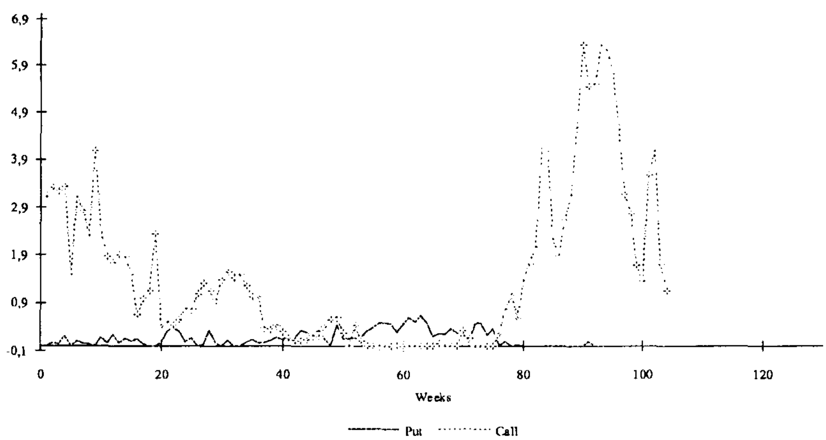

Figure 5.1 Call-parity relation for CTO13068 (January 1991- May 1993).

6 Conclusions

In this paper we use the Ho and Lee model to price puttable bonds (CTOs) together with the options embedded in them. We show how the valuation of the interest-rate contingent claims depends crucially on modelling the term structure movements. To implement it we estimate:

a) the current term structure using Italian Treasury bond market prices;
b) the two parameters which, together with the observed term structure, fully specify the model.

This paper should be viewed as an initial step to acquire more empirical insight into pricing Italian interest rate contingent claims related to the term structure. Further research is in progress to better understand the role of π and δ parameters in the Italian market and also to check the possibility to extend the binomial model to a multinomial model in the Ho-Lee framework.

References

[1] Ball C.A., Torous W.N. (1983) "Bond price dynamics and options" Journal of Financial and Quantitative Analysis, Vol. 18, n.4.
[2] Barone E., Cuoco D., Zautzik E. (1989) "La struttura dei rendimenti per scadenza secondo il modello di Cox; Ingersoll e Ross: una verifica empirica" Temi di discussione del Servizio Studi, Banca d'Italia.
[3] Barone E., Cuoco D. (1991) "La valutazione delle obbligazioni e delle opzioni su obbligazioni" Quaderni di Ricerca N.10 Roma, Università Luiss.
[4] Barone E., Cuoco D., (1991), "The valuation of bond and bond options: some empirical evidence" in Large-scale economic and financial applications: new tools and methodologies, M. Bertocchi and L. Stefanini eds., Franco Angeli, Milano, 11-40.
[5] Bertocchi M., Sergi P. (1992), "Parallel globaloptimization over continuous domain by simulated annealing" in Parallel Computing: problems, methods and applications, A. Murli and P. Messina, eds., Elsevier, 87-98.
[6] Bliss R. R., Ronn E.I. (1989), "Arbitrage-based estimation of non stationary shifts in the term structure of interest rates" The Journal of Finance, vol. XLIV, n.3, 589-610.
[7] Brennan M.J., Schwartz E.S. (1979) "A continuous time approach to the pricing of bonds" Journal of Banking and Finance, n.3.
[8] Cox J.C., Ingersoll J.E., Ross S.A.(1985) "An intertemporal general equilibrium model of asset prices" Econometria, Vol 53 N.2.
[9] Cox J.C., Ross S.A., Rubinstein (1985) "Option pricing: a simplified approch" Journal of Financial Economics, 7, n.3.
[10] Courtadon G. (1982) "The pricing of options on defaul-free bonds" Journal of Financial and Quantitative Analysis, Vol XXVII, N.1.

[11] Giacometti R., Nuzzo C. (1994) "Embedded option pricing on interest-rate securities in the Italian market" EWGFM Proceedings., D'Ecclesia R. and Zenios S. eds, Springer-Verlag, 1994.

[12] Heath D., Jarrow R., Morton A. "Bond pricing and the term structure of interest rates" Working Paper, Cornell University.

[13] Ho T.S.Y, Lee S. (1986) "Term structure movements and pricing interest rate contingent claims" The Journal of Finance, Vol.XLI N.5.

[14] Hull J., White A. (1990) "Pricing interest-rate derivative securities" The Review of Financial Studies, n.3, 573-592.

[15] Litzenberger R., Rolfo J. (1984) "An international study of tax effects on the government bonds" The Journal of Finance, vol. XXXIX,n.3.

[16] Ritchken P., Boenaan K. (1990) "On Arbitrage free pricing of interest-rate contingent claims" The Journal of Finance, vol. XLV, n.1, 259-264.

[17] Schaefer S.M., Schwartz E.S. (1987) "Time dependent variance and the pricing of bond options" The Journal of Finance, vol,XLII, n.5.

[18] Vasicek O. (1977) "An equilibrium characterization of the term structure" Journal of Financial Economics, 177-178.