THE DYNAMICS OF INTEREST RATE MOVEMENTS AND INFLATION ON ASSETS, LIABILITIES AND SURPLUS FOR PROPERTY AND CASUALTY INSURERS

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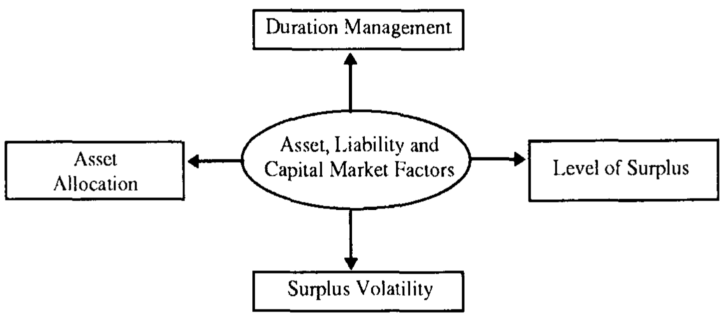
Abstract

This paper analyzes the complexities of investing for property and casualty insurers. We assess the dynamics of interest rate movements and inflation utilizing a variant of the two factor Brennan-Schwartz yield model to generate stochastic simulations of yield curve movements that forecast asset returns and correlations. We then combine the asset forecast with estimated liability cash flows in a Markowitz Efficient Frontier framework to develop optimal asset allocations to maximize surplus. This paper focuses on assessing the interrelationships of asset, liability and capital market factors (A-L-C) and implications of their variability on investment strategy and economic surplus, both in terms of the absolute level of surplus and surplus volatility.

The factors reviewed are divided into three categories : asset, liability and capital market.

Asset Factors *(asset constraints, income requirements, inflation sensitivity of pricing, surplus, FAS 115 classification)*
Liability Factors *(retention ratio, business mix, inflation sensitivity of loss payouts, loss ratio variability)*
Capital Market Factors *(economic scenario, time horizon, asset class assumptions)*



Investing for property and casualty insurers is a complex process that takes into consideration the dynamics of asset, liability and capital market factors (A-L-C Factors). Assessing the relationships of these A-L-C Factors, and the implications of their variability on investment strategy, is critical to developing a rational investment model. This paper discusses in detail how these key A-L-C Factors impact asset allocation and economic surplus.

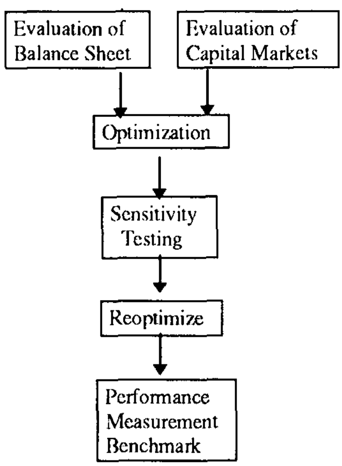
In an earlier paper, "Asset-Liability Management And Asset Allocation For Property And Casualty Companies - The Final Frontier",[1] we discussed a five

[1]Salvatore Correnti and John Sweeney, "Asset Liability Management And Asset Allocation For Property And Casualty Companies - The Final Frontier", 4th AFIR's Colloquium

step process toward asset allocation for property and casualty companies. We call this process the Asset-Liability Management Efficient Frontier (ALMEF):

1. Economic evaluation of the balance sheet which considers the ongoing nature of the business;
2. Evaluation of capital markets employing a stochastic economic simulation model;
3. Surplus optimization utilizing a multi-time-period non-linear optimization model which develops efficient frontier portfolios that explicitly consider the liability cash flows and characteristics and is dynamically linked to changing capital markets;
4. **Sensitivity testing of key asset, liability, and capital market factors;** and
5. A performance measurement system that culminates in a liability benchmark index.

The five step ALMEF process is depicted in the diagram below:



The subject of the prior paper was comprehensive and did not provide a detailed discussion of step four, sensitivity analysis. Step four refers to the sensitivity of

optimal asset allocations and resulting surplus volatility to the variables comprising the assets, liabilities, and capital market factors.
The (A-L-C) factors will be divided into three categories: asset, liability and capital market.

Asset Factors *(asset constraints, income requirements, inflation sensitivity of pricing, surplus, FAS 115 classification)*
Liability Factors *(retention ratio, business mix, inflation sensitivity of loss payouts, loss ratio variability)*
Capital Market Factors *(economic scenario, time horizon, asset class assumptions)*

These A-L-C factors have varying degrees of impact on asset allocation, ending surplus and surplus volatility. Before we discuss these factors, an overview of the simulation model and efficient frontier concept is presented. It is critical to understand these concepts in order to evaluate the sensitivity of A-L-C factors.

Simulation Model

The stochastic economic simulation model[1] produces a set of equally probable economic outcomes or simulations, based on user-defined economic assumptions. The model creates a reasonable range of possible outcomes to aid in decision-making under uncertainty. The model allows up to 500 simulations and up to a 20 year time horizon. Economic assumptions can include initial conditions and user-defined expected conditions. Each simulation is independent of every other simulation, however, the overall pattern of results is consistent with the user-defined inputs.

The model has a cascade, or top-down, structure (see Appendix A) and uses a combination of diffusion, econometric and conditional lognormal models in each

[1] Towers Perrin's Cap:Link System is the economic simulation model. Appendix A reviews the cascade structure. Appendix B reviews the general structure of the formulas employed.

of its phases. We briefly discuss the yield curve and inflation models since they are critical to the sensitivity analysis discussed herein.

The yield curve model is a variant of the two factor Brennan-Schwartz model. The two factor model controls the short and long rates in an interconnected process. The short and long simulated spot rates are connected both to each other and to their prior values. The formulas for the short and long spot rates are expressed as stochastic differential equations consistent with the Brennan-Schwartz model. The simulated process also includes mean reversion.

The simulation model uses a cascade structure where inflation is modeled directly below the yield model. Simulation of inflation is based on a stochastic diffusion model and derived from previously simulated yields. The model also allows inflation spikes through a stochastic volatility parameter and includes mean reversion to keep inflation within reasonable bounds.

Efficient Frontier

The objective of our asset-liability modeling work is to produce the most efficient portfolio for each liability (line of business). The efficient frontier curve represents the optimal asset mix for each level of risk (standard deviation of surplus or return), where "optimal" means the maximum reward (surplus) for each risk level. An infinite number of lesser returning portfolios for each corresponding level of risk are located below the efficient frontier curve. In constructing this curve, the model[1] starts by finding the minimum risk portfolio that satisfies the user input constraints. A similar process is followed to find the portfolio with the highest reward and lowest risk for that reward. The range between the lowest and highest risk portfolios is divided by the number of portfolios requested by the user. Then the model solves for the highest reward portfolios at each risk level. Specifically, the model uses a FORTRAN computer system called GRG2[2] (generalized reduced gradient method) to solve non-linear optimization problems. Exhibit 1 below details the algebraic formulas employed:

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[1] Towers Perrin OPT:Link, Asset Allocation (Efficient Frontier), User Manual, September 1993

[2] The GRG2 non-linear optimization code was developed by Leon Lasdon, University of Texas at Austin, and Allan Waren, Cleveland State University.

Exhibit 1: GRG2 stands for Generalized Reduced Gradient Method (a type of Numerical Analysis) which tries to solve problems of the form:

maximize or minimize $g_P(x)$

subject to $g_i(x) = b_i$ *for some values of* i

$gl_i \leq g_i \leq gu_i$ *for other values of* i

and

$xl_i \leq x_i \leq xu_i \quad i = 1, \ldots, n$

In the above, x is a vector of n variables. The g functions are real-valued functions of x. There are m such functions, one of which is the objective. The other functions are constraints. The problem may have both equality and inequality constraints. The inequalities may impose upper and lower bounds gu and gl on the g functions. There may also be bounds on the x variables. GRG2 uses first partial derivatives of each function g with respect to each x variable to solve a variant of Newton's method.

Therefore, the portfolios identified are optimal only at the specified risk levels. The mere selection of a portfolio alone is not indicative of its merits. Consideration must be given to its risk level and how it relates to the company's risk tolerance. A key distinction of our modeling is the merger of economic simulation of capital markets and asset class forecasts with liability cash flows and characteristics to solve for the maximum level of surplus at the end of the stated horizon. We utilize three methods to determine an acceptable level of surplus risk, i.e., which point on the curve (asset mix) to select: (1) match duration to the going concern liability duration, (2) plot the current mix and select the highest returning portfolio for that risk level which equates to the current risk tolerance, and (3) the area of the yield curve with the steepest slope or an approximate equivalent to a Sharpe Measure.[1]

[1] Sharpe Measure is defined as return per unit of standard deviation.

LIABILITY FACTORS

We begin our discussion with the liabilities since the asset allocation process is liability driven. We will discuss four liability factors. The first two factors, business mix and retention ratio, affect the duration of liabilities. The duration of liabilities in turn affects the surplus volatility.

The remaining two factors, inflation sensitivity and loss ratio variability, primarily impact the level of surplus volatility. Before we discuss these factors, a brief discussion of how duration is used in the optimization model is needed.

Going Concern Duration Calculation of Liability Flows

The calculated going concern duration is effectively a weighted duration of the following two components of the liability flows:

1) existing loss payouts resulting from existing loss and loss adjustment expense reserves for premiums earned to date, and
2) expected losses from renewals and/or new business.

Appendix C provides the mathematical formula for the calculation of duration. Duration, as referred to herein, is modified duration. Mathematically, the relationship between Macauley duration and modified duration is derived from the first term of a Taylor Series of the price function.[1] The going concern duration of the liability cash flows equals the modified duration of the sum of the existing liability cash flows plus the new liability cash flows resulting from renewals and/or new business.

existing liability cash flows	+	new liability cash flows	=	total liability cash flows

The going concern duration liability flows are implicitly considered in the optimization process. Surplus risk is reduced when longer duration cash flows are matched with longer duration asset classes. For example, in the case of Workers'

[1] Frank J.Fabozzi, Fixed Income Mathematics, Revised Edition, 1993, page 164.

Compensation where the liability stream can have a duration of seven years, long Treasurys are chosen in the lower risk Efficient Frontier portfolios. However, long Treasurys would be less appropriate in other lines where the duration is five or less.

Another example of duration's impact is evident when a duration constraint is placed on the optimization. A constraint adds a selection criteria for asset classes because the weighted duration of the portfolios cannot violate the constraint. Since duration is a measure of a portfolio's sensitivity to interest rate movements, a duration constraint defines the range of interest rate risk that a line of business may assume.

Both the retention ratio and business mix affect duration as follows :

Retention Ratio:

For this example, we assume new premium in year 2 is a function of the current year's premiums multiplied by a retention ratio (RR) where (year 2 premium = year 1 premium x RR). A higher ratio extends the life of premiums and corresponding losses, while a lower ratio has the reverse effect, resulting in a longer or shorter maturity of new asset flows (new premium - expenses) and new liability flows (earned premium x loss ratio). The retention ratio's impact on duration of total liabilities (existing and new liabilities) is mitigated if the existing reserve component is large, i.e. new losses would have less influence on the weighted duration calculation. For example, if 70% of Workers' Compensation loss payouts come from the existing reserves and only 30% are from new business, the retention ratio affects only the new business or the 30% portion. The variability of the retention ratio helps define the duration range for a line of business. For example, if the retention ratio for Personal Lines varies between 70% and 90%, the liability duration can range from 3.0 to 6.0. It is critical to work with each line of business to develop a range of acceptable retention ratios as a function of business plan, prior experience, and competitive issues. The retention ratios primary impact is on the duration of the liabilities. Independent of other factors, a higher retention ratio will result in an increase in the duration of the resulting asset allocation.

Business Mix:

Different liability subsegments (e.g., commercial liability versus commercial property) have significantly different payout patterns, durations, loss ratio variability, and inflation sensitivity. An increase or decrease in forecasted premiums in one subsegment will allow it to exert more or less influence on the total duration for that segment. Our forward-looking process focuses on a segment's forecasted business mix to accurately assess its going concern duration. Different subsegments have varying degrees of inflation risk; surplus volatility is affected by business mix assumptions. The business mix assumption is critical to developing accurate liability flows and the resultant durations, loss ratios, and inflation sensitivity. Assessing a company's current business mix and future plans is critical in determining an appropriate asset allocation and assessing potential surplus volatility.

Inflation Sensitivity:

The third liability factor is inflation sensitivity and its primary effect on surplus volatility. What is the sensitivity of the liability flows to inflation? When the actuarial department sets reserves, an inflation rate is assumed in order to estimate future payouts of losses. Inflation sensitivity of liability flows refers to the impact if future inflation is different from the implied inflation rate forecast in the reserves. Future inflation is stochastically generated by an economic simulation model and is a function of the initial inflation rate and the user's inflation outlook. Liability flows are then adjusted for the difference between the actuarial forecast and the impact of the stochastically modeled future inflation. The degree of the adjustment is specified by the user as a factor between 0 and 1 (0: no adjustment, 1: full adjustment). For example, if reserves are sensitive to unanticipated inflation in a one-to-one relationship, a 1% unanticipated increase in inflation above the rate implied in the reserves will cause liability payouts to increase 1%.

Although asset factors are discussed later, we discuss the sensitivity of new asset flows to inflation at this point. Inflation sensitivity of new asset flows refers to adjustments to the company's pricing practices as a result of unanticipated movements in inflation. Does the company adjust pricing as a result of adverse or positive effects of inflation on loss payouts? Again, the degree of the adjustment is specified by the user as a factor between 0 and 1 (0: no adjustment, 1: full adjustment). Additionally, the user may specify the direction of price sensitivity, (upward, downward or both). For example, if pricing is sensitive to unanticipated inflation in a one-to-one relationship, and if inflation increases 1% above the anticipated inflation rate, pricing or new premiums will also increase 1%. Again, the model's stochastic inflation feature allows one to assess the random impacts of inflation on pricing.

To the extent the asset and liability sensitivity factors are different and inflation moves in an adverse direction, asset classes that provide a hedge against this movement are favored in the optimal allocations. Both historical analysis and intuition are utilized to develop line of business loss reserve and pricing sensitivity to inflation. Inflation will have a significant impact on the volatility of surplus return and the asset mix selected.

Loss Ratio Variability:

The final liability factor is the loss ratio variability. As with inflation sensitivity, the major impact of loss ratio variability is on surplus volatility. An independent shock to loss ratios has been modeled with the major impact affecting the dollar value of ending surplus, not asset allocation. The main relevance of loss ratio to the optimization is in its deviation as a function of unexpected movement of some capital market variable(s). At this point, neither random loss ratio variability nor a link to a capital market factor is factored into our model. We are in the process of determining if any of the loss ratio variability can be attributed to a capital market factor so as to link the economic simulation with the loss ratio variability. If any relationship is found, then a link can be factored in the model accordingly. The loss ratio variability would have a similar effect as the inflation sensitivity in that assets that hedge any adverse movement will be favored to optimize the portfolio's risk/return tradeoff.

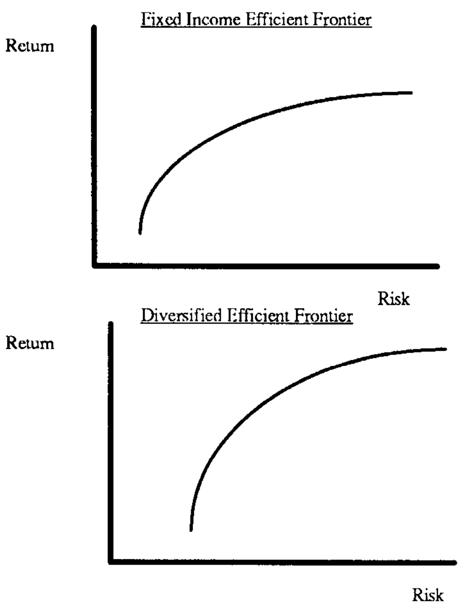
ASSET FACTORS

Five asset factors: asset constraints, income requirements, inflation sensitivity, surplus, and FAS 115 classification are tested to assess their sensitivity and the resulting impact on the asset allocation.

Asset Constraints:

Suppose corporate policy and regulatory considerations limit investment in "higher risk assets", defined as Equities and Real Estate. This constraint leaves primarily fixed income alternatives for selection in the optimization. Exhibit 2, below, compares a Fixed Income Efficient Frontier and a Diversified Efficient Frontier.

Exhibit 2: Fixed Income vs. Diversified Efficient Frontier



In a Fixed Income Efficient Frontier, there is not as much incremental return for taking on additional risk, i.e., the more constrained one is the less return one is able to get for taking on additional risk. As a result, the Fixed Income Efficient Frontier curve has a very steep slope at the low end of the risk range, and it is almost flat for the remainder of the risk range. Returns of a fixed income portfolio are much more limited than those of a diversified portfolio which contains a broader mix of assets. Therefore, the range of returns (Y-axis) for the efficient frontier is smaller than it would be if "riskier" assets that provide diversification benefits were included. The latter part of the curve is almost flat because for the same high risk points, the fixed income portfolio provides less return than a portfolio with a mix of diversified assets. Each company needs to determine the asset categories permitted and the maximum exposure to each category. This is why the development of an investment policy and guidelines is the first step in the asset allocation/ALM process. Modeling the results of various asset class constraints can prove to be a valuable tool in determining an investment policy and understanding risk.

Income Requirements:

The second asset factor is a minimum income requirement. Setting a minimum investment income constraint is similar to setting an asset mix constraint. The model is forced to select higher yielding, and not necessarily higher returning, asset classes. For example, mortgage-backed securities, with higher nominal yields but potential for extreme negative convexity, become a larger percent of the allocation the higher the minimum income constraint. Again, it's a company's specific decision as to the desired level and stability of investment income. One approach is to set the income constraint equal to a Treasury yield equivalent of the minimum target yield tested by the National Association of Insurance Commissioners. The tradeoff between statutory operating income needs and long term economic returns must be tested based on each company's goals.

Inflation Sensitivity:

The third asset factor is inflation sensitivity of new asset flows, or the pricing response to the impact of adverse or positive inflation on loss payouts. As previously discussed in the liability factors section, the degree of the pricing response to inflation and the relative value of new premiums versus loss payouts results in the selection of asset classes that provide varying levels of a hedge against the movement. This factor is driven by the company's business mix, pricing practices and competitive forces. Again a combination of historical analysis and judgment is required to model this factor. The resultant asset allocation and surplus return volatility will be impacted by the extent of the sensitivity of pricing to inflation.

Surplus:

The fourth asset factor is the level of economic surplus in relation to the present value of loss payouts. A higher level of starting surplus provides a larger cushion and results in an efficient frontier that moves toward an asset-only efficient frontier, with less emphasis on the liability flows. The higher a company's level of surplus, the less concern it will have about meeting liability obligations. The higher the company's surplus, the greater the investment risk the company can withstand.

FAS 115 Classification

The final asset factor is the impact of FAS 115. For United States companies reporting on a GAAP basis, bonds must be classified as held-to-maturity, available-for-sale or trading. Held-to-maturity bonds are carried at amortized cost and may not be sold, except for very specific reasons identified in the regulation. Trading bonds are carried at market value with gains and losses flowing through the income statement. Available-for-sale bonds are carried at market value with unrealized gains or losses impacting only shareholder's equity. Since this analysis is market value based, we must adjust for cash flows based on held-to-maturity assets. This process requires extensive analysis which we will not discuss in this paper. Our analysis is then completed on the remaining starting assets (available-for-sale and trading) and incorporates held-to-maturity cash flows as they mature. Generally, having a portion of assets in a held-to-maturity classification results in a lower risk, lower return asset allocation as compared to a 100% available-for-sale/trading classification.

CAPITAL MARKET FACTORS

Our final set of sensitivity factors is called capital market factors (economic scenario, time horizon and modeling of the asset classes).

Economic Scenario:

The economic scenario or economic input is critical to the simulation results for asset classes and the ultimate asset mix. Key inputs are short and long Treasury rates, inflation, dividend yield and growth rates. We look at two economic scenarios and weigh each based on our viewpoint to consider the potential impact if our economic forecast is incorrect. In scenario one, the economic inputs equal current conditions, and the model simulates from the starting (current) assumptions. In scenario two, the user inputs his view for "normalized" assumptions or expectations for the mean reversion process. The model then moves from current economic conditions toward this view. We then run up to 500 simulations using both basic sets of economic inputs to assess the sensitivity of asset class results to changing the economic assumptions. Each company needs to determine its core economic assumptions for simulation modeling since the asset class results will be dependent on these assumptions.

Time Horizon:

The second factor is the time horizon. We optimize the ending surplus for a five year time horizon. How does one determine the appropriate time horizon? Our method is to assess the duration of the going concern liabilities. Consideration must also be given to business issues such as, competitive issues, tax issues, and the current state of the company. A stable company with a large market share could look at a longer horizon. A company in the midst of a restructuring may want to select a shorter horizon. Teivo Pentikainen, a retired CEO and professor raised an interesting question at a Wharton Conference where we presented our first paper, "Asset-Liability Management and Asset Allocation for Property/Casualty Companies." How do we assess the probability of ruin in any intermediate year before year five? Although our objective function does not consider year-to-year variability, we do test the selected five year portfolio under

1, 2, 3, and 4 year horizons using the same economic simulation. This process helps ensure that our results are acceptable for all five time horizons.

Asset Class Assumptions:

The final asset factor tested is the modeled asset classes. Our fixed income asset classes are modeled as a function of interest spread to relevant Treasurys. We further model the expected impact of default, duration and convexity parameters. In order to test these factors we re-run the optimization with reasonable changes to these basic assumptions. We test the sensitivity of the spreads, duration, convexities, and the resulting asset allocation. Testing of the asset class assumptions allows us to make an informed decision as to when an asset class is "cheap" for our specific line of business.

CONCLUSIONS

The sensitivity testing of the key A-L-C factors allows us to develop a model portfolio range for each line of business and a better understanding of the business risk. Our testing allows us to quantify the impact of these sensitivity factors. The portfolio range selected for each line of business provides acceptable results under numerous economic scenarios. Our portfolio management process then operates within this framework. The "views" of our investment committee must stay within the product lines' model portfolio ranges. A summary of the A-L-C factors and their strategic impacts follows in Exhibit 3.

Exhibit 3:

	A-L-C Factors	Strategic Impact
Liability	• Retention ratio	Portfolio duration
	• Business mix	Duration, surplus volatility
	• Inflation sensitivity	Surplus volatility
	• Loss ratio variability	Surplus volatility, short term assets
Asset	• Asset constraints	Limits or expands the efficient frontier
	• Income requirement	Emphasis yield, not economic return
	• Inflation sensitivity	Pricing sensitivity
	• Level of surplus	Risk tolerance, ALM match
	• FAS 115 classification	Ending surplus, and risk tolerance
Market	• Economic scenario	Performance of asset classes
	• Time horizon	Optimization, surplus volatility
	• Asset class assumptions	Performance of asset classes

We believe this rigorous analysis by product line gives management a better understanding of how to efficiently allocate assets by line of business. More importantly, management can assess the volatility of the results and the tradeoffs from alternate product line or asset class decisions under up to 500 simulations. This process can assist in key marketing decisions. Do we enter or exit a line of business? The process allows you to evaluate a new line of business before you start. If the profile is positive, how can we increase market share? If the profile is negative, do we maintain market share, and reduce profits or take actions to reduce share and maintain profits. A final step in the decision making process would be to complete a full dynamic financial projection assessing the relevant statutory, GAAP and tax implications of the suggested asset mix. Our ALMEF process discussed in the first paper and the sensitivity analysis discussed here allow for superior prospective investment decision-making that maximizes surplus and improves shareholder wealth.

Acknowledgement

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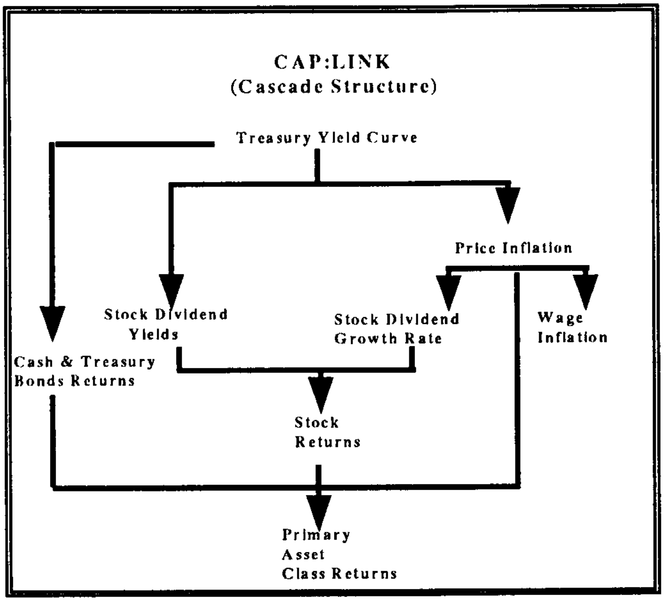
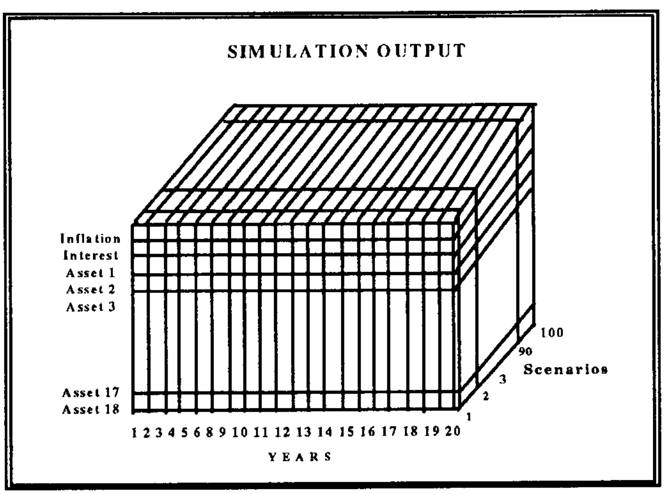
APPENDIX A: STOCHASTIC SIMULATION MODEL: CAP:LINK





APPENDIX B: GENERAL STRUCTURE OF CAP:LINK

CAP:Link uses diffusion models to generate stochastic projections of economic and capital market variables. A diffusion model is simply a process which attributes the change in a particular variable to two separate components. These components include an expected change and a random shock term, both of which are functionally related to the time interval involved. More specifically, the diffusion models used within CAP:Link are variations of general stochastic models where only the present state of the process is relevant for projecting the future. This type of stochastic process is referred to as a Markov process.

The random shock term used within CAP:Link's diffusion models follows a Wiener process. A Wiener process (also referred to as Brownian motion) is a particular type of Markov process such that if z follows a Wiener process, then:

$$\Delta z = \epsilon \sqrt{\Delta t}$$

$$dz = \epsilon \sqrt{dt} \qquad \text{as } \Delta t \rightarrow 0$$

Where,

ϵ is a random sample from a standardized normal distribution and values of Δz are independent for any two intervals of time, Δt

The full form of the diffusion models used within CAP:Link are derived in full or in part by variations to one of the following processes:

Generalized Wiener Process (Brownian Motion with Drift):

$$dx = \mu \, dt + \sigma \, dz$$

Geometric Brownian Motion:

$$\frac{dx}{x} = \mu \, dt + \sigma \, dz$$

Ito Process:

$$dx = \mu\,(x, t)\,dt + \sigma\,(x, t)\,dz$$

Ornstein-Uhlenbeck Process:

$$dx = K\,(\mu - x)\,dt + \sigma\,dz$$

where,

μ = mean drift rate
σ = instantaneous volatility
K = mean reversion rate
dz = a Wiener process

APPENDIX C:

Exhibit 2: Duration Calculations[1]

Mathematically, Macaulay duration is computed as follows:
Macauley duration (in periods) =

$$\frac{(1)PVCF_1+(2)PVCF_2+(3)PVCF_3+\ldots+(n)PVCF_n}{PVTCF}$$

where

$PVCF_t$ = The present value of the cash flow in period t discounted at the prevailing period yield;

t = The period when the cash flow is expected to be received $(t = 1,\ldots,n)$;

n = Number of periods until the final maturity or payout (specifically, number of years to maturity times k, rounded to the nearest whole number);

k = Number of periods, or payments, per year (i.e., $k = 2$ for semiannual-pay bonds, $k = 12$ for monthly pay bonds);

$PVTCF$ = Total present value of the cash flows where the present value is determined using the prevailing yield to maturity.

The formula for Macaulay duration gives a value in terms of periods. Dividing by the number of payments per year converts Macaulay duration to years. That is,

$$\text{Macaulay duration (in years)} = \frac{\text{Macaulay duration (in periods)}}{k}$$

Macaulay duration was developed in 1938 by Frederick Macaulay to use a proxy for the length of time a bond investment was outstanding. This duration is effectively the weighted average term-to-maturity of the bond's cash flows.

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[1] Frank J. Fabozzi, Fixed Income Mathematics, Revised Edition, 1993, Chapter 11, page 164

The significance of a duration calculation is in measuring price volatility. The link between price volatility and Macauley duration is given by the modified duration.

$$\% \text{ change in price} = -\left[\frac{1}{(1+y/k)}\right] \times \text{Macaulay duration} \times \text{yield change}$$

where y = yield to maturity

restated,

$$\text{modified duration} = \frac{\text{Macaulay duration}}{(1+y/k)}$$

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