Optimal consumer behaviour in a jump-diffusion environment

Sachi Purcal
Thomas Huan Wang
School of Actuarial Studies
University of New South Wales
Sydney, Australia

September 2005

E-mail: s.purcal@unsw.edu.au

Overview

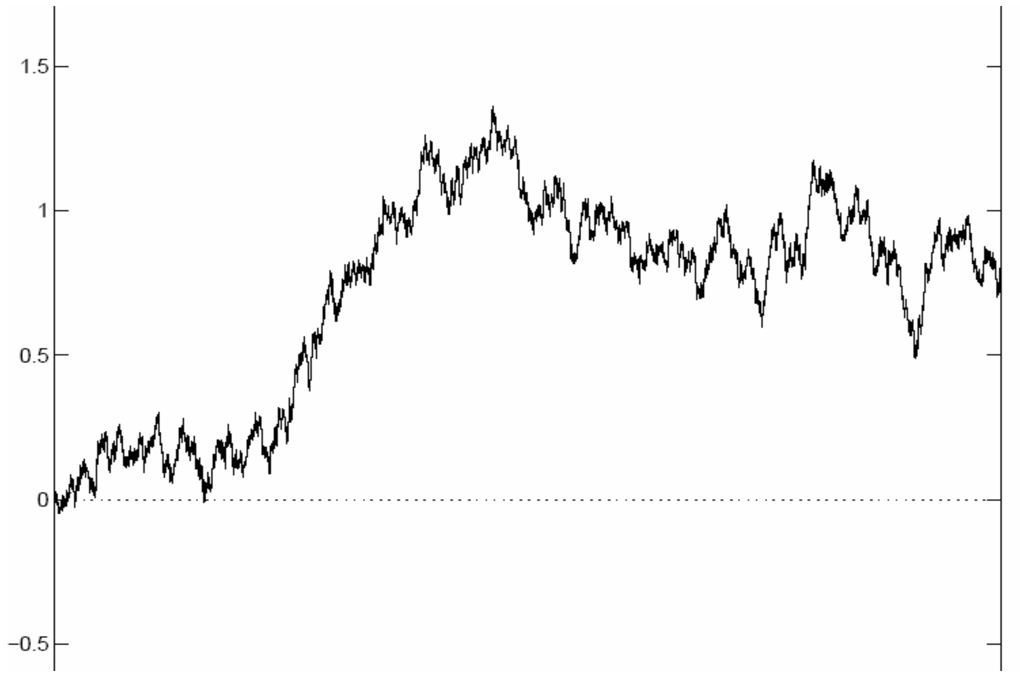
1. Motivation
2. Literature
3. Extension to Richard (1975): jumps
4. Solution method
5. Results

1 Motivation

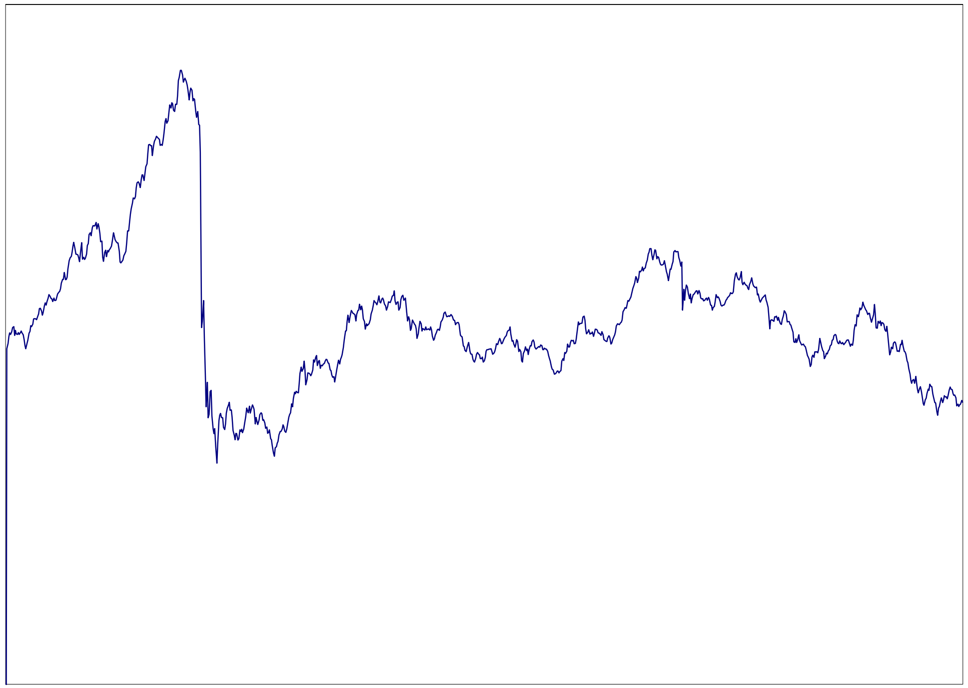
Better fit to data

- problems with geometric Brownian motion (Gallant & Tauchen, 1997)

Market incompleteness—economic implications



Geometric Brownian motion.



All ordinaries index 1987–1990.

2 Literature

- Optimal investment/consumption modelling
  - Merton (1969, 1971)
  - Richard (1975)
- Event risk
  - Merton (1973)
  - Liu, Longstaff & Pan (2003)
  - Emmer & Klüppelberg (2004)
- Economic implications
  - Campbell & Viciera (2002)
  - Gourinchas & Parker (2002)

3 Extension to Richard (1975): jumps

$$\max_{C,\pi,Z} \mathbf{E}\left[\int_{\tau}^{T} U(C(t),t)dt + B(Z(T),T)\right] \tag{1}$$

$$\frac{dQ(t)}{Q(t)} = \alpha dt + \sigma dq(t) + \xi dJ \tag{2}$$

$$\begin{aligned} dW(t) \quad = \quad & -C(t)dt - P(t)dt + Y(t)dt + rW(t)dt \\ & +(\alpha - r)\pi(t)W(t)dt + \sigma\pi(t)W dq(t) \\ & +\sigma\pi(t)\xi W dq(t) \end{aligned} \tag{3}$$

4 Solution method

Unable to find closed-form solution

Numerical solution: *Markov Chain approximation technique* of Kushner & Dupuis (2001)

Replace partials in HJB equation with finite difference approximations

5 Results

1. Low return setting
2. High return setting
3. Expected consumption

Age	Expected no. of market crashes every decade	Value of HJB equation	Optimal controls Consumption	Risky investment	Desired bequest
30	0.0	-264.47	0.17810	33.08%	7.5132
	0.1	-264.50	0.17809	31.34%	7.5126
	0.5	-265.32	0.17772	24.51%	7.4969
	1.0	-267.81	0.17661	16.15%	7.4503
50	0.0	-173.67	0.23579	33.13%	7.8957
	0.1	-173.68	0.23578	31.37%	7.8953
	0.5	-174.04	0.23545	24.52%	7.8845
	1.0	-175.12	0.23449	16.16%	7.8522
70	0.0	-92.73	0.35838	33.16%	8.4783
	0.1	-92.74	0.35837	31.42%	8.4781
	0.5	-92.83	0.35812	23.55%	8.4722
	1.0	-93.12	0.35738	16.17%	8.4545
90	0.0	-32.21	0.72548	33.17%	9.1052
	0.1	-32.21	0.72546	31.43%	9.1051
	0.5	-32.22	0.72531	24.56%	9.1031
	1.0	-32.25	0.72484	16.17%	9.0971

Table 4: Control variables: jump frequencies vary.

	Complete market			Market crash frequency at 10%		
Age	Consumption	Risky investment	Bequest	Consumption	Risky investment	Bequest
30	0.2973	44.5%	7.1663	0.2967	43.0%	7.1531
40	0.3190	45.0%	7.3224	0.3186	43.5%	7.3113
50	0.3510	45.5%	7.5442	0.3506	44.0%	7.5353
60	0.3992	46.0%	7.8420	0.3988	44.3%	7.8353
70	0.4744	46.2%	8.2143	0.4741	44.6%	8.2097
80	0.6024	46.4%	8.6564	0.6022	44.8%	8.6536
90	0.8453	46.4%	9.0003	0.8451	44.8%	8.9988
100	1.5171	46.4%	9.0174	1.5169	44.8%	9.0166

Table 8: High return setting

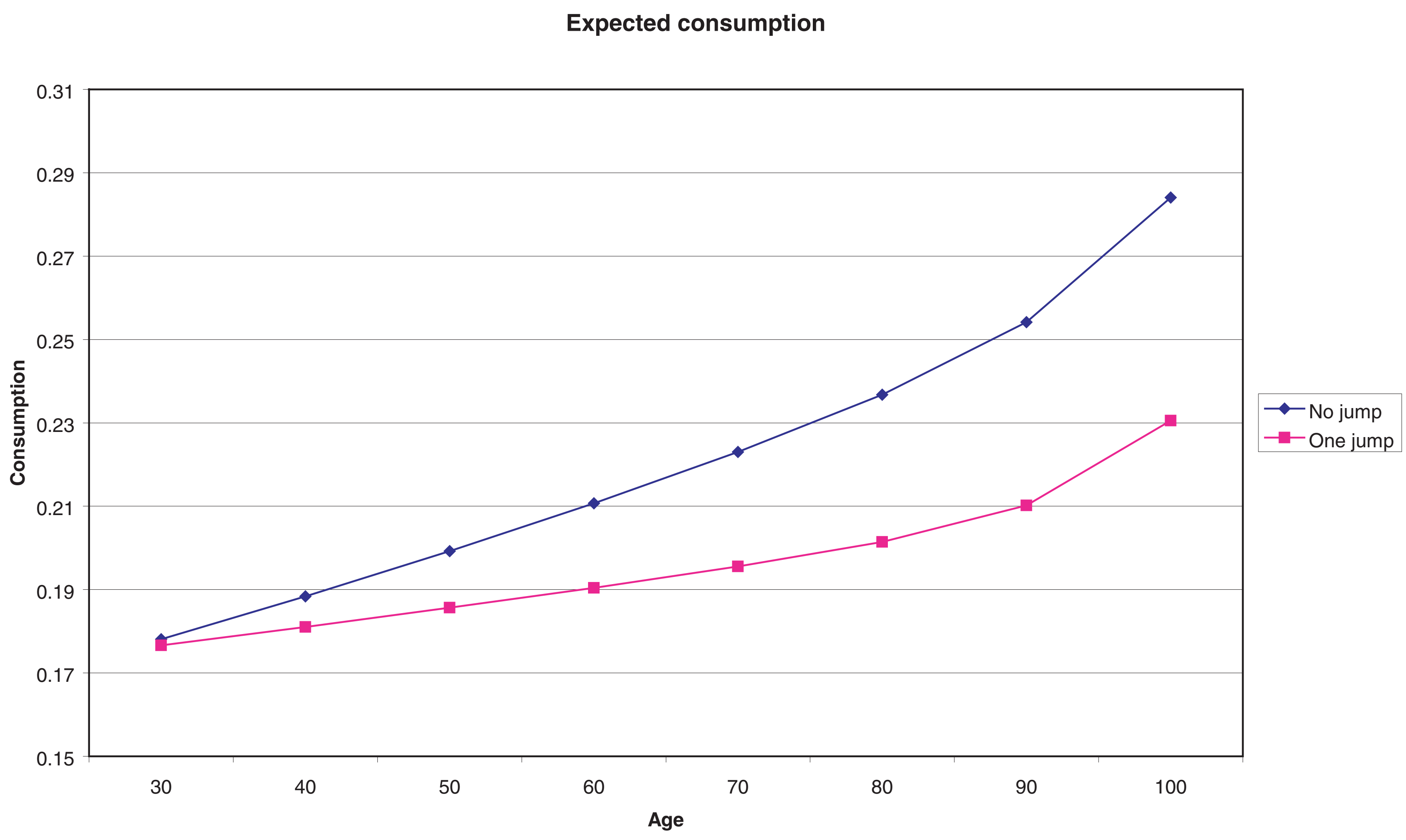


Figure 1: Expected optimal consumption paths

Optimal consumer behaviour in a jump-diffusion environment

Sachi Purcal
Thomas Huan Wang
School of Actuarial Studies
University of New South Wales
Sydney, Australia

September 2005

E-mail: s.purcal@unsw.edu.au