SOME OBSERVATIONS ON THE BLACK-SCHOLES OPTION PRICING FORMULA

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Abstract

The mathematics of option pricing and of dynamic hedging strategies using options and other derivatives is largely based on the Black-Scholes option pricing formula, which assumes that a risk-free hedge can be set up at any instant in time without affecting the underlying market prices. This paper goes back to first principles in terms of discussing in outline a framework for option pricing and dynamic hedging strategies using the downside approach to risk described in Clarkson (1990). It is suggested that this approach will lead to conclusions that differ radically from those based on the classical Black-Scholes approach.

1. Introduction

The option pricing formula derived by Black and Scholes (1973) is undoubtedly one of the most important mathematical tools in the modern theory of finance. However, over the twenty years since the formula first made its appearance in practical financial management, surprisingly little attention has been paid to the simplifying assumptions behind its derivation, with the result that the formula may not be as universally applicable in the real financial world as is often thought. The purpose of this paper is to discuss in very general terms the practical implications of these crucial simplifying assumptions.

2. The Assumption of Equilibrium

The central role of this assumption is highlighted in the first two sentences of the authors' Summary at the beginning of their seminal paper:

"If options are correctly priced in the market, it should not be possible to make sure profits by creating portfolios of long and short positions in options and their underlying stocks. Using this principle, a theoretical valuation formula for options is derived."

The manner in which this principle of "correct pricing" is employed is described very clearly in the final paragraph of the section that discusses previous work on the pricing of warrants and other options:

"One of the concepts that we use in developing our model is expressed by Thorp and Kassouf (1967). They obtain an empirical valuation formula for warrants by fitting a curve to actual warrant prices then they use this formula to calculate the ratio of shares of stock to options needed to create a hedged position by going long in one security and short in the other. What they fail to pursue is the fact that in equilibrium, the expected return on such a hedged position must be equal to the return on a riskless asset. What we show below is that this equilibrium condition can be used to derive a theoretical valuation formula."

The reference to "the return on a risk-free asset" indicates that a crucial assumption is the existence not only of an asset which in some respect has "zero risk" but also of a trade-off between "excess return" (over the "risk-free rate") and "risk". The definition of risk employed is the symmetric one of variability of return rather a "downside" one relating to the undesirable consequences of a lower than expected return. These considerations lead to the first two of the "ideal conditions" assumed by Black and Scholes:

"a) The short-term interest rate is known and is constant through time.

b) The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of any finite interval is lognormal. The variance rate of the return on the stock is constant."

3. Numerical Verification of the Black-Scholes Formula

It is interesting to note that, if we assume a pure diffusion process of the above type, the very general measure of downside risk first suggested in Clarkson and Plymen (1989) and then developed in Clarkson (1989) gives results that are for all practical purposes identical to those obtained using the Black-Scholes formula. For example, in Clarkson (1990) I obtain, for various values K of the exercise price as a multiple of the current stock price, the following results for a one year call option:

K	Calculated value	Black-Scholes value
0.8	0.281	0.281
0.9	0.212	0.212
1	0.157	0.156
1.1	0.113	0.111
1.2	0.080	0.078

4. The Assumption of Continuous Rebalancing

The creation of a "risk-free" hedged position which can be continuously rebalanced is perhaps the most restrictive assumption behind the derivation of the Black-Scholes option pricing formula. The formal "ideal conditions" introduced in this connection are as follows:

"c) There are no transaction costs in buying or selling the stock or the option.

f) It is possible to borrow any fraction of the price of a security to buy or hold it, at the short-term interest rate.

g) There are no penalties to short selling. A seller who does not own a security will simply accept the price of the security from a buyer, and will agree to settle with the buyer at some future date by paying him an amount equal to the price of the security on that date."

However, accepting that continuous rebalancing is completely impractical, Black and Scholes defend the general applicability of their methodology in the following terms:

"Even if the hedged position is not changed continuously, its risk is small and is entirely risk that can be diversified away, so the expected return on the hedged position must be at the short term interest rate. If this were not true, speculators would try to profit by borrowing large amounts of money to create such hedged positions, and would in the process force the returns down to the short term interest rate."

The latter sentence is strongly reminiscent of the following type of general statement which has been used by financial economists to rationalise what has become known as "The Efficient Market Hypothesis":

"If anomalies existed, alert traders would notice them, and - by attempting to exploit them - would remove them."

5. The Assumption of Stationarity

Many of the mathematical models of the modern theory of finance assume not only that residuals follow a normal distribution but also that the means and variances of these distributions are, in the terminology of the statistician, "stationary". The Black-Scholes option pricing formula is no exception in this regard in view of the assumption that "the variance rate of the return on the stock is constant."

The potentially disastrous practical consequences of an erroneous assumption of "constant variance of return" are vividly spelled out in Pepper's contribution to the discussion on Clarkson and Plymen (1989) and Clarkson (1989):

"When a large Securities House is dealing in many products, ranging right across the foreign exchange markets and debt markets, from overnight money to long-term bonds, from sterling bonds to dollar bonds and deutchmark bonds, and including equities, there is a very strong case for having a consistent system of risk control and for setting limits. The initial approach in each of these markets was to examine the percentage daily changes in price and from these figures to calculate the standard deviations. An extension of that was to do so on a rolling basis. I understand in the middle of 1987 it was common practice in the United States primary government bond market to measure the standard deviation of daily price changes of bonds over the last 30 business days. In the middle of 1987 some US bond houses were arguing that the bond market had become a lot less volatile in the last year and, therefore, they could run much larger positions on the same amount of capital as the year previously without incurring any greater risk. The extraordinary rise in the bond market when the equity market crashed in October 1987 then occurred ..... As far as these price changes over the last 30 business days were concerned, any practical market man knows that the market is well aware of events over this period but it is unexpected events which cause the biggest price movement and the last period you really want to examine, the least important period, is the last 30 days. As far as genuine risk is concerned, surely it is not the central part of the frequency distribution which is important; it is what happens at the tails."

6. Anomalous Evidence Regarding Equilibrium and Efficiency

In recent years various researchers have compiled large bodies of evidence which suggest that the cornerstone "equilibrium" and "efficiency" assumptions of the theory of finance are inconsistent with empirical observations. In particular, Shiller (1989) summarizes his and his coauthors' work in the following terms:

"I present here evidence that while some of the implications of the efficient markets hypothesis (that speculative prices always represent the best information about the true economic value) are substantiated by data, investor attitudes are of great importance in determining the course of prices of speculative assets. Prices change in substantial measure because the investing public en masse capriciously changes its mind."

Shiller also emphasizes the profound practical implications of his findings:

"That prices change for no good reason is of great importance for many purposes. Prices of speculative assets guide very many economic activities in our society. When an asset is underpriced, incentives are created to neglect or abuse it. When it is overpriced, incentives are created to invest too much resources in it."

Peters (1989) pursues a not dissimilar theme using chaos theory as one of his main investigative tools, and concludes that current theories are inadequate:

"We have seen evidence that the capital markets are non-linear systems, and we have seen that current capital market theory does not take these effects into account. Because of this omission their validity is seriously weakened."

7. Anomalous Evidence regarding Continuous Rebalancing

While capital market prices can generally be assumed to be continuous when trading volumes are light, the underlying market mechanisms might break down on occasional chaotic events that take all market participants by surprise. A very clear account of the underlying fragility which can lead to acute disorder was given by George Soros in his evidence to the U.S. House Banking Committee in April 1994:

"The trouble with derivative instruments is that those who issue them usually protect themselves against losses by engaging in so-called delta, or dynamic hedging. Dynamic hedging means, in effect, that if the market moves against the issuer, the issuer is forced to move in the same direction ... and thereby amplify the initial price disturbance.

"As long as price changes are continuous, no great harm is done except perhaps higher volatility, which, in turn, increases demand for derivative instruments.

"But if there is an overwhelming amount of dynamic hedging to be done in the same direction, price movements may become discontinuous.

"This raises the specter of financial dislocation. Those who need to engage in dynamic hedging, but cannot execute their orders, may suffer catastrophic losses .... That is what happened in the stock market crash of 1987."

In short, attempts to "rebalance" portfolios on either a sharp fall or a sharp rise in the market could shatter the theoretical "equilibrium" on which the rebalancing strategy was based.

8. Anomalous Evidence Regarding Stationarity

The non-linear behavior cited by Peters and the discontinuous behavior described by Soros are inconsistent with the assumed stationarity of capital market theories generally and of the "constant variance" assumption underlying the Black-Scholes formula.

The dichotomy between "orderly" and "disorderly" periods of capital market behavior is described very clearly by Mandelbrot (1963) :

"Broadly speaking, the predictions of my main model seem to be reasonable. At closer inspection, however, one notes that large price changes are not isolated between periods of slow change; they rather tend to be the result of several fluctuations, some of which "overshoot" the final change. Similarly, the movement of prices in periods of tranquility seems to be smoother than predicted by my process. In order words, large changes tend to be followed by large changes - of either sign - and small changes tend to be followed by small changes."

9. The Definition of Risk

The presumptions of "equilibrium" and "efficiency" in capital market theory can be traced back to empirical tests such as those carried out by Jensen (1968) in which it is assumed that risk is equivalent to short term variability of return. An implicit corollary is that all stocks are homogeneous in terms of short term variability of return, in that there is no attribute other than risk as perceived by an investor which influences the variability of return.

In Clarkson (1994) I examine the analysis in Jensen (1968) on the basis that short term price variability is a function of the frequency with which important new information arrives that has not already been discounted in the consensus forecasts. The exceptionally high risk-adjusted return of the one utilities fund and the very low risk-adjusted returns of the "science" (i.e. "high technology") funds suggest that Jensen has overlooked a source of extreme heterogeneity and hence that his inference of "strong level efficiency" is unsound. Similar criticisms can be made of many other investigations that were influential in the formulation of "The Efficient Market Hypothesis".

10. An Alternative Assumption

To begin rebuilding the theory of option pricing on more solid foundations it is necessary to abandon the assumptions of "equilibrium" and "efficiency". An alternative and much more realistic assumption is as follows:

"Capital market prices successively overshoot and undershoot some central value which could be regarded as a long term equilibrium value."

This assumption is consistent not only with the anomalous evidence assembled by Shiller, Peters and others but also with the practical Mean Absolute Deviation approach to security analysis described in Clarkson (1978), Clarkson (1981) and Clarkson and Plymen (1989). Furthermore, the Mean Absolute Deviation multiplier of 1.6 which is found to work best in practice suggests that, as a first approximation, a suitable non-linear model is simple harmonic motion with short term random noise superimposed.

11. General Observations

Rebuilding the theory of option pricing in the absence of the Black-Scholes assumptions regarding "equilibrium", "risk", "efficiency" and "stationarity" is clearly too massive a task to be attempted in this brief exploratory paper. However, several features of a plausible new framework can be described in outline.

(i) As demonstrated in Section 3 above, the Black-Scholes formula can in certain instances be regarded as the special case where the underlying stock is standing at its "central" value and in this respect is "fairly valued".

(ii) The downside approach to risk developed in Clarkson (1989) and Clarkson (1990) will result in option prices which vary with the financial circumstances and risk preferences of individual market participants.

(iii) The rebalancing of portfolios between stock and options will depend on the perceived cheapness or dearness of both the stock and options as measured against appropriate "central" values.

(iv) One-way "dynamic hedging" driven by mechanistic rules will be replaced by two-way arbitrage on both stock and option prices guided by medium term fundamental value, thereby introducing more elasticity into the system and reducing the risk of financial dislocation.

12. Conclusion

Despite the somewhat restrictive simplifying assumptions required, the Black-Scholes option pricing formula has become a vital tool in financial management and risk control. This paper begins the search for an even more general framework for option pricing by examining which of the Black-Scholes assumptions should be replaced by alternative assumptions which are more in accord with the non-linearity and non-stationarity observed in capital market behavior.

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