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An Investigation of the loss of an CAPM-portfolio

Considerations on risk, measurement of risk and safeguarding risk of investments

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Motivation: Risk of a pension fund

- pension funds: Assets (e.g. shares, bonds) used for paying benefits
- Risk of a pension funds: Rate of return is to low to pay benefits; insolvency of funds or employer
- Germany: Funds or assets of employer are safeguarded by Pensionssicherungsverein auf Gegenseitigkeit PSVaG
- PSVaG demands a premium proportional to investment (book reserve) from all companies owing funds; pay as you go finance
- Wealthy companies pay for poor companies (insolvency)
- Thesis: With such a premium a prudent man will invest in the assets with highest rate of return not considering the risk of this investment

Risk of an investment: definition of the loss

- Loss of an investment: rate of return is below an expected value R_{nec}
- Measures of loss:
  - Noise
  - Shortfall
  - Variance: (CAPM, (Sharpe, W.F. The Journal of Finance 19, 425-42 (1964))
  - Value at risk (1-α of all rates of return are larger than R_{VaR})
  - All these measures give a value of the probability of a loss or a value of the maximal loss (with 1-α confidence)

- Alternative measure?
- lower partial moment? as Fishburn, American Economic Review 67, 116-26 (1977).

Risk of an Investment: average loss L_A

- Definition of the average loss based on lower partial moments

$$L_A = I \int_{-\infty}^{Rnec} (R_{nec} - R)\, p(R)\, dR$$

- R_{nec}: rate of return, which is necessary for the investor; *I*: amount of investment, *R*: rate of return of the investment, *p(R)*: probability density
- The probability to obtain a loss larger than 0 is:

$$\int_{-\infty}^{R_{nec}} p(R)\, dR \qquad \text{with} \qquad \int_{-\infty}^{\infty} p(R)\, dR = 1$$

- upper boundary R_{nec} instead of ∞, because we measure the loss, not the gain of the investment
- L_A is a measure of the expected loss: i.e.

$$L_A(R_I) > L_A(R_{II})$$

means, that using portfolio I, a larger loss must be expected and has to be safeguarded than using portfolio II

Risk of an Investment: average loss L_A

- L_A is a function of R, R_{nec} and p(R)
- L_A is not additive with respect to R, p(R) and R_{nec}.
- Using L_A:
  - measurement of L_A of different investments or portfolios
  - optimization of the composition of different portfolios with respect to
    - average loss
    - expected rate of return, when the average loss can be safeguarded.

Average loss L_A of a CAPM-portfolio

- CAPM portfolio consists of “bonds” and “shares”, (see cash and tangential portfolio of the CAPM theory)
- Bond: $\sigma = 0$, i.e. no variance, fixed rate of return R_0. Shares: $\sigma_{TP} > 0$, variable rate of return R_{TP}
- The rate of return of this (pension fund) portfolio is:

$$R_{PF} = \alpha R_0 + (1-\alpha) R_{TP}$$

and the expected rate of return

$$\mu_{PF} = \alpha R_0 + (1-\alpha)\mu_{TP};$$

and the standard deviation

$$\sigma_{PF} = (1-\alpha)\sigma_{TP};$$

with μ_{TP} the expected rate of return. We assume $0 \le \alpha \le 1$ (investment on the capital market line).

Average loss L_A of a CAPM-portfolio

- We obtain for L_A:

$$L_A = I \int_{-\infty}^{R_{nec}} (R_{nec} - \alpha R_0 - (1-\alpha)R_{TP})\, p(\alpha R_0 + (1-\alpha)R_{TP})\, d(\alpha R_0 + (1-\alpha)R_{TP})$$

we define:

$$p_{PF}(R_{TP}) \equiv p(\alpha R_0 + (1-\alpha)R_{TP})$$

We will use only $p_{PF}(R_{TP})$ and we call this $p(\mathrm{R}_{TP})$. We conclude:

$$L_A = I(1-\alpha) \int_{+\infty}^{\frac{R_{nec}-\alpha R_0}{1-\alpha}} (R_{nec} - \alpha R_0 - (1-\alpha)R_{TP})\, p(R_{TP})\, dR_{TP}$$

- L_A can not been calculated straight forward, neither the derivate of L_A as function of α
- Assuming that R_{TP} is normally distributed, we can calculate upper limits for L_A in an analytic manner
- For $R_0 > R_{nec}$, there is a minimum ofl L_A at $\alpha = 1$

Average loss of a CAPM-portfolio. Example MSCI

- Shares described by MSCI-Europe-ET-performance-index
- Assummtions: rate of return is normally distributed, investment period of 1 year:
- average rate of return of 10.8 % and variance of 20 %; investment period of 1 year (average rate of return of 0.64% and variance of 7.90 %: investment period of 1 month)
- R_{nec}, = 6 % and 3.25%; R_0 = 3 %, or 1.5 %
- L_A as a function of the composition of the portfolio is not monotone, but has a minimum for $\alpha = 0.8$

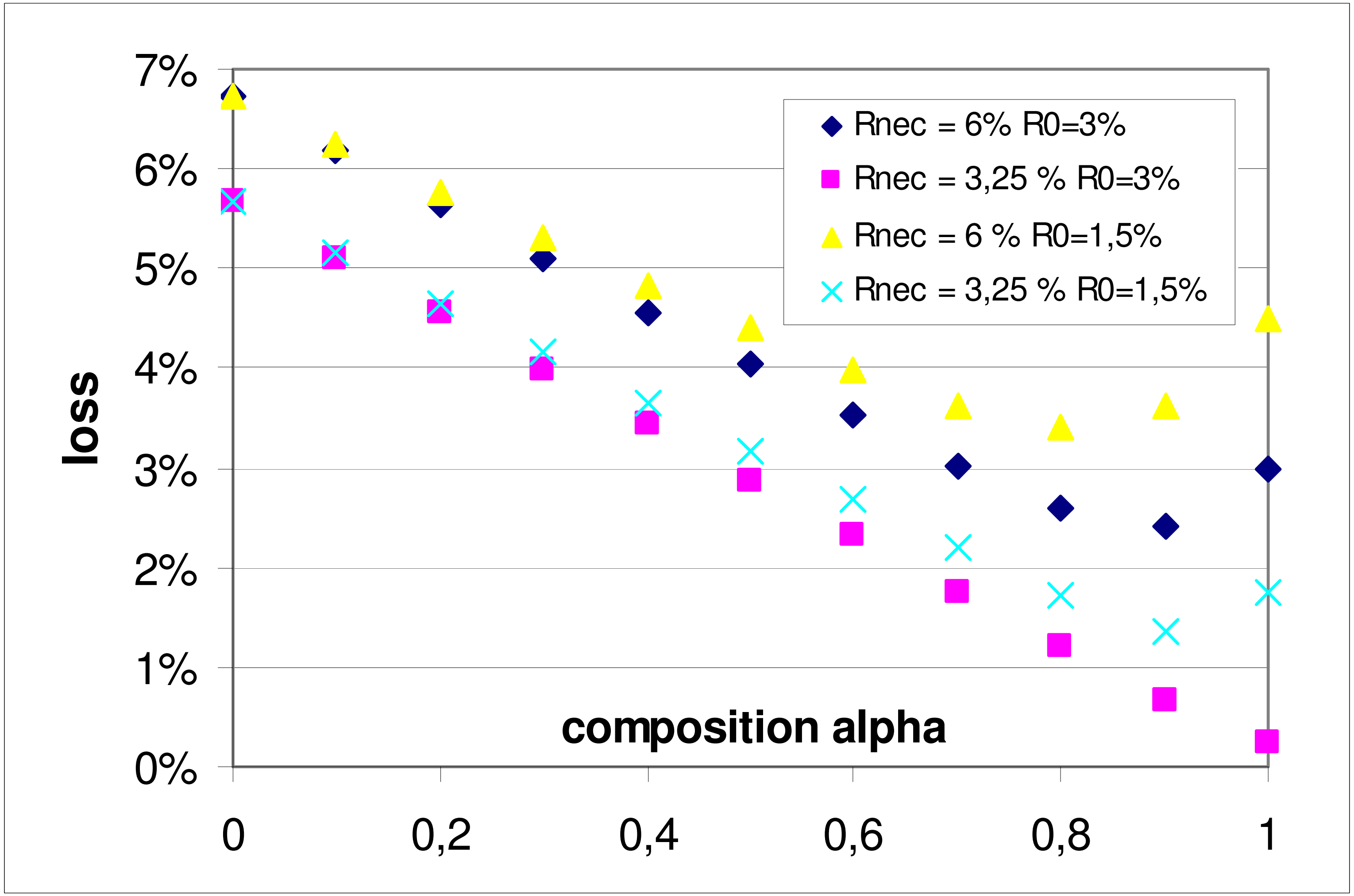

Fig. 1 L_A of a CAPM-portfolio (shares described by MSCI EUROPE Performance Index)

Safeguarding of the loss and optimization of an CAPM-portfolio

- We undewrite the average loss by an insurance.
- Concerning the premium of the insurance contract, we investigate three scenarioes:
- A: the premium is proportional to the investment: $P = \pi I$
- B: the premium is proportional to the investment in shares: $P = \pi I (1-\alpha)$
- C: the premium is proportional to the loss: $P = \pi L_A$
- We define an utility function *U* as sum of the expected return of the investment minus the premium.

$$U = I\,(aR_0+(1-\alpha)R_{TP})-P$$

- The investor builds his portfolio, that he obtains the maximum in the utility function.

Safeguarding of the loss of a portfolio and building of the portfolio

- Scenario A: $P = \pi I$:

$$U = I\,(aR_0+(1-\alpha)R_{TP})-I\,\pi = I\,(aR_0+(1-\alpha)R_{TP}-\pi)$$

- For $R_{TP} > R_0$, the investor will always choose a portfolio which contains only shares.
- Scenario B:

$$U = I\,(aR_0+(1-\alpha)R_{TP})-(1-\alpha)I\,\pi = I\,(aR_0+(1-\alpha)(R_{TP}-\pi))$$

- An investor will invest in shares, when

$(R_{TP} - \pi) > R_0$ and in bonds, when $(R_{TP} - \pi) < R_0$

- Scenario C:

$$U = I\,(\alpha R_0+(1-\alpha)R_{TP})-L_A\,\pi$$

- Maximization of U by variation of α can only be done by numerical methods.
- different premiums π, i.e. 0.1, 0.5, 1 and 2
- For $\pi > 1$, local maximum for $0 < \alpha < 1$

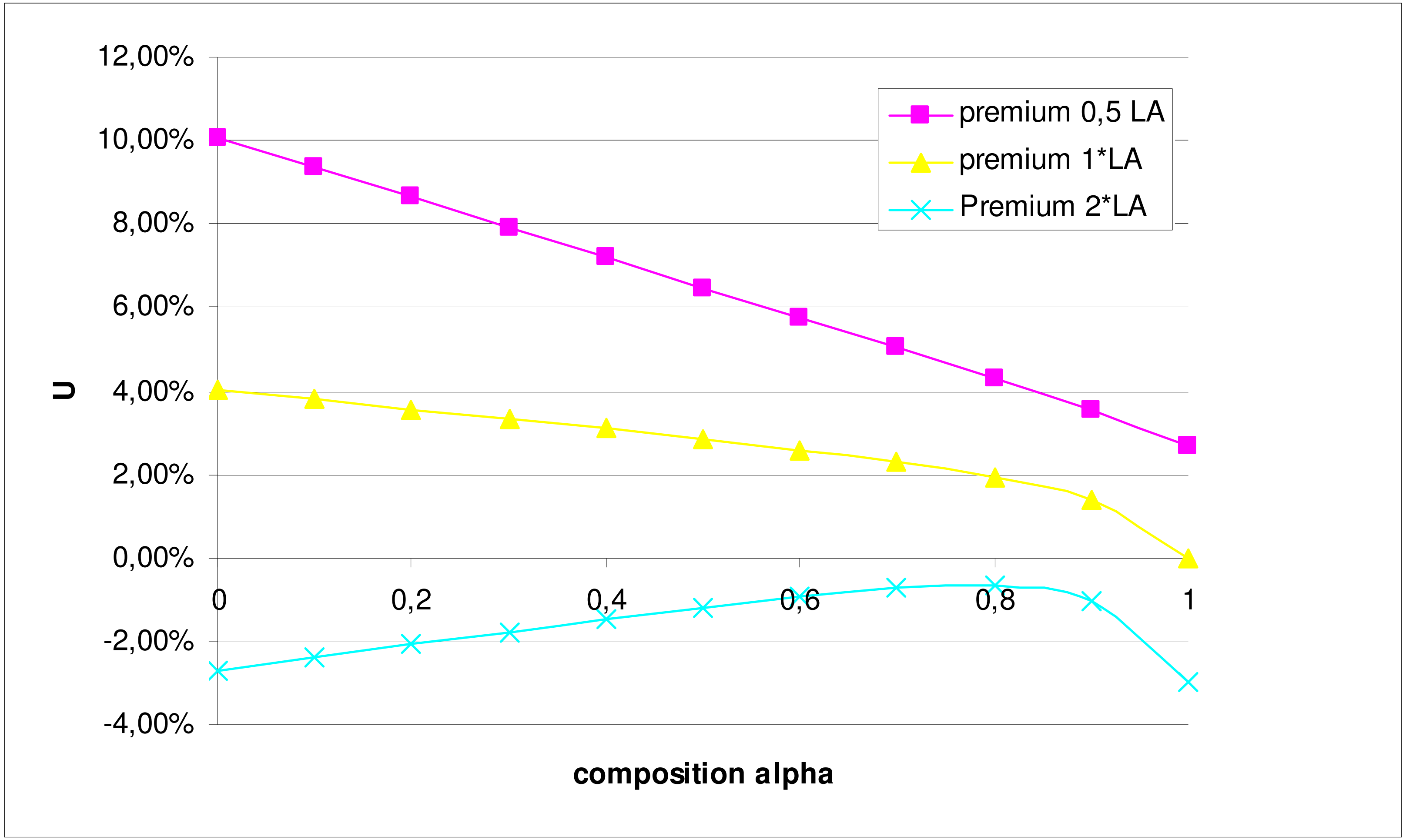

Figure 2: Utility function of a CAPM portfolio, if the average loss L_A is safeguarded by different insurance premiums, R_{nec} 06%, R_0 = 3%

Conclusion

- Average loss L_A is appropriate to measure the loss of a portfolio
- Using the average loss L_A we can optimize the portfolio with respect to L_A
- Calculating L_A and safeguarding it, we can optimize the portfolio with respect to the utility function U
- U is the expected return of the investment minus the insurance premium.
- Parameters and values, used to calculate the premium are important, for the portfolio composition, considering maximal values of U ; i.e. maximal rate of return and no loss.