PERFORMANCE CONCENTRATION : « LESS IS MORE »

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Topic of the paper

- Importance of performance measurement for evaluating an investment process. Value added : where ?
- An old debate : top down or bottom up approach ?
- Aim of the paper : to address this issue with a new performance measure : a measure of **performance concentration**.
- Why to introduce a new performance indicator ? Because of the flaws of CAPM-based, APT-based, or model-based performance measures. This indicator is **model-free**.
- Stylized fact : « less is more ».

Methodology (1) : contribution of stocks

Let R_P be the total performance of a given portfolio over one periode (one day, one week, one month...). We introduce the stock j contribution, denoted C_j, such as

$$R_P = \sum_{j=1}^{N} C_j$$

Next, we split the whole portfolio into two subportfolios, and we consider the n positive contributions and the n' negative contributions : $n + n' = N$. We rank the subsample by decreasing order.

Notation 1 $C_{(k)}$ *is the* k*-th upper positive contribution.*

We obtain the ordered sample

$$C_{(1)} > C_{(2)} > C_{(3)} > \cdots > C_{(n)}$$

where $C_{(1)} = \max(C_1, \cdots, C_n)$, and $C_{(n)} = \min(C_1, \cdots, C_n)$.

Methodology (2) : concentration index

To exhibit the positive performance concentration on few stocks, we consider the contribution of the $100\,p\%$ upper positive contributions, compared to 100% of the total positive performance.

Définition 1 (Stock picking performance concentration index)
The contribution of the $100\,p\%$ upper positive contributions to the total performance is the ratio

$$T_+(p) = \frac{C_{(1)} + \cdots + C_{([np])}}{C_{(1)} + \cdots + C_{(n)}} \qquad \frac{1}{n} < p < 1 \tag{1}$$

where $[np]$ is the integer part of np.

The quantity $T_+(p)$ is the contribution of the $[np]$ upper positive contributions of stocks over the period.

It is a concentration measure in the exact sense where the value of $T_+(p)$ quantifies the way of which the $100\,p\%$ upper positive contributions contribute to 100% of the aggregate positive performance.

For example, if $p = 0,2$, and if $T_+(0,2) = 0,8$, it means that 20% of the individual positive specific securities returns are responsible for more than 80% of the total positive return (the "law of 80/20").

Methodology (3) : tracing out the Lorenz curve

To build the Lorenz curve, we use the empirical index

$$T_n(k) = \frac{C_{(1)} + \cdots + C_{(k)}}{C_{(1)} + \cdots + C_{(n)}}$$

The table below details the method for tracing out the curve. The same methodology is used for tracing out the Gini-Lorenz curve of negative stocks contributions.

SET cum. frequency (X-axis)	TOTAL POSITIVE PERFORMANCE cumulative positive ordered contributions	RELATIVE MASS cum. frequency (Y-axis)
$1/n$	$C_{(1)}$	$T_n(1)$
$2/n$	$C_{(1)} + C_{(2)}$	$T_n(2)$
$\vdots$		$\vdots$
k/n	$C_{(1)} + C_{(2)} + \cdots + C_{(k)}$	$T_n(k)$
$\vdots$		$\vdots$
1	$C_{(1)} + C_{(2)} + \cdots + C_{(k)} + \cdots + C_{(n)}$	1

Methodology (4) : total concentration

We now address the cumulative impact of *both* positive and negative concentration of specific securities returns.

The total performance is the sum of all positive and negative contributions :

$$R_P = \underbrace{C_{(1)} + C_{(2)} + \cdots + C_{(n)}}_{\text{ordered positive contributions}} + \underbrace{C_{(n+1)} + C_{(n+2)} + \cdots + C_{(N)}}_{\text{ordered negative contributions}}$$

We assess the impact of the **withdrawal** of the best stock picking bets.

We drop out the upper contribution, and recaculate the new total performance as

$$R_P - C_{(1)} = C_{(2)} + C_{(3)} + \cdots + C_{(N)}$$

and for the two upper contributions

$$R_P - \left(C_{(1)} + C_{(2)}\right) = C_{(3)} + \cdots + C_{(N)}$$

etc.

And the same for the negative contributions.

Methodology (5) : returns

$\{V_t,\ t \geq 0\}$ is the process of market value of the portfolio.
$\{R_t,\ t \geq 0\}$ is the continuously compounded process such that

$$R_t = \ln V_t - \ln V_0 \tag{2}$$

Periodic continuous return for an interval of length τ

$$\Delta R(t,\tau) = R_t - R_{t-\tau} \tag{3}$$

where the notation Δ is choosen to emphasize the periodic nature of return over this interval of length τ.

Notation 2 $\Delta R(t,1) = \Delta R_k = R_k - R_{k-1}$ *is the daily return.*

Between the two dates 0 et T : sample $\{\Delta R_1, \cdots, \Delta R_N\}$ of size N.

As previously $\Delta R_{(k)}$ is the k-th upper positive return. $\Delta R_{(1)} = \max(\Delta R_1, \cdots, \Delta R_n)$, and $\Delta R_{(n)} = \min(\Delta R_1, \cdots, \Delta R_n)$. The ordered positive sample is

$$\Delta R_{(1)} > \Delta R_{(2)} > \Delta R_{(3)} > \cdots > \Delta R_{(n)}$$

Cumulative positive performance of the k highest positive returns :

$$R_{(k)} = \Delta R_{(1)} + \Delta R_{(2)} + \cdots + \Delta R_{(k)} = \sum_{i=1}^{k} \Delta R_{(i)}$$

Methodology (6) : concentration index

To exhibit the positive performance concentration on few days, we consider the contribution of the $100\,p\%$ upper returns, compared to 100% of the aggregate positive performance.

Définition 2 (Market timing performance concentration index)
The contribution of the $100\,p\%$ *upper positive returns to the total performance is the ratio*

$$J_+(p) = \frac{\Delta R_{(1)} + \cdots + \Delta R_{([np])}}{R_{(n)}} \qquad \frac{1}{n} < p < 1 \qquad (4)$$

where $[np]$ *is the integer part of* np.

The quantity $J_+(p)$ is the contribution of the $[np]$ upper positive returns over the period.

It is a concentration measure in the exact sense where the value of $J_+(p)$ quantifies the way of which the $100\,p\%$ upper positive returns contribute to 100% of the aggregate positive performance.

For example, if $p = 0,2$, and if $J_+(0,2) = 0,8$, it means that 20% of the individual positive returns are responsible for more than 80% of the total positive return (the "law of 80/20").

Results. Managed portfolio

- Portfolio of the french insurance company SMA BTP between 31/12/2001 and 23/09/2004.
- Why to choose SMA BTP ?
  - Style of the investment process. SMA BTP has choosen an actively managed style with high level of stock picking operations, **deliberately not indexed on any type of a given benchmark**.
  - The high overperformance of SMA BTP against the two main benchmarks for French diversified portfolios : SMA BTP =-3,76%. CAC40 = -21%, Eurostoxx = -28,13%. **CAC + 17%, ESX+19%**.
- To analyze and to detect the sources of this overperformance.

Result (1) : contributions of stocks
Lorenz curves « law of 30/75 » or « 50/90 »

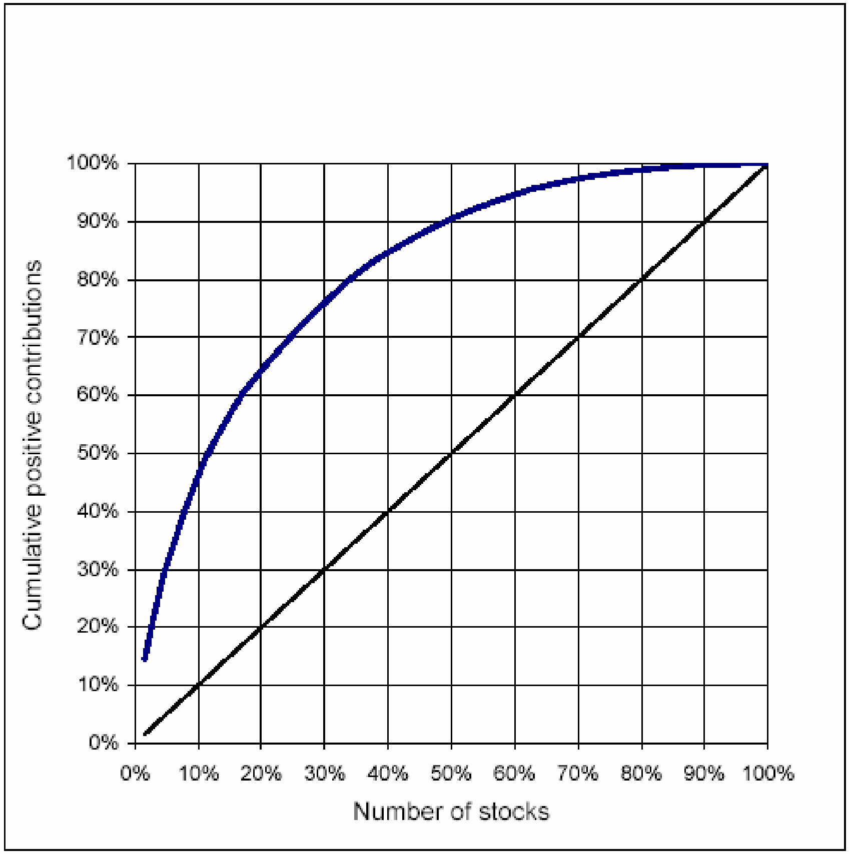

Figure 5 :
Stock picking concentration of positive performance of managed portfolio of SMA BTP

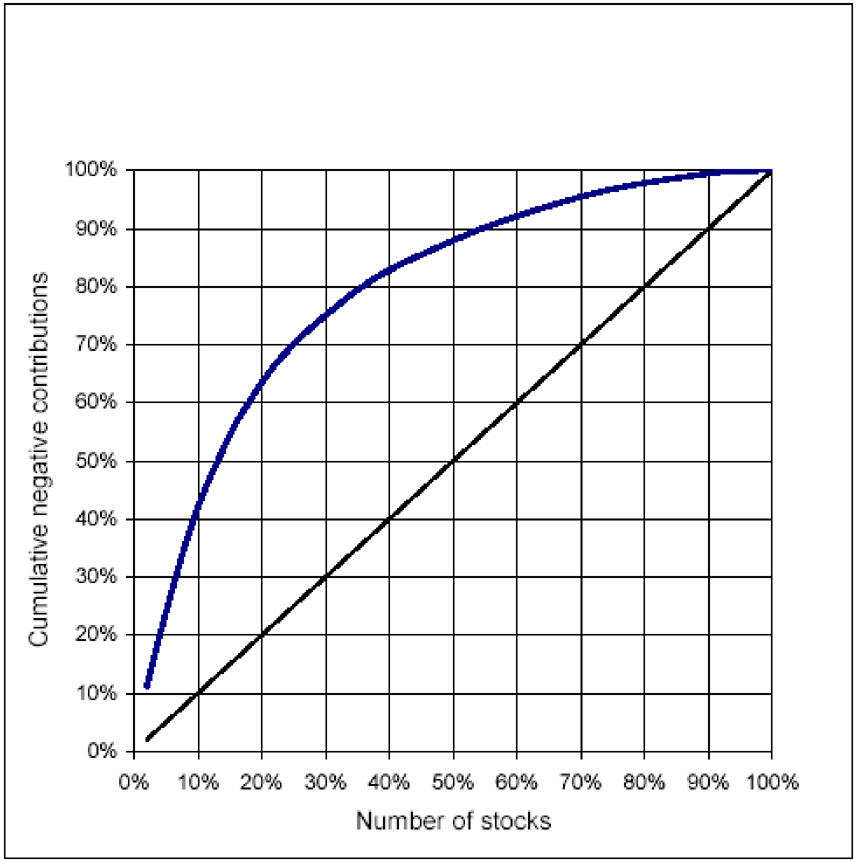

Figure 6 :
Stock picking concentration of negative performance of managed portfolio of SMA BTP

Result (2) : the upper contributions

Rank	STOCK	Ordered contributions	IMPACT OF CONCENTRATION : New total performance after withdrawal of cumulative contribution
1	Areva	$+1,94\%$	$-5,70\%$
2	Pernod Ricard	$+1,08\%$	$-6,78\%$
3	Essilor	$+0,92\%$	$-7,70\%$
4	Crédit Lyonnais	$+0,70\%$	$-8,40\%$
5	Technip	$+0,68\%$	$-9,08\%$
6	Spir comm.	$+0,59\%$	$-9,67\%$
7	Vinci	$+0,57\%$	$-10,24\%$
⋮	⋮	⋮	
108	Lafarge	$-0,79\%$	$+5,13\%$
109	France Telecom	$-0,92\%$	$+4,34\%$
110	Suez	$-1,11\%$	$+3,42\%$
111	Aventis	$-1,32\%$	$+2,31\%$
112	Scor	$-1,37\%$	$+0,99\%$
113	Vivendi	$-1,48\%$	$-0,38\%$
114	Sanofi	$-1,90\%$	$-1,86\%$
	TOTAL	$-3,76\%$	

→ 7 stocks of 114 = 6% of portfolio

$$R_P = \sum_{j=1}^{114} C_j = -3,76\%$$

Result (3) : stock picking and extreme concentration

Extreme concentration of the performance on very few stocks which capture the main part of the global gain (loss).

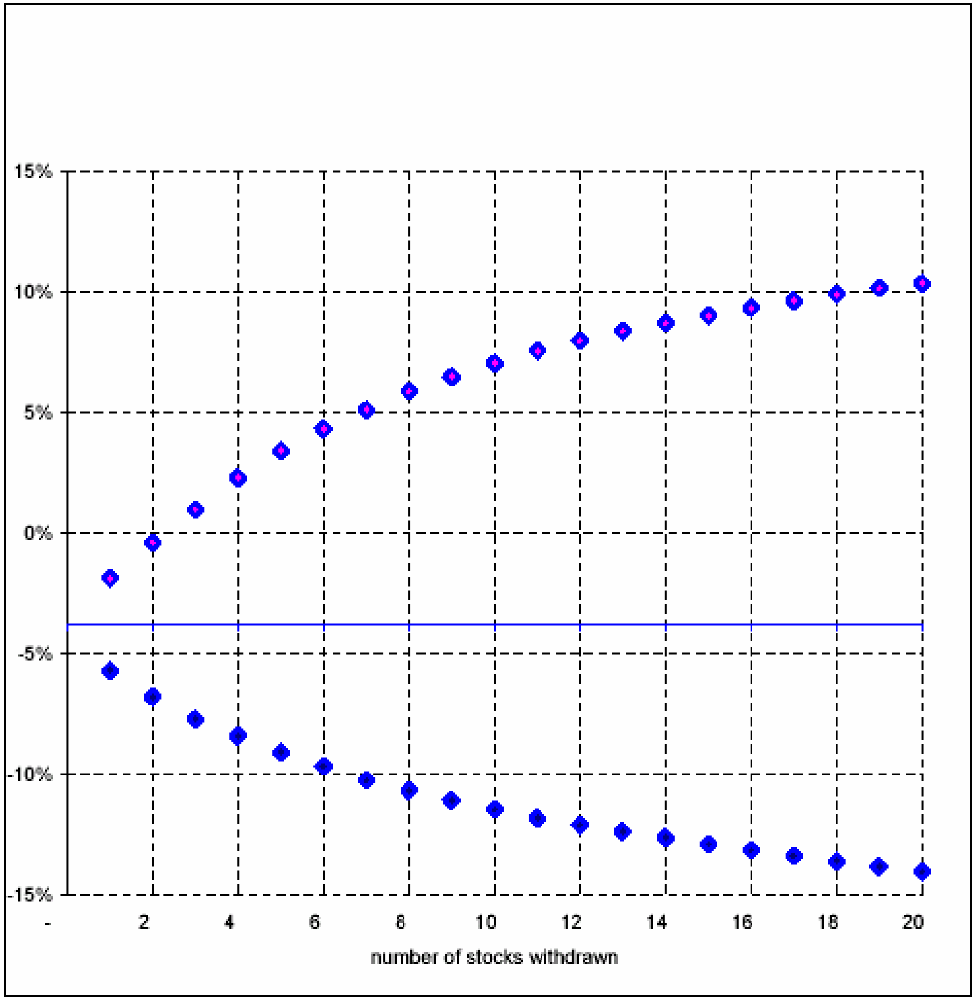

Figure 7 :
Stock picking concentration of total performance of managed portfolio of SMA BTP

Result (4) : contribution of days
Lorenz curves « law of 30/65 » or « 50/85 »

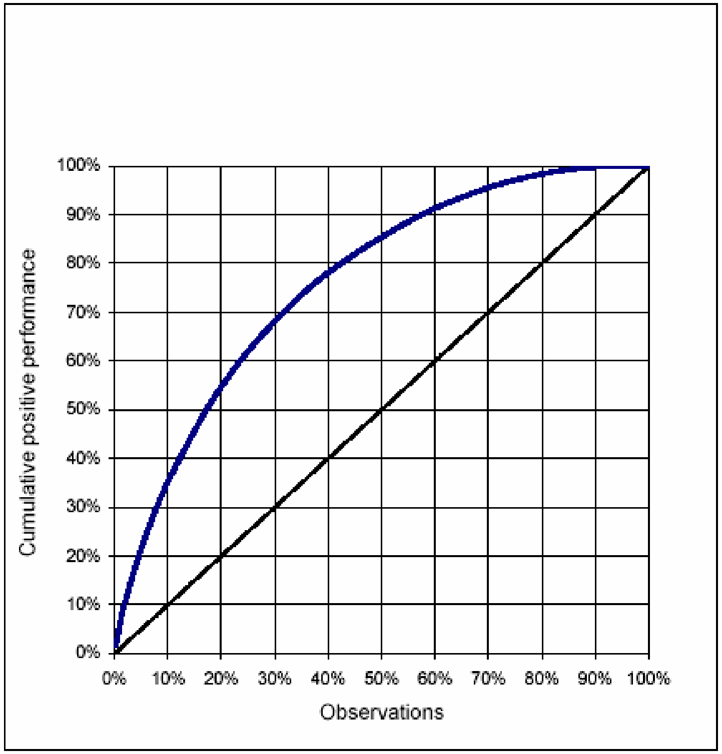

Figure 3 :
Market timing concentration of positive performance of managed portfolio of SMA BTP

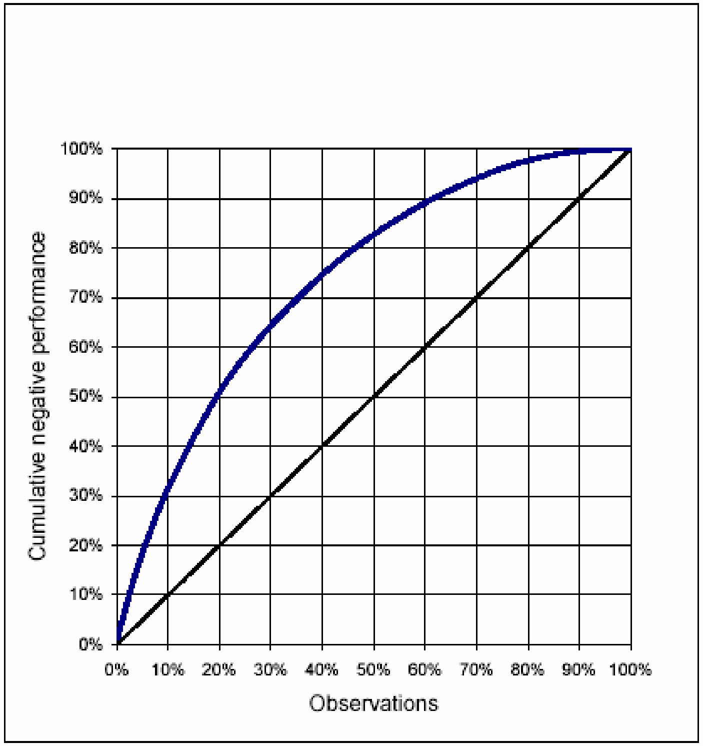

Figure 4 :
Market timing concentration of negative performance of managed portfolio of SMA BTP

Analysis (5) : persistence of concentration

	overall period		subperiod 1		subperiod 2		subperiod 3	
	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$
$J(0,10)$	0,30	0,35	0,30	0,35	0,30	0,30	0,30	0,25
$J(0,30)$	0,65	0,68	0,63	0,70	0,62	0,65	0,60	0,58
$J(0,50)$	0,85	0,85	0,83	0,87	0,82	0,85	0,80	0,80
$R(5)$	+8%	−8%						
$R(10)$	+15%	−14%						
$R(15)$	+20%	−20%						

This table presents the results of market timing performance concentration for the four periods analyzed : overall (31/12/2001 – 23/09/2004), subperiod 1 (year 2002), subperiod 2 (year 2003), subperiod 3 (31/12/2003 – 23/09/2004). For each period, we give the results of $J_-(p)$ and $J_+(p)$ for $p = 10\%, 30\%, 50\%$. We note a relative stability of results : it appears that 10% of positive (resp. negative) returns are responsible for more than 30% of the aggregate positive (resp. negative) performance, 30% of positive (resp. negative) returns are responsible for more than 65% of the aggregate positive (resp. negative) performance, and 50% of positive (resp. negative) returns are responsible for more than 85% of the aggregate positive (resp. negative) performance. The last three lines $R(.)$ give the value of annualized performance recalculated after the withdrawn of the 5, 10, and 15 upper (resp. lower) returns of the overall period.

Matching the stylized facts : first look on the tails

The relationship between the order statistics and the empirical distribution function of a sample is well known. The empirical df $F_n(x)$ of a random variable X is defined by

$$F_n(x) = \mathrm{Fr}(X \leq x) = \frac{1}{n}\mathrm{card}\{i : 1 \leq i \leq n,\ X_i \leq x\} = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}_{\{X_i \leq x\}}, \quad x \in \mathbb{R}$$

where $\mathbf{1}_A$ denotes the indicator function of the set A. Hence, for the market timing concentration, we have

$$F_n(\Delta R_{(k)}) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}_{\{\Delta R_i \leq \Delta R_{(k)}\}} = \frac{n-k}{n}$$

therefore

$$\mathrm{Fr}(\Delta R_i \geq \Delta R_{(k)}) = 1 - F_n(\Delta R_{(k)}) = 1 - \frac{n-k}{n} = \frac{k}{n} \qquad (5)$$

where Fr is the empirical cumulative distribution function of ΔR_k. A **rank/ordering technique** can now be used to trace out the charts of the two tails.

Result (7) : Generalized Pareto Distribution ?

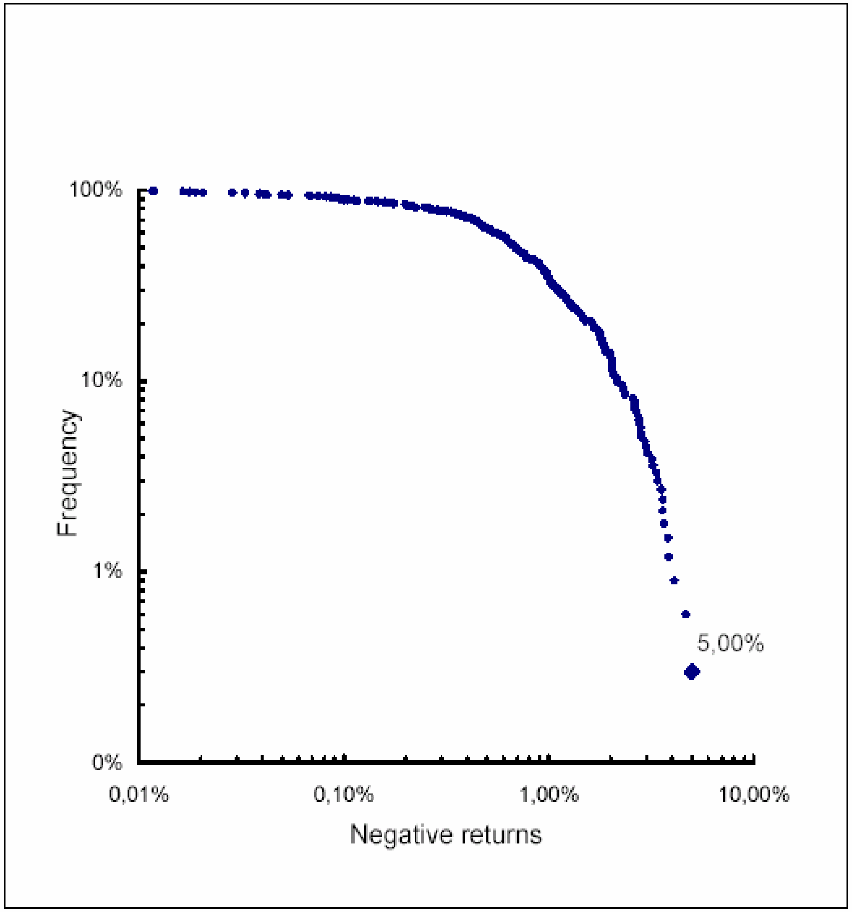

Figure 2 : Negative tail of the managed portfolio of SMA BTP

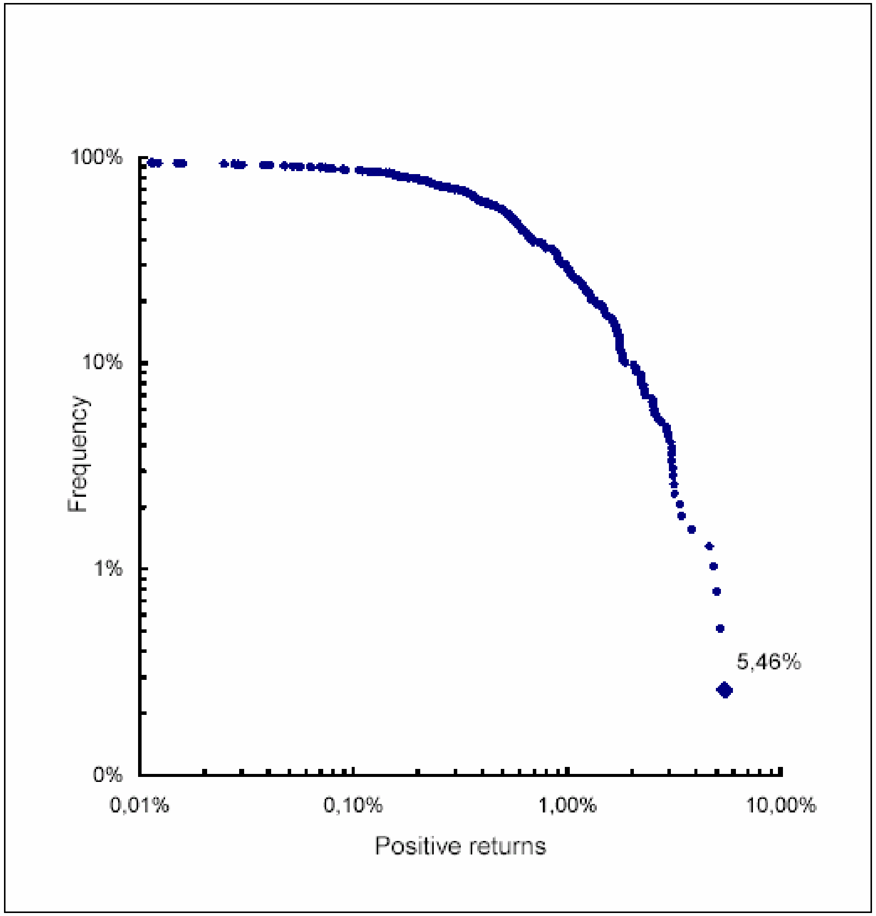

Figure 1 : Positive tail of the managed portfolio of SMA BTP

Conclusion

- New type of performance measure : concentration index, model-free.
- New type or performance chart : Lorenz curve.
- Stylized fact : « less is more »

- Debates…

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