RETURN AND SHORTFALL RISKS OF ROLLOVER HEDGE-STRATEGIES WITH OPTIONS

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Abstract

On the basis of a (partially) historical simulation approach the authors evaluate return and shortfall risks of various hedge strategies with options (put hedge: fixed percentage and ratchet strategy; covered short call; collar) which are performed in a roll-over design. The shortfall-risk measures considered are: shortfall probability, shortfall expectation, 5 %-percentile and 1 %-percentile. Transaction costs are included in the analysis.

Keywords

shortfall risk; rollover hedge strategies

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Résumé

Sur la base d'une simulation (partiellement) historique, les auteurs evaluent le rendement et le risque de shortfall des stratégies de hedging par options (put hedge: fixed percentage and ratchet stratégie; covered short call; collar) employée de façon roulante. Les mesures du risque de shortfall mentionnées dans cet article sont: shortfall-probabilité et shortfall-moyenne, 5%-percentile et 1%-percentile. Les côutes de transactions sont inclus dans l'analyse.

1. Introduction

Due to the publication of FIGLEWSKI et al. (1993) the performance of rollover put-hedge strategies has recently attracted increased interest. Rollover strategies with options are characterized by the fact that the maturity of the option position is shorter than a fixed planning horizon making it necessary to buy (or sell) a *sequence* of short-maturity options. In contrast to that static hedge strategies are characterized by an identical maturity of the option position and the planning horizon. FIGLEWSKI et al. (1993) show that rollover hedge strategies give a reasonable protection, but do not reduce return as much as the corresponding static hedge strategies.

The present paper continues the analysis of rollover option strategies. There are the following differences compared to the paper of FIGLEWSKI et al. (1993):

1) The analysis is (at least partially) based on empirical data. FIGLEWSKI et al. (1993) use a stochastic simulation approach.
2) In addition to alternative put-hedge strategies we analyze covered-short-call strategies as well as collar strategies.
3) Transaction costs are included in the analysis.
4) We use (inter alia) two measures of shortfall-risk, shortfall probability and shortfall expectation to quantify the risk of the strategies considered.

2. Data basis and strategies

As data basis we use the 408 end-of-month values of the DAFOX from 1960 - 1993. The DAFOX, cf. GÖPPL/SCHÜTZ (1992), is a performance index for the German stock market, especially designed for research purposes. The DAFOX time series mirrors the performance of an index strategy with respect to a highly diversified portfolio of German stocks. The DAFOX is representing an unprotected investment strategy in stocks and defines the benchmark for the performance of the analyzed hedge strategies. At the beginning of each month (date of rollover) one 1-month-put or/and one 1-month-call on the DAFOX is bought resp. sold. All option prices are calculated using the BLACK-SCHOLES-formula. The volatility parameter is estimated as in HULL (1993, p. 215) on the basis of the differences of the logarithms of the past end-of-month values of the DAFOX, using the last 12 values in each case. As riskless interest rate per month we use the average of the day-to-day money rates of the corresponding month as published by the German Central Bank.

Contrary to FIGLEWSKI et al. (1993) we do not finance option premiums at the riskless interest rate or do invest eventual proceeds at the riskless interest rate. According to our experience this would unreasonably bias the results. Instead we perform an *operation blanche* by investing eventual proceeds additionally in the DAFOX and by financing capital requirements by selling a portion of the DAFOX.

Transaction costs of 1 % of the option premium are assumed when buying or selling an option. When exercising an option at the end of the month we additionally assume 0.2 % of the inner value as transaction costs. According to our investigations these assumptions are corresponding to a realistic order of magnitude of the transaction costs of institutional investors.

We consider the following hedging strategies with options in our analysis:

I. Rollover Put-Hedge Strategies

A. Rollover Fixed-Percentage Strategies

At every rollover date we buy puts on the basis of a fixed-percentage strategy, i.e. the exercise prices X_t are corresponding to a fixed percentage rate p of the price S_t of the DAFOX, the underlying security, i.e. $X_t = (p/100)S_t$. We consider one in-the-money strategy ($p = 102$), one at-the-money strategy ($p = 100$) as well as three out-of-the-money strategies ($p = 94, 96, 98$). At each time one put is bought, however this does not exactly correspond to a 1 : 1 strategy.

B. Ratchet Strategies

Ratchet strategies, cf. FIGLEWSKI et al. (1993), are characterized by $X_t = \max(X_{t-1}, p\, S_t) = p \max(S_{t-1}, S_t)$ which implies $X_t = \max(X_0, X_1, ..., X_{t-1}) = p \max(S_0, ..., S_{t-1})$. The ratchet strategy allows the investor to lock in early gains without accepting the downside risk of the fixed percentage strategy.

II. Rollover (Fixed Percentage) Covered-Short-Call Strategies

Following this strategy the investor sells a call at each rollover date, the exercise price being a fixed percentage of the price of the underlying. Again we consider one in-the-money strategy (p = 98), one at-the-money strategy as well as three out-of-the-money strategies (p = 102, 104, 106).

III. Rollover (Fixed Percentage) Collar Strategies

Following this strategy at each rollover date one put is bought and one call is sold as well. The exercise prices of both the put and the call follow fixed percentage strategies, where we only consider the symmetrical strategies (94, 106), (96, 104) and (98, 102).

3. The evaluation criteria

We consider the sequence $\{r_t\}_{t=1, \ldots, T}$ of monthly returns of the various (protected or unprotected) strategies as being a realization of a corresponding sequence $\{R_t\}_{t=1, \ldots, T}$ of random variables. All monthly returns are calculated on a *continuous* basis. In case the $\{R_t\}$ are an independent sequence of random variables identically distributed as a random variable R the sample estimators

$$\bar{R} := \frac{1}{T}\sum_{t=1}^{T} R_t \tag{1}$$

and

$$S_R^2 := \frac{1}{T-1}\sum_{t=1}^{T} (R_t - \bar{R})^2 \tag{2}$$

are distribution free and unbiased estimators of E(R) resp. Var(R).

While E(R) is commonly accepted as a measure of (average) return, the adequacy of Var(R) as a measure of risk is criticized to an increasing extent. Alternatively measures of shortfall risk, cf. HARLOW (1991) or ALBRECHT (1994), are proposed. Especially when analyzing positions with options symmetrical measures of risk, like the variance or the standard deviation are not very suitable, cf. already BOOKSTABER/CLARKE (1985), as option positions typically follow an asymmetrical risk-return-profile. Looking at the put-hedge for example, the downside risk of the investor is limited to an absolute extent. On the other hand the investor participates in increases of the price of the underlying object to an unlimited extent (only reduced by the option premium). It is almost evident that variance in case of a put hedge is not a measure of risk but a measure of investment chances (in the sense of "upward" volatility).

Taking into account the asymmetrical nature of positions with options we will measure risk by four alternative asymmetrical measures.

At first we consider two measures of shortfall risk. A general class of shortfall risk measures is given by the *lower partial moments* of degree $n \geq 0$ of R, cf. ALBRECHT (1994):

$$\begin{aligned} LPM_n(r_Z) &= E[\max(r_Z - R,\ 0)^n] \\ &= \int_{-\infty}^{r_z} (r_Z - r)^n f(r) dr \ . \end{aligned} \qquad (3)$$

The quantity r_z is a target return, which can be interpreted as a specified minimum return level. In the following we will consistently use a target of -5% (on a continuous return basis). As risk measures we use the *shortfall probability*, which corresponds to the case $n = 0$, and the *shortfall expectation*, which corresponds to the case $n = 1$.

The other two asymmetrical measures of risk are based on the ε-percentile r_ε of the distribution F (additionally we assume the existence of a density function f) of R, defined by

$$\varepsilon = \int_{-\infty}^{r_\varepsilon} f(r) dr = F(r_\varepsilon) \ . \qquad (4)$$

Obviously shortfall-probability and ε-percentile are inversely related measures of risk. In case of the shortfall probability the target return r_z is given, in case of the ε-percentile it has to be determined according to a fixed shortfall probability of ε.

In the present paper we use estimators of the 1 %- and the 5 %-percentile as measures of "desaster risk", cf. FIGLEWSKI et al. (1993).

Now, how can we estimate the lower partial moments resp. the ε-quantiles based on the observed returns $\{r_t\}$? According to (3) we have to estimate the n-th absolute moment of the random variable max $(r_z - R, 0)$. The corresponding sample counterpart is $\frac{1}{T}\sum_{t=1}^{n} \max(r_Z - R_t, 0)^n$.

Defining the indicator variable $I_z(R) = 1$, in case $R < r_z$ and $I_z(R) = 0$, in case $R \geq r_z$, then the last expression is identical to

$$\hat{\text{LPM}}_n(r_Z) := \frac{1}{T}\sum_{t=1}^{T} (r_Z - R_t)^n I_Z(R_t) \; . \tag{5}$$

In case of independent and identically distributed $\{R_t\}$ expression (5) gives us a distribution free and unbiased estimator of the lower partial moment $\hat{\text{LPM}}_n(r_z)$. The cases $n = 0, 1$ give the corresponding estimators for the risk measures shortfall probability and shortfall expectation.

According to (4) we obtain estimators for the ε-percentile by replacing the right-hand side of (4) by $\hat{\text{LPM}}_0(r_z)$ according to (5) and "solving" the resulting identity for r_z.

4. Statistical problems of rollover option strategies

As can be verified mathematically, cf. ALBRECHT/MAURER/STEPHAN (1994), even in case of independent and identically distributed returns $\{R_t\}$ of the underlying, the DAFOX, the corresponding sequence of returns for rollover option strategies are *neither independent nor identically distributed.* FIGLEWSKI et al. (1993) characterize rollover option strategies on a more intuitive basis as *path-dependent strategies.* This however does imply, that the estimators (1), (2) and (5) are loosing their properties in the i.i.d.-case. The estimators now are only simple *descriptive* statistical measures and no more unbiased estimators for the moments of a parent distribution. This has to be borne in mind, when interpreting the various estimators used in sequel.

5. The average return of rollover hedge strategies

The average return of the underlying, the DAFOX, as well as of the alternative rollover hedge strategies is estimated on the basis of (1). All returns calculated are monthly returns on a continuous basis. As a benchmark the average monthly return of the DAFOX is used, which is 0.6283 %.

The following table contains the average return values for the rollover put-hedge strategies for the different exercise prices expressed as a percentage of the price of the underlying, the DAFOX:

	Exercise Price				
	94	96	98	100	102
Av. Return	0.6325	0.6133	0.58	0.5533	0.4974

Table 1: Put-Hedge-Fixed-Percentage

One recognizes that higher levels of protection are corresponding to lower average returns which is according to intuition as well as theory. Remarkably, however, in case of deep-out-of-the-money puts the average return is slightly higher compared to the unprotected position, the DAFOX (although we have included transaction costs!) This is in contrast to theory and we will analyze this phenomenon at the end of this paragraph.

The table for the ratchet strategy is:

	Exercise Price				
	94	96	98	100	102
Av. Return	0.5417	0.558	0.5442	0.5117	0.4548

Table 2: Ratchet-Strategy

Contrary to a first intuition the ratchet strategy dos *not* lead to a higher realized average return. This is caused by the fact, that the ratchet strategy achieves a higher level of protection which implies higher "insurance costs" and therefore a higher reduction of the average return compared to the fixed-percentage strategy.

The table for the covered short call is:

	Exercise Price				
	98	100	102	104	106
Av.Return	0.47029	0.5008	0.5475	0.5992	0.6408

Table 3: Covered-Short-Call

Again we notice a trade-off between level of protection and average return as in case of the put-hedge strategies. Again the average return is reduced compared to the unprotected position with the exception of a deep-out-of-the-money call.

Finally the table for the collar strategy:

	Exercise Prices		
	94/106	96/104	98/102
Av. Return	0.6467	0.5858	0.4933

Table 4: Collar-Strategy

According to the design of the collar the effects of the put-hedge and the covered-short-call are reflected simultaneously. Narrowing the collar to (98%, 102%) implies an almost dramatic reduction of the average return so that an extreme strategy of this kind has to be well-thought.

Finally we have to analyze the "deep-out-of-the-money phenomenon" resulting in average returns which - although transaction costs are included - are higher than in the unprotected case, which is contrary to theory. An explanation for this anomaly is given by analyzing the deviation of the empirical distribution function of the continuous returns of the underlying from the normal distribution. The normal distribution for the continuous returns is implied, cf. HULL (1993, p. 212), by the assumption of a Brownian motion process for the stock prices in the BLACK/SCHOLES-model. Using the ANDERSON-DARLING goodness-of-fit test for the normal distribution, cf. D'AGOSTINO (1986), the (composite) hypothesis of normally distributed (continuous) DAFOX returns is very clearly rejected. The test statistic gives a value of 1.895 which distinctly exceeds the critical value even in case of a very low level of significance of 0.5 %, which gives a critical value of 1.159. The deviations from the normal distribution are caused by some very negative returns, which imply a significant skewness to the left of the empirical distribution. By eliminating the five lowest returns from the sample, however, the ANDERSON-DARLING-test (value of the test statistic: 0.862) now does not reject anymore the normal distribution at a level of significance of 10 % (critical value: 0.873).

This leads to the following explanation of the observed phenomenon. The *buyer* of deep-out-of-the-money puts takes profit from the shortfall risks which are empirically higher than implicitely assumed by the BLACK/SCHOLES-formula. The *writer* of deep-out-of-the-money calls takes profit from the lower return chances of the underlying compared to the normal distribution because of the skewness to the left of the empirical returns.

6. The risk of rollover option strategies

We first give the frequency diagram for the put-hedge fixed-percentage strategy in case of p = 96:

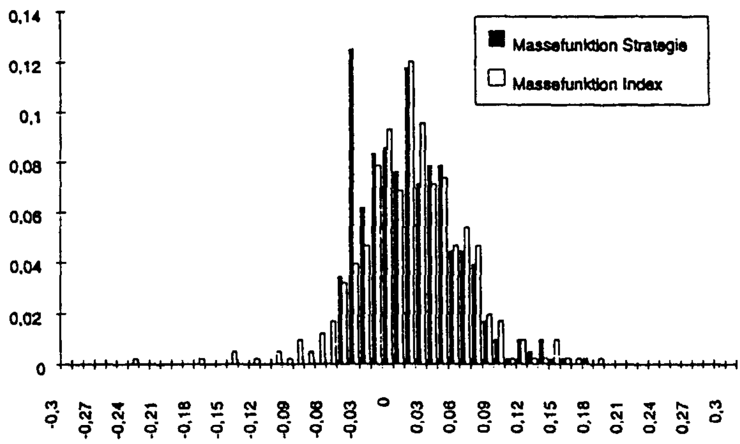

Figure 1: Frequency Diagram Put-Hedge Fixed-Percentage 96%

The figure clearly shows the asymmetry of the frequency distribution as well as the reduction of risk, as intended by the strategy. More detailed insight into the risk of the strategy is given by table 6. All numerical values are given as a percentage with respect to continuous monthly returns.

	Exercise Price				
	94	96	98	100	102
Std. Deviation	4.37	4.09	3.68	3.11	1.89
Shortfall-Frequency	10.54	3.43	0	0	0
Average Shortfall	0.13	0.01	0	0	0
1%-Percentile	-6.78	-5.37	-4.40	-3.61	-2.44
5%-Percentile	-6.32	-4.78	-3.68	-2.91	-1.42

Table 5: Put-Hedge Fixed-Percentage

The relevant benchmark figures again are given by the corresponding values for the unprotected investment in the DAFOX as shown by table 6.

Std. Deviation	4.94
Shortfall-Frequency	9.56
Average Shortfall	0.34
1%-Percentile	-15.82
5%-Percentile	-6.91

Table 6: DAFOX

One recognizes that the asymmetrical risk measures are very sensitive to a change of the desired level of protection. In addition we notice a significant reduction of desaster risk especially when compared to the covered-short-call strategy (as presented below).

The next figure gives the frequency diagram for the ratchet strategy with $p = 96$.

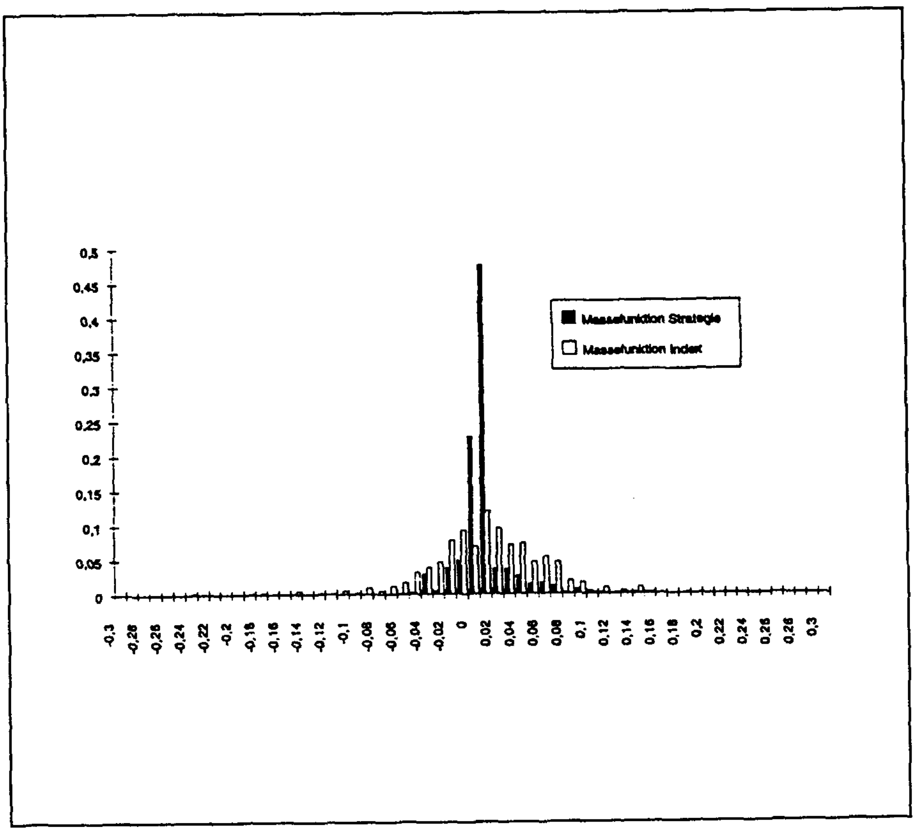

Figure 2: Frequency Diagram Ratchet-Strategy 96 %

Next we have the corresponding table:

	Exercise Price			
	94	96	98	100
Std. Deviation	2.70	2.44	2.13	1.8
Shortfall-Frequency	2.21	0.25	0	0
Average Shortfall	0.026	0.0003	0	0
1%-Percentile	-6.37	-4.60	-3.25	-2.12
5%-Percentile	-3.12	-2.57	-2.20	-1.35

Table 7: Ratchet-Strategy

Comparing table 7 to table 5 we notice a generally higher level of risk reduction for the ratchet strategy. This confirms FIGLEWSKI et al. (1993, p. 56), who state: "Our results show that the ratchet's advantage is not a higher mean, but its protectiveness, especially against a desaster."

Figure 3 gives the frequency diagram for the covered-short-call-strategy with $p = 104$.

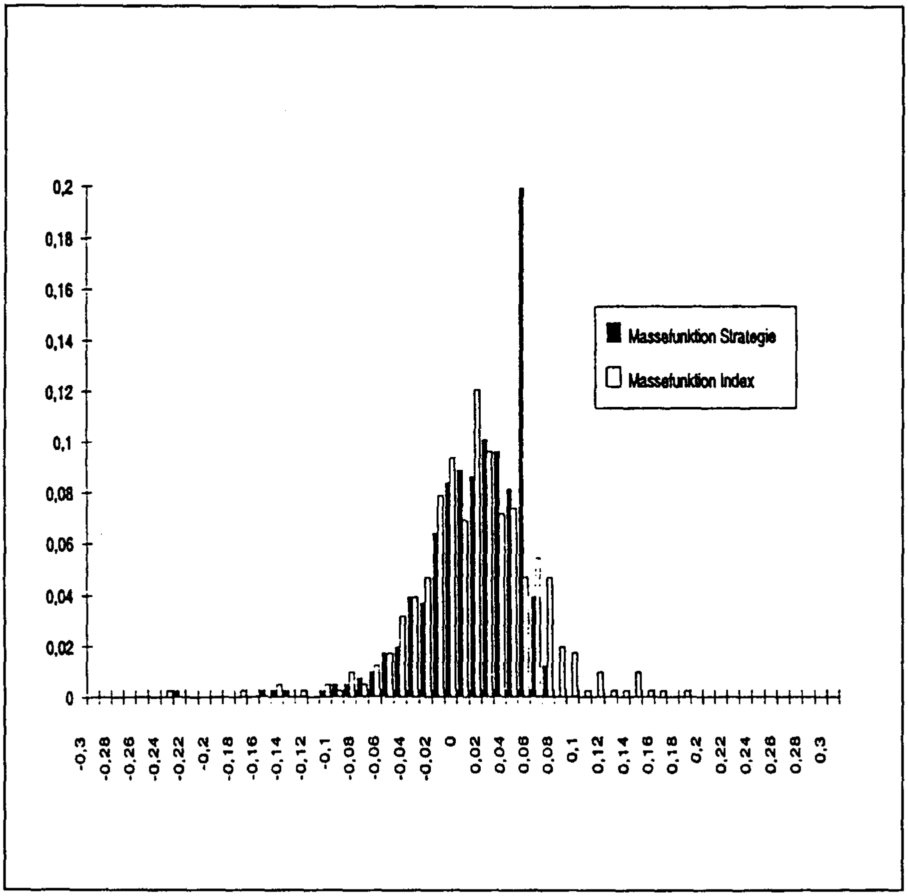

Figure 3: Frequency Diagram Covered-Short-Call Fixed-Percentage 104 %

One recognizes that compared to the put-hedge strategy there is practically no protection against desaster risk. This is confirmed by table 8.

	Exercise Price				
	98	100	102	104	106
Std. Deviation	0.74	2.9	3.09	3.59	4.32
Shortfall-Frequency	2.94	4.90	6.37	7.60	8.82
Average Shortfall	0.14	0.18	0.23	0.27	0.30
1%-Percentile	-12.05	-13.32	-14.27	-14.93	-15.34
5%-Percentile	-3.83	-5.11	-6.02	-6.60	-6.80

Table 8: Covered-Short-Call Fixed-Percentage

The reduction in variance is almost entirely explained by the reduction of "upward volatility" which again stresses the inadequacy of variance as a measure of risk in case of positions with options.

Finally we give the corresponding frequency diagram and the corresponding table for the (96, 104) collar strategy:

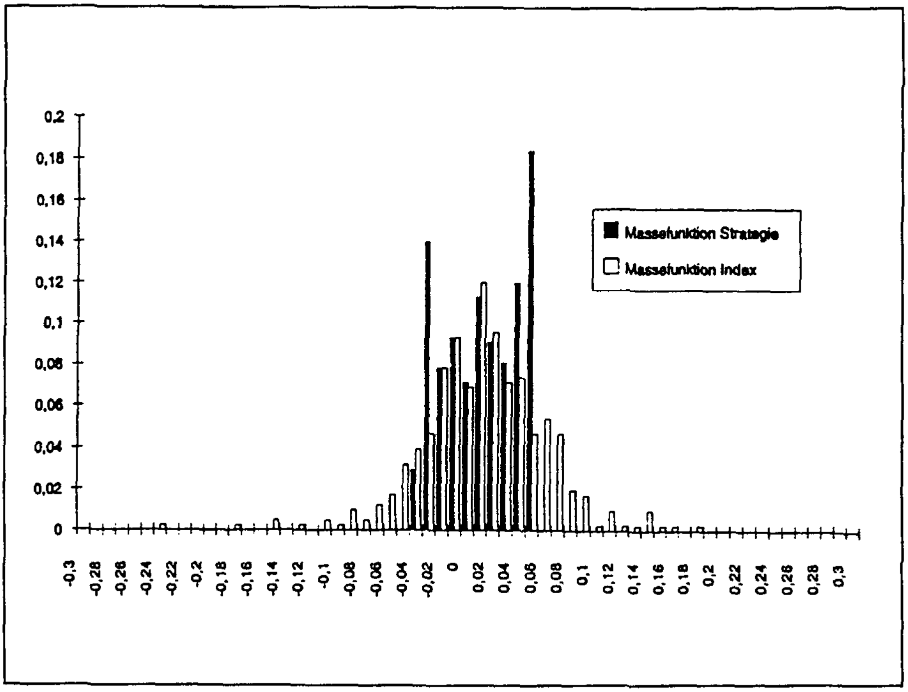

Figure 4: Frequency Diagram Collar-Strategy (96 %, 104 %)

	Exercise Price		
	94/106	96/104	98/102
Std. Deviation	3.67	2.89	1.72
Shortfall-Frequency	9.56	0	0
Average Shortfall	0.09	0	0
1%-Percentile	-6.19	-4.05	-1.97
5%-Percentile	-6.02	-3.94	-1.87

Table 9: Collar Fixed-Percentage

The collar strategy gives the highest risk reduction for all measures of risk. However, as we have seen in paragraph 5, this is also true for the reduction of the average return.

7. Rollover hedge strategies in "bull markets"

The preceeding results show that the collar and the versions of the put hedge have advantageous risk reduction features. Therefore we are interested, in the sense of a worst case scenario analysis, in the performance of these strategies from January 1980 to September 1987, i.e. in a bull market phase. Table 10 gives the values for the unprotected investment, the DAFOX.

Average Return	1.3593
Std. Deviation	4.52
Average Shortfall	8.6
Shortfall-Mittel	0.18
1%-Percentile	-11.74
5%-Percentile	-5.85

Table 10: DAFOX (sub-period)

The following tables give the corresponding figures for the rollover hedge strategies.

	Exercise Price			
	94	96	98	100
Av. Return	1.3088	1.2981	1.2116	1.1120
Std. Deviation	4.32	4.06	3.7	3.17
Shortfall-Frequency	9.68	4.30	0	0
Average Shortfalll	0.10	0.01	0	0
1%-Percentile	-6.76	-5.36	-4.00	-2.91
5%-Percentile	-6.29	-5.03	-3.62	-2.50

Table 11: Put-Hedge Fixed-Percentage (sub-period)

	Exercise Price			
	94	96	98	100
Av. Return	1.0139	0.9662	0.9880	0.9686
Std. Deviation	3.79	3.45	2.99	2.44
Shortfall-Frequency	4.30	0	0	0
Average Shortfall	0.05	0	0	0
1%-Percentile	-6.55	-4.78	-3.36	-2.36
5%-Percentile	-5.62	-4.45	-2.99	-1.77

Table 12: Ratchet-Strategy (sub-period)

	Exercise Price		
	94/106	96/104	98/102
Av. Return	1.1702	1.0159	0.7769
Std. Deviation	3.59	2.79	1.67
Shortfall-Frequency	8.60	0	0
Average Shortfalll	0.06	0	0
1%-Percentile	-6.19	-4.04	-1.84
5%-Percentile	-5.78	-3.92	-1.80

Table 13: Collar Fixed-Percentage (sub-period)

We see that the collar and the put hedge ratchet strategy lead to a significantly reduced average return.This is because of the high level of protection implied by these strategies. On the other hand for the put-hedge fixed-percentage strategy we interestingly only have a reduction in average return which is moderate.

8. Conclusions

The results presented show that the collar and the versions of the put-hedge have advantageous risk reduction features. As to be expected we have a trade off between level of protection and average return. The advantage of the ratchet strategy is not a higher average return but its increased protectiveness, especially against a desaster. The increases in average return observed when analyzing deep-out-of-the-money positions are not of a systematic nature. In the "bull market phase" 1980 - 1987 we observe that the fixed-percentage strategy leads only to a moderate reduction of the average return.

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